ARAB REPUBLIC OF EGYPT

THE INSTITUTE OF NATIONAL PLANNING

Memo No (754)

Productivity,
Concepts and Measurment

By

Karl Heinz Horn

April, 1967

جمهورية مصر العربية – طريق صلاح سالم – مدينة نصر – القاهرة – مكتبب ريد رقم ١١٧٦٥

A.R.E Salah Salem St. Nasr City , Cairo P.O.Box : 11765

"Opinions Expressed and Positions Taken by Authors are Entirely their Own and do not Necessarily Reflect the Views of the Institute of National planning".

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Introduction

Introductions usually point out the importance of the topic. I am sure that nobody can deny the outstanding role of increasing labour productivity for the development of the economy. Despite that we know many arguments about the contrary consequences of rising labour productivity especially in developing countries. The main problem in this connection in countries suffering under a population pressure is that of the danger of a growing unemployment. Under certain conditions machines and all kinds of modern , i.e. labour-saving, technologies can push away manpower not only in a relative, but also in an absolute extent. The argument of unemployment created by technological progress is one of the reasons to investigate the question of what technology must be chosen by developing countries in their process of industrialization.

I would not like to answer this question in an extreme and absolute sense. So far as possible most advanced technology must be applied, so far as necessary less advanced techniques too. It depends on the political, social and economic circumstances of each of the projects. At any rate all the given opportunities, all the existing resources must be used. This has to be implemented in a balanced way resulting in an organic growth of all interlaced spheres of the economy. The newly constructed and modern productive branches must be rooted in widespread economic activities aimed to develop trade, small industries, and agriculture. In particular the traditional skills of the artisans might be used and supported

to meet the requirements of local markets, to create jobs and incomes, to exploit local resources and initiatives in order to raise national income. In many cases artisans have an i portant share in exports.

Partly we have to look upon small industries as instruments used to achieve necessary preconditions to apply advanced technologies, such as an enlarged and growing internal marker or improved skills of the workers. In this sense small industries are also sources of accumulation. On the other hand we have to keep in mind that even the highly developed industrial countries are in need of certain branches of small industries, for example in the field of repairs and many kinds of services. Most advanced technologies, such as automation, are normally connected with a necessary reduction of the assortment produced in the automated enterprises. Consequently even automation given space for the development of certain small and medium sized industries, specialized in manufacturing articles which cannot be produced in an automated manner.

Nevertheless we would like to emphasise the necessity to concentrate the main efforts on the introduction of highly or even most advanced technology. The United Arab Republic, after a period of the extension of the economy by setting up new capacities, is now confronted with the goal to intensify the use of these new enterprises. This includes the application of productivity raising methods. Growing labour productivity is the sole, and practically unlimited, method to improve the effectivity of the economy and to reduce the cost per unit of an article. So it is the crucial point to solve the problems the develping countries, enabling them

1. To increase investments and so to improve the growth rate of production;

2. to improve the utilization of the existing capacities, and, consequently, to raise the profitability of the economy;

3. to increase the standard of living including all spheres of social life, such as education and training as a precondition to further development;

4. to compete on the world marked;

5. to stabilize the internal political situation as a result of all the points mentioned above;

6. to achieve economic independence as a prerequisite to lasting political independence.

As to the danger of growing unemployment as a result of increasing labour productivity: I would not say that this problem cannot exist even under the conditions of planned economy in certain periods. But I am convinced that a planned economy has the instruments to overcome this problem, at any rate it has the better instruments than an unplanned system. At last it is the proclaimed goal of labour productivity development under socialism to liberate workers at first from all kinds of monotonous and unhealth. labour, furthermore to reduce working days and weeks, to enlarge vacations, to give the population the time needed for activities in social, cultural, etc. fields. The reduction-at first the relative reduction- of the number of workers needed in productive spheres of the economy gives way to an increasing employment in secondary branches, such as education, science, health service, culture, etc.

Those structural changes of employment and professions is an expression of social progress. One century ago the population of countries which are now industrialized was mainly connected with agriculture - today only about 12 or 15 per cent of the manpower is sufficient to meet the labour requirements in agriculture. In developed countries today industry employes most of the workers, but in the last years unproductive branches, such as commerce, monetary and insurance system, public and private services, are going to surpass industry in the aspect of employment, and after some decades 15 or 20 per cent of the working people will be able to cover the manpower needs of a highly automated industrial production. This includes on the one hand a lot of difficult social problems, but on the other hand we may keep in mind that this is the way of human progress, indicated and caused by the development of the productive forces, by growing labour productivity.

The right of existence of any a social and political system is based on its ability to forward mankind, i.e. at last to improve the effectivity of human labour.

I. Concepts.

The term labour productivity is closely connected with the term productive labour. According to the labour-value theory human labour is the sole productive factor in the process of manufacturing goods. In this sense we cannot speak about productivity of machines, capital, or soil, as regularly done by economists of capitalism.

Generally speaking, labour productivity is the efficiency of labour, defined as the volume of output per worker in a certain unit of time, or the other way round, time needed by one worker for producing one unit of output. Although we consider in the definition only the "living" labour, compared with the input of dead, enbodied, or incorporated labour, we cannot neglect the effectiveness of the materials and equipments. Firstly the level of labour productivity depends on the quality and quantity of the used machines. Secondly it is necessary to measure productivity also in a broader sense, as the effectivity of the national economy or of economic units, sometimes called the productivity of "social" labour in general. In this sense Amina El Hefny for instance defined "productivity" as the "ratio of actual output to input".[2] This is without doubt a practicable synthetic indicator. It includes all input element, similar to the rate of profitability.

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1) Partly based on Dr. Sack/Dr. Linsel, contents and Measuring of Labour Productivity in Socialist Industry, Memo. 595, INP, October 1965.

2) Amina El Hefny, Objectives of Productivity Measurement and Prediction in U.A.R., Paper submitted to the Seminar on Manpower Planning, Cairo 23-26. Nov., 1966, p. 1.

John W. Kendrick mentions both possibilities: "The term productivity may be defined as the ratio of output to any or all of the inputs employed in production." But he continues: "The most commonly used productivity measure is output per man-hour'. This is a 'partial productivity' measure since it is the ratio of output to only one class of input; although labor is the most important input ..."[1)]

The reasons why we are emphasizing the productivity of labor are the following:

1. We want to stress that the workers are the decisive productive forces;
2. Materials and machines are dead things, unable to create new value. Their value is incorporated and only transferred to the new product by the workers. In economic fields we use the term productive only in the sense of creative, i.e. to to creat new value;
3. At last it is important to distinguish between the human and the material elements of the production process, because the factors affecting their efficiency are different. Hence, the measures implemented to improve the effectiveness of the varies elements are different too.

As to the last point: Technological progress leads to a changing structure of the value of the commodities, and consequently the structure of the whole economy will alter. As a rule we can state that the input of dead labour will increase faster than the input of living labour - seen from the point of view of the whole economy. So the living labour is carried

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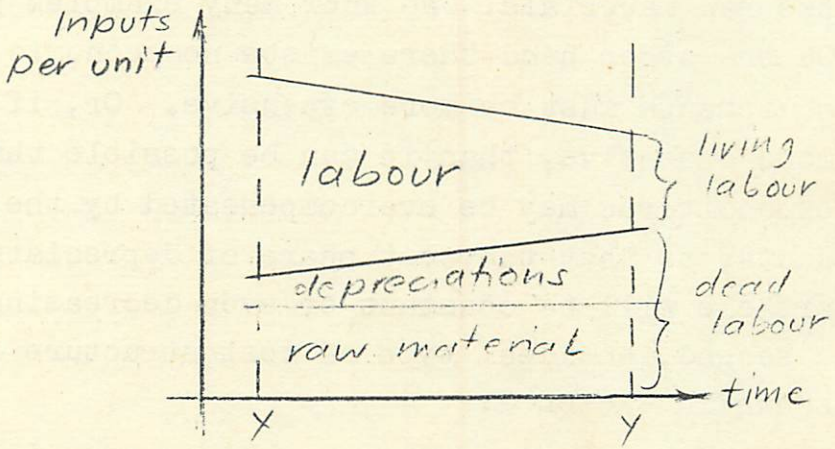
1) John W. Kendrick, Productivity, Costs and Prices: Concepts and Measures, in "Readings in Labor Economic", Homewood/Illinois, 1963.

out by the workers it follows that the reproduction of this productive element requires consumer goods, on the other hand the dead labour existing in raw materials and machines must be replaced by new materials and equipments, i.e. by capital goods. The result is, that the growth rate of the production of capital goods will exceed the growth of consumer goods. This basic relation between the two main spheres of production reflects the development of labour productivity: One unit of human labour is able to process a growing amount of raw material by using larger and often more expensive equipment, hence, producing a greater volume of articles in every time period. Every productive worker tackles and rules an ever growing amount of capital. This is, briefly spoken, the result of growing labour productivity showing the structural consequences for the entire economy.

Of course the cost and value composition of every single article must be also affected by increasing labour productivity. At any rate the labour costs per unit of an article will be reduced. In many cases the reduction of labour input is caused by additional inputs of capital, mainly equipments. The result is illustrated in the following sketch 1:

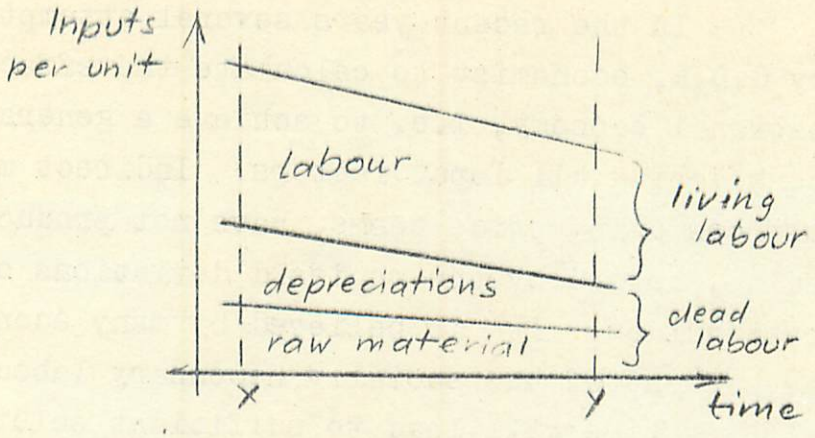


Sketch 1

The changing input structure as a result of growing labour productivity, first variant.

It is obvious, that the introduction of new and expensive equipments must result in a reduction of the total cost per unit. The central problem is, that in particular the reduction of labour costs per unit of an article has to overcompensate the additional capital costs, otherwise it is nonsense to apply more expensive new equipments. From this point of view the wage level of a country is of importance. High wages always stimulate the application of advanced technologies, and vice verca, a low wage level hampers the introduction of expensive machines, of automation, etc. In the last case it is more economically to prefer the use of cheap human labour instead of machines - from a pure economic point of view. The consequences might be illustrated by many examples taken out of the history of the former colonies, which were characterized by extremely low wage levels.

Experiences of the recent years show, that the cost structure may change even in a quite ideal way: our assumption, that the raw material input per unit of an article will be constant must not stand the test. Advanced technologies, connected with large scale production, open the way to a better use of the raw materials. We know many examples proving that fact. On the other hand there exists no economic law that more advanced equipments must be more expensive. Or, if the new machines are more expensive, than it can be possible that the additional total expenditures may be overcompensated by the greater volume of products, so that the cost share of depreciations per unit of an article will be constant or even decreasing. The outcome is a second, an ideal type of cost structure development, as it is shown in sketch 2:



Sketch 2:
The changing input structure as a result of growing labour productivity, second variant.

If the second variant becomes the dominant one, the consequence for the entire economy would be, that the two main departments, production of capital goods and of consumer goods, will approach toa parallel growth.

We can summarize that the effectiveness of the economy depends on two main factors:

1. Productivity of labour, and
2. Effectiveness of capital, i.e. fixed assets as well as raw materials and intermediate goods.

The productivity of labour can be expressed by the following formula:

$$L_p = \frac{Q}{T}$$

L_p labour productivity
Q Quantity of products
T Time needed to produce Q

The efficience of the used fixed assets is the contents of the

$$\text{rate of fixed assets} = \frac{\text{gross production}}{\text{gross volume of fixed assets}}$$

At last the standards of material consumed deal with the economical utilization of raw material and intermediate goods.

In the recent years several attempts have been carried out by G.D.R. economist to calculate the effectivity of the entire national economy, i.e. to achieve a general input-output ratio considering all input factors. Indirect methods, that means methods using price terms, have not produced satisfactory results, caused by the manifold deviations of the prices from the real values. It is believed by many economists, that alone the assessment of the socially necessary labour input directly in working time will lead to sufficient solutions. First experimental calculations of the full labour input for the gross national product of the country have been carried out by research teams under the supervision of the Economic Research Institute of the State Planning Commission. For this purpose it was necessary to express the inputs of raw material and equipments, i.e. the inputs of materialized labour, also in working time, in hours needed to reproduce these elements. A second problem was to make several kinds and qualities of labour comparable.

For this reason qualified and complicated labour must have been converted to simple and unified labour, so that qualified labour appears as multiplied simple labour. This could be done by using, for example, the wage structure and qualification structure data as yardsticks. The mathematical solution of the problem required comprehensive matrix models in order to consider the complicated interlacements between the branches, that means to trace the production flow from raw materials over intermediate goods to final products. Changes, mainly reductions, of the labour input per unit of each product resulting out of alterations of the labour input into the product in question as well as of the preceding stage products, which will be consumed by the product in question, must be evident and computable. Similar calculations have been carried out also on branch and enterprise

level.[1] The results are not only important for the ascertainment of level and growth of labour productivity, but also for more accurate investigations in the fields of price fixing, of interbranch and international comparisons, and for an exact determination and planning of proportionality of the economy.

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1) See e.g. J.Rudolph, The First Experimental Calculation of the Full Labour Input for the Aggregate Social Product in the G.D.R., in "Wirtschafswissenschaft" 211965; and W. Marschall, Interlacement Models to Calculate and Analyse Labour Productivity, Berlin 1966.

II. Measurement of labour productivity

Scientific management is impossible without quantifying and measuring economic events and processes. Especially the measurement of the development of labour productivity is a prerequisite to better use of social labour, to improve management and planning, and to reduce the cost per article. The methods of measuring labour productivity must enable us to analyze the factors affecting labour productivity in order to utilize these factors, to concentrate our efforts upon them.

Furthermore, analysis and measurement of productivity are prerequisites to calculate and plan the increase of national income, to fix the relations between accumulation and consumption, to determine the development of the wage fund as well as the average wages and salaries.

So far as possible we measure level and growth of productivity in physical volume terms, since the purpose is to know the efficiency with which resources are utilized. Since in every productive unit (enterprise, department) the produced articles are different, the problem arises how to aggregate and to compare the figures. To solve this problem we have to use time-and price terms.

1. Calculation of labour productivity on the basis of physical units.

In consistency with our definition the level and the growth of labour productivity can be expressed best by using physical terms, such as meters, tons, etc., produced in a unit of working time. This is not only the most precise but also the simplest method of planning and calculation of labour productivity. If for

instance a miner produces in one shift of eight hours 24 tons of coal, than the level of his productivity is expressed by

$$L_p = \frac{Q}{T} = \frac{24 \text{ tons}}{8 \text{ hours}} = 3 \text{ tons/hour.}$$

In this example it may be posible to use as time unit not one hour but one shift, so we may say that the level of labour productivity is characterized by 24 tons per man and shift.

Since we want to follow-up the development of labour productivity, we compere the level of productivity of the period under report with that of the basic period by using the following simple formula:

$$L_p^{1} = \frac{\frac{Q_n}{T_n}}{\frac{Q_o}{T_o}} = \frac{Q_n}{T_n} \times \frac{T_o}{Q_o} = \frac{T_o}{T_n} \times \frac{Q_n}{Q_o} = \frac{\frac{Q_n}{Q_o}}{\frac{T_n}{T_o}}$$

The signs stand for

Index of labour productivity development : L_p^{1};

Labour productivity level in the basic period: $\frac{Q_o}{T_o}$

Labour productivity level in the report period : $\frac{Q_n}{T_n}$

To give an example : Our miner has improved his preformances to 30 tons per shift, hence we calculate

$$L_p^{1} = \frac{30}{10} \times \frac{10}{24} = \frac{30}{24} = 1{,}25$$

Multiplying this coefficient with 100 we obtain that the labour productivity rises from 100 in the basic period to 125 in the period under report or by 25 per cent - a result which we are able to know even without any formula and calculation.

The other way round it is in many cases interesting to as certain how many time is necessary to produce one unit of the artical in question. For this purpose we calculate the coefficient reciprocally :

$$\frac{T_o}{Q_o} = \frac{8}{24} = \frac{1}{3}$$ hours or 20 minutes per ton, and,

respectively, $\frac{T_n}{Q_n} = \frac{8}{30} =$ 0,267 hours or 16 minutes per ton.

Of course also these figures can be used to calculate the development of labour productivity.

$$L_p{}^1 = \frac{\frac{Q_n}{Q_o}}{\frac{T_n}{T_o}} = \frac{\frac{1}{1}}{\frac{16}{20}} = \frac{20}{16} = \underline{1,25 = 125 \text{ per cent.}}$$

That means, if our starting point is a given time unit, than we have to compare the different amounts (tons etc.) of the article produced in that time unit , and, vice versa, if we start with a given amount of a certain article, than we have to compare, i.e. to divide, the different time periods needed to produce the given article unit reciprocally.

The advantage of the described method consists of their exactness. International comparisons might be based upon labour productivity measurements using physical units and terms. At last it is a relative simple method. On the other hand we have to keep in mind that this method requires comparable products. Even the same products will differ from time to time from the point of view of their quality. The improved quality of any a raw material or intermediate good which may be an important benefit for the whole economy, can appear as a decrease of labour productivity in

the enterprise manufacturing the improved article, if that enterprise needs a little bit more of working time to achieve a better quality. We are able to overcome this problem in such cases, in which we can use natural parameters to express the quality of the articles, such as the calorific value of coal or other kinds of fuel, horse power units of motors. Theoretically it is possible to introduce also general quality coefficients to multiply the quantity of goods produced in a time unit, hence, the achieved qualitative improvements can be estimated, valued and rewarded like a quantitive enlargement of production, respectively like an increase of labour productivity.

But if an enterprise produces several quite different articles, than the application of methods using natural units is impossible.

2. Calculation of labour productivity on the basis of fixed and comparable prices.

The central idea is to add different products based upon fixed prices. This method is mainly used for the calculation of labour productivity of whole branches, and of the entire economy. The basic formula stating the level of productivity is shaped as follows :

$$L_p = \frac{\text{gross production in constant prices}}{\text{number of productive workers}}$$

The index of growth is than the result of a comparison between these relations achieved in a basic and a plan period, respectively a period under report :

$$L_p^{\,1} = \frac{\sum P_n \times Q}{T_n} : \frac{\sum P_o \times Q}{T_o} \text{ or } \frac{\sum P_n \times Q}{\sum P_o \times Q} : \frac{T_n}{T_o}$$

P = Products
Q = Prices, constant and comparable
T = Number of productive workers
The basic period is marked by 0, the report period by n.

To give a numerical example we presume that in the basis period 100 workers produce a gross product of 1000 price units, and in the reporting period 120 workers an amount of 1400 price units:

$$L_p^{\,1} = \frac{1400}{120} : \frac{1000}{100} \text{ or } \frac{1400}{1000} : \frac{120}{100}$$

$$L_p^{\,1} = 11{,}67 : 10 = \underline{1{,}167}$$

That means labour productivity increased from 100 in the basic period to nearly 117 in the plan period.

We have to consider that the development of labour productivity cannot be expressed exactly by this method caused by the following factors:

1 - The real conditions and proportions are often distorted by prices deviating from real values.

2 - Gross production includes not only the results of the activities of the enterprise in question but also the use of prefabricated products, of strange performances. So an increasing labour productivity can be calculated with this formula although in practice nothing happened than the enterprise in question bought more prefabricated goods or more expensive materials.

3 - Since the value composition differs concering the various products, any change of the assortment will affect the index of labour productivity without a corresponding development of the performances of the enterprise.

Preferences of the constant price method are, that all the needed data normally are available not only for each enterprise, but also concerning branches and the economy as a whole. We can apply this method especially to compute the entire productivity of the whole economy, but we have to consider that all factors affecting labour productivity, such as cooperation and division of labour, structural changings of the assortments, etc., are reflcted by the received index. Besides this our calculation cannot be free of double countings. To overcome some of the disadvantages of the constant price method several experiments have been made to substitute the gross production by net production or by socalled self performances, that means to eliminate the inputs of raw material, prefabricated goods, and all kinds of services and performances of other enterprises.

3. The time summing-method

Since the beginning of the year 1963 the time-summing method to measure productivity growth is the obligatory method for all public owned industrial enterprises, after the procedure had been proved by experiments in several branches. The time-summing method is an essential precondition to a scientific based planning and analysing of labour productivity.

As expenditure of living labour is considered to be the input of the main productive forces, the working time needed to produce a certain article serves as the basis of the time-summing method. The method expresses the ability to produce a certain volume of articles in a given time period. This shows, that there exists a direct relation to above mentioned calculation technique based upon physical terms.

We obtain the index of productivity growth by comparing the development of the production volume by the development of the working time in the following way :

$$L_p^{\,1} = \frac{\sum Q_n \; T_o}{\sum Q_n \; T_n}$$

The symbols stand for :

Q_n quantity of products (in pieces etc.) in the planned period or the period under report.

$T_{n,o}$ working time (living labour) needed per unit of the article in the planned resp. reporting period (n) and in the basic period (o).

The difference to the other methods is, that various products can be added, weighted by their labour input requirements or actual labour inputs, instead of weighting them by prices.

The following example will explain how the calculation is carried out.[1)]

article	Q_n	T_o	T_n	$\sum Q_n T_n$	$\sum Q_n T_n$
	(piece)	1000 hours	1000 hours	1000 hours	1000 hours
A	50	21	20	1050	1000
B	100	18	16	1800	1600
C	200	10	10	2000	2000
Total				4850	4600

$$L_p^{\,1} = \frac{\sum Q_n T_o}{\sum Q_n T_n} = \frac{4850}{4600} = 1{,}054 = \underline{105.4 \text{ per cent}}$$

That means it is planned (or achieved) to raise labour productivity 105.4 per cent, or the needed working time to produce the same volume is cut down by 250 000 working hours (4850 - 4600). Caused by the fact, that all influences of incorrect or changing prices as well as altering value compositions as a result of changing inputs of raw materials and strange performances are eliminated, the index is able to express the real improvements of the performances of the enterprise in question by means of better technologies or intensified labour. Hence, it is easier to quantify and analyse the factors influencing the growth of productivity. Furthermore this method enables us to link up planning and accounting of labour productivity with, the production plan, and the cost analysis.

1) The example is based upon KELLER / MAIER, Productivity, Berlin 1965, p. 59.

If we apply the time-summing method to the whole industry, it must be considered that the manufacturing times per unit are related to a certain enterprise, i.e., they are individual times. Consequently we have to use the average manufacturing time per unit related to the whole branch in order to calculate the index.

One of the difficulties of the application of time-summing method is the determination of the effective time needed to produce the articles. In many cases the effective times are not or not complete available, in other cases it may be impossible to attribute working times to a certain product, for instance in the chemical industry. These are the reasons why some approaches are necessary to get approximate values by using standard times instead of effective times. Several variants are practicable.

1. The norm or standard time per unit will be divided by the average fulfilment of the norms. The productivity index is than shaped like follows :

$$L_p^{\,1} = \frac{\frac{\sum Q_n T_{N,o}}{E_o}}{\frac{\sum Q_n T_{N,n}}{E_n}} \approx \frac{\sum Q_n T_o}{\sum Q_n T_n}$$

T_N norm or standard time

E average fulfilment of the time standards

This formula is based upon the fact, that the standard time divided by the fulfilment of the standards leads to a result which equals the approximately used up manufacturing time :

$$\frac{T_n}{E} \approx T$$

If the effective times are known regarding the period under report but not for the basic year, than the formula is the following :

$$L_p^{\,l} = \frac{\frac{\sum Q_n T_{N;o}}{E_o}}{\sum Q_n T_n}$$

In the process of the introduction of the time-summing method this formula is used very often, avoiding the time-consuming and complicated procedure to determine the effective times of the gone basic year. The standard times and the average fulfilment of the norms as a basis for the wage calculations are in most of the cases available also for the recent years.

A second variant is based upon the fact, that the sum of all reductions of standard times equals the difference between the norm times of the report period and the basic period, each of them multiplied by the volume of production in the reporting period:

$$\sum Q_n T_{N,n} - \sum Q_n T_{N,o} = \pm E_{N,n}$$

E_N = increase or decrease of norm times

Or : $\sum Q_n T_{N,n} \pm E_{N,n} = \sum Q_n T_{N,o}$.

It follows :

$$L_p^{\,l} = \frac{\frac{\sum Q_n T_{N,n} \pm E_{N,n}}{E_o}}{\frac{\sum Q_n T_{N,n}}{E_n}}$$

Also this formula is applied very often, because it is relative simple to get the difference between the standard times of the compared years. But we have to consider that their use includes deviations from the real situations, caused by the use of the average fulfilment of the standards as well as by the procedure of the determination of E_N .

Of course the application of the time-summing method in practice requires some preconditions, such as a well organized and unified accounting system, the technological documents about the production process must be up to date, scientific based labour standards, etc. Although the introduction of this method will require additional efforts in the first time, it is proved by our experiences that the time-summing method is a very suitable one connected with a series of advantages.

4. Calculation of Labour Productivity with the Aid of Input-Output Tables. (1)

As mentioned earlier first experiences are made in the field of the use of interlacement models to calculate labour productivity. The general introduction of a system of input - output tables should be connected with a corresponding new method to determine growth and level of productivity. One of the basic ideas of this method is, that also the alterations of material inputs are taken into consideration by converting and expressing them in time units needed to produce these materials. Hence we follow the production flow up to a certain degree over several stages, investigating and determining all the existing interlacements. The used balance is shaped like follows:

Production interlacements between the stages of the process	market production
Labour expenditure in the stages	sum of labour expenditure

Or, expressed as a system of equations :

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(1) See Wolfgang Marshall, Interlacing Models to Calculate and Analyse Labour Productivity, Berlin 1966.

$$+ m_{11}\, q_1 - m_{12}\, q_2 - \ldots\ldots\ldots\ldots\ldots - m_{1n}\, q_n = W_1$$

$$- m_{21}\, q_1 + m_{22}\, q_2 - \ldots\ldots\ldots\ldots\ldots - m_{2n}\, q_2 = W_2$$

$$\vdots \qquad\qquad\qquad\qquad\qquad\qquad \vdots$$

$$- m_{n1}\, q_1 - m_{n2}\, q_2 - \ldots\ldots - m_{n1\,n-1}\, q_{n-1} + m_{nn}\, q_n = W_n$$

$$-t_1\, q_1 - t_2\, q_2 - \ldots\ldots\qquad\qquad - t_n\, q_n = T$$

The symbols :

$w_1 \ldots\ldots\ldots w_n$	Volume of marked production of commodity 1..n,
$q_1 \ldots\ldots\ldots q_n$	Total volume produced in the stages 1.....n;
$-m_{ij}$ $(i \neq j)$	consumption of article i to produce one unit of article j;
$+ m_{ii}$	is always equal to 1, in connection with the production of the articles;
t	input of labour in the stages;
T	sum of labour input.

m_{ij} and t_j are the coefficients of the equation system, they are assumed to be constant in a certain period.

We can write the linear equations as follows:

$$\left(\frac{\underline{B}}{\underline{t}}\right) \underline{d} = \left(\frac{\underline{w}}{T}\right)$$

with the symbols :

$$\underline{B} = \begin{matrix} m_{11} \ldots\ldots m_{1n} \\ \vdots \qquad\quad \vdots \\ m_{n1} \ldots\ldots m_{nn} \end{matrix}$$

Matrix of the material interlacement;

$\underline{t} = (t_1 \ldots\ldots t_n)$ vector of the direct labour input;

$\underline{d} = \begin{pmatrix} q_1 \\ \vdots \\ q_n \end{pmatrix}$ vector of total production of the several stages of the process;

$\underline{w} = \begin{pmatrix} w_1 \\ \vdots \\ w_n \end{pmatrix}$ vector of market production of the articles 1.....n;

T is a real figure (scalar) expressing the sum of labour input in all stages.

A picture-like model showing the material flow through all departments (stages) of the production process is normally the starting point of the implementation of this method in practice . It is suitable to register volumes and levels of inputs and outputs of all stages in this production flow picture. The next step is the elaboration of the interlacement balance, the construction of it may be illustrated by the following simplified example :

stages / Products	s_1	s_2	s_3	s_4	s_5	market production
w_1	1000	-500		-200		300
w_2		500	-100		-400	0
w_3			3000	-1000	-2000	0
w_4				-200		200
w_5					100	100
B	-50		200			150
T	-200	-100	-300	-50	-100	-750

The balance means, that for instance the total volume of product 1 equals 1000, but 500 and 200 of product 1 are consumed to produce the goods 2 and 4, thus the marked production amounts to 300. In stage 3 a by-product with a volume of 200 emerges, of which 50 are required to produce article 1, so tha 150 of B, the by-product, are available as market production. The row T consists of the inputs of living labour in each stag of the process, for instance article 1 requires 200 (hours or other time units) of living labours, in materialized form part of this labour input are transferred to article 2 and 5, etc., over all stages of the interlinked process. The total amount of labour input, 750, appears in the last column of the last now.

The elements of the obtained balance are effective production - or plan - figures, that means, the balance is a special case out of a large number of possible variants, which may be considered in the process of planning. The desired model mu be a general one, applicable to each variant. Thus our last step is the construction of a coefficient model by dividing every element of each column by the element of the main diagona We obtain the coefficients expressing the inputs per unit, the inputs of prefabricated products respectively the labour inputs into one unit of the article which we well find in the main di gonal. Naturally the coefficient model consists only of the interlacement matrix and the matrix of labour inputs, the sum column is eliminated.

The application of interlacement balances in productivity calculations enables us to analyse the productivity development for all stages of the process. It becomes evident whic departments of the enterprise contribute to the productivity an which departments not. Hence, such models are ideal instrument for all kinds of decision taking, of planning and management.