

**APPLICATIONS OF MEASURES OF NONCOMPACTNESS TO  
THE INFINITE SYSTEM OF DIFFERENTIAL EQUATIONS IN  
 $bv_p$  SPACES**

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**ABSTRACT.** In this study, we apply the technique of measures of noncompactness to the theory of infinite system of differential equations in the Banach sequence spaces  $bv_p$  ( $1 \leq p < \infty$ ). Our aim is to present some existence results for infinite system of differential equations formulated with the help of measures of noncompactness.

1. PRELIMINARIES, BACKGROUND AND NOTATION

By  $\omega$ , we shall denote the space of all complex valued sequences. Any vector subspace of  $\omega$  is called as a *sequence space*. We shall write  $\ell_\infty, c$  and  $c_0$  for the spaces of all bounded, convergent and null sequences, respectively. Also by  $bs, cs, \ell_1$  and  $\ell_p$ ; we denote the spaces of all bounded, convergent, absolutely and  $p$ - absolutely convergent series, respectively;  $1 < p < \infty$ . By  $e$  and  $e^{(n)}$  ( $n = 0, 1, 2, \dots$ ), we denote the sequences such that  $e_k = 1$  for  $k = 0, 1, \dots$ , and  $e_n^{(n)} = 1$  and  $e_k^{(n)} = 0$  for  $k \neq n$ . For any sequence  $x = (x_k)$ , let  $x^{[n]} = \sum_{k=0}^n x_k e^{(k)}$  be its  $n$ -section.

An FK space is a complete linear metric sequence space with the property that convergence implies that coordinatewise convergence; a BK space is normed FK space. A BK space  $X \supset \phi$  is said to have AK if every sequence  $x = (x_k) \in X$  has a unique representation  $x = \sum_{n=0}^{\infty} x_n e^{(n)}$ . Note that the sequence spaces  $\ell_\infty, c$  and  $c_0$  are BK-spaces with usual sup-norm given by  $\|x\|_{\ell_\infty} = \sup_k |x_k|$ , where the supremum is taken over all  $k \in \mathbb{N}$ . Also, we write

$$\ell_p = \left\{ x = (x_k) \in \omega : \sum_{k=0}^{\infty} |x_k|^p < \infty \right\}; \quad (1 \leq p < \infty)$$

and the space  $\ell_p$  is a BK space with the usual  $\ell_p$ -norm defined by  $\|x\|_{\ell_p} = (\sum_{k=0}^{\infty} |x_k|^p)^{1/p}$ , [1].

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If we denote by  $A = (a_{nk})_{n,k=0}^{\infty}$  an infinite matrix with complex entries and by  $A_n$  its  $n$ th row, we write

$$A_n(x) = \sum_{k=0}^{\infty} a_{nk}x_k \quad \text{and} \quad A(x) = (A_n(x))_{n=0}^{\infty}; \quad (1)$$

then  $A \in (X, Y)$  if and only if  $A_n(x)$  converges for all  $x \in X$  and all  $n$  and  $A(x) \in Y$ . The set

$$X_A = \{x \in \omega : A(x) \in X\} \quad (2)$$

is called the matrix domain of  $A$  in  $X$ . Bařar and Altay [23] recently defined the space of sequences of  $p$ - bounded variation, which is the difference spaces of the sequence spaces  $\ell_p$  and  $\ell_{\infty}$ , as follows:

$$bv_p = \left\{ x = (x_k) \in \omega : \sum_k |x_k - x_{k-1}|^p < \infty \right\} \quad (1 \leq p < \infty),$$

and

$$bv_{\infty} = \left\{ x = (x_k) \in \omega : \sup_{k \in \mathbb{N}} |x_k - x_{k-1}| < \infty \right\}.$$

With the notation of (2), we may redefine the spaces  $bv_p$  and  $bv_{\infty}$  by

$$bv_p = \{l_p\}_{\Delta} \quad \text{and} \quad bv_{\infty} = \{l_{\infty}\}_{\Delta},$$

where the matrix  $\Delta = (\delta_{nk})$  as defined

$$\delta_{nk} = \begin{cases} (-1)^{n-k}, & n-1 \leq k \leq n, \\ 0, & 0 \leq k < n-1 \quad \text{or} \quad k > n. \end{cases}$$

Let  $X$  be a normed space. Then, we write  $S_X$  and  $B_X$  for the unit sphere and the closed unit ball in  $X$ , that is,  $S_X = \{x \in X : \|x\| = 1\}$  and  $B_X = \{x \in X : \|x\| \leq 1\}$ . If  $X$  and  $Y$  are Banach spaces, then  $B(X, Y)$  denotes the set of all bounded linear operators  $L : X \rightarrow Y$ .  $B(X, Y)$  is a Banach space with the operator norm given by  $\|L\| = \sup_{x \in S_X} \|L(x)\|$ . In particular, if  $Y = \mathbb{C}$  then we write  $X^*$  for the set of all continuous linear functionals on  $X$  with the norm  $\|f\| = \sup_{x \in S_X} |f(x)|$ .

Infinite systems of ordinary differential equations describe numerous real world problems which can be encountered in the theory of branching processes, the theory of neural nets, the theory of dissociation of polymers and so on [2, 3, 4, 5]. Let us also mention that several problems investigated in mechanics lead to infinite systems of differential equations [6, 7, 8]. Moreover, infinite systems of differential equations can also be used in solving some problems for parabolic differential equations investigated via semidiscretization [9, 10, 17].

Recently the theory of measures of noncompactness have been used in determining the compact operators of matrices on various BK spaces [11, 12, 13, 14, 15].

In this paper we apply the technique of measures of noncompactness to the theory of infinite systems of differential equations in Banach sequence spaces  $bv_p$ . Our aim is to present some existence results for infinite systems of differential equations formulated with the help of measures of noncompactness. We determine the sufficient conditions for the solvability of infinite systems of differential equations in  $bv_p$  ( $1 \leq p < \infty$ ) analogous to those of Banař and Lecko [16] who considered the classical Banach sequence spaces  $c_0$ ,  $c$  and  $l_1$ .

## 2. HAUSDORFF MEASURE OF NONCOMPACTNESS

The theory of FK spaces is the most important tool in the characterization of matrix transformations between certain sequence spaces. The most important result is that matrix transformations between FK spaces are continuous. It is quite natural to find conditions for a matrix map between FK spaces to define a compact operator. This can be achieved by applying the Hausdorff measure of noncompactness. The Hausdorff measure of noncompactness was defined by Goldenštein, Gohberg and Markus in 1957, later studied by Goldenštein and Markus in 1968 [19].

Here, we will recall some basic definitions and results. More results about measure of noncompactness can be found in [19, 20].

By  $\mathcal{M}_X$ , we denote the collection of all bounded subsets of a metric space  $(X, d)$ . Let  $Q$  be a bounded subset of  $X$  and  $K(x, r) = \{y \in X : d(x, y) < r\}$ . Then the Hausdorff measure of noncompactness of  $Q$ , denoted by  $\chi(Q)$ , is defined by

$$\chi(Q) = \inf \left\{ \varepsilon > 0 : Q \subset \bigcup_{i=1}^n K(x_i, r_i), x_i \in X, r_i < \varepsilon (i = 1, 2, \dots), n \in \mathbb{N}_0 \right\}.$$

If  $Q, Q_1$  and  $Q_2$  are bounded subsets of the metric space  $(X, d)$ , then we have

$$\chi(Q) = 0 \text{ if and only if } Q \text{ is a totally bounded set,}$$

$$\chi(Q) = \chi(\overline{Q}),$$

$$Q_1 \subset Q_2 \text{ implies } \chi(Q_1) \leq \chi(Q_2),$$

$$\chi(Q_1 \cup Q_2) = \max \{ \chi(Q_1), \chi(Q_2) \}$$

and

$$\chi(Q_1 \cap Q_2) \leq \min \{ \chi(Q_1), \chi(Q_2) \}.$$

If  $Q, Q_1$  and  $Q_2$  are bounded subsets of the normed space  $X$ , then we have

$$\chi(Q_1 + Q_2) \leq \chi(Q_1) + \chi(Q_2),$$

$$\chi(Q + x) = \chi(Q) \quad (x \in X)$$

and

$$\chi(\lambda Q) = |\lambda| \chi(Q) \text{ for all } \lambda \in \mathbb{C}.$$

Let  $X$  and  $Y$  be Banach spaces and  $\chi_1$  and  $\chi_2$  be the Hausdorff measures of noncompactness on  $X$  and  $Y$ , respectively. Then the operator  $L : X \rightarrow Y$  is called  $(\chi_1, \chi_2)$ -bounded if  $L(Q)$  is a bounded subset of  $Y$  for every bounded subset  $Q$  of  $X$  and there exists a positive constant  $K$  such that  $\chi_2(L(Q)) \leq K \cdot \chi_1(Q)$  for every bounded subset  $Q$  of  $X$ . If an operator  $L$  is  $(\chi_1, \chi_2)$ -bounded then, the number

$$\|L\|_{(\chi_1, \chi_2)} = \inf \{ K > 0 : \chi_2(L(Q)) \leq K \cdot \chi_1(Q) \}$$

for all bounded  $Q \subset X$ , is called  $(\chi_1, \chi_2)$ -measure of noncompactness of  $L$ . In particular, if  $\chi_1 = \chi_2 = \chi$ , then we write  $\|L\|_{(\chi, \chi)} = \|L\|_\chi$ .

The most effective way in the characterization of compact operators between the Banach spaces is by applying the Hausdorff measure of noncompactness. This can be achieved as follows. Let  $X$  and  $Y$  be Banach spaces and  $L \in B(X, Y)$ . Then, the Hausdorff measure of noncompactness of  $L$ , denoted by  $\|L\|_\chi$ , can be determined by

$$\|L\|_\chi = \chi(L(S_X)), \quad (3)$$

and we have that  $L$  is compact if and only if

$$\|L\|_\chi = 0. \quad (4)$$

Now, the following result gives an estimate for the Hausdorff measure of noncompactness in Banach spaces with Schauder basis. It is known that if  $(b_k)$  is a Schauder basis for a Banach space  $X$ , then every element  $x \in X$  has a unique representation  $x = \sum_{k=0}^{\infty} \alpha_k(x)b_k$ , where  $\alpha_k$  ( $k \in \mathbb{N}$ ) are called the basis functionals. Moreover, for each  $n \in \mathbb{N}$ , the operator  $P_n : X \rightarrow X$  defined by  $P_n(x) = \sum_{k=0}^n \alpha_k(x)b_k$  ( $x \in X$ ) is called the projector onto the linear span of  $\{b_0, b_1, \dots, b_n\}$ . Besides, all operators  $P_n$  and  $I - P_n$  are equibounded, where  $I$  denotes the identity operator on  $X$ .

**Theorem 2.1.** [19, Theorem 2.23] Let  $X$  be a Banach space with Schauder basis  $\{b_0, b_1, \dots, b_n\}$ ,  $Q$  be a bounded subset of  $X$ , and  $P_n : X \rightarrow X$  be the projector onto the linear span of  $\{b_0, b_1, \dots, b_n\}$ . Then we have

$$\frac{1}{a} \limsup_{n \rightarrow \infty} \left( \sup_{x \in Q} \|(I - P_n)x\| \right) \leq \chi(Q) \leq \limsup_{n \rightarrow \infty} \left( \sup_{x \in Q} \|(I - P_n)x\| \right), \quad (5)$$

where  $a = \limsup_{n \rightarrow \infty} \|I - P_n\|$ .

**Theorem 2.2.** [21, Theorem 2.8] Let  $Q$  be a bounded subset of the normed space  $X$ , where  $X$  is  $\ell_p$  for  $1 \leq p < \infty$  or  $c_0$ . If  $P_n : X \rightarrow X$  is the operator defined by  $P_n(x) = (x_0, x_1, \dots, x_n, 0, 0, \dots)$  for  $(x_0, x_1, \dots, x_n, 0, 0, \dots) \in X$ , then

$$\chi(Q) = \lim_{n \rightarrow \infty} \left( \sup_{x \in Q} \|(I - P_n)x\| \right). \quad (6)$$

It is easy to see that for  $Q \in \mathcal{M}_{\ell_p}$

$$\chi(Q) = \lim_{n \rightarrow \infty} \left( \sup_{x \in Q} \sum_{k \geq n} |x_k|^p \right). \quad (7)$$

Here we give a theorem to find the Hausdorff measure of noncompactness of bounded subsets in matrix domains of triangles.

**Theorem 2.3.** [22, Theorem 2.18] Let  $X$  be a normed sequence space and  $\chi_T$  and  $\chi$  denote the Hausdorff measures of noncompactness on  $\mathcal{M}_{X_T}$  and  $\mathcal{M}_X$ , the collection of all bounded sets in  $X_T$  and  $X$ , respectively. Then

$$\chi_T(Q) = \chi(T(Q))$$

for all  $Q \in \mathcal{M}_{X_T}$ .

The following result shows how to compute the Hausdorff measure of noncompactness in the spaces  $c_0$  and  $bv_p$  ( $1 \leq p < \infty$ ).

**Corollary 2.4.** [22, Example 2.19] Let  $T = \Delta$  and  $X = \ell_p$  for  $1 \leq p < \infty$  or  $X = c_0$ . Then it follows from Theorems 2 and 2 that

$$\begin{aligned}\chi_T(Q) = \chi(T(Q)) &= \lim_{n \rightarrow \infty} \sup_{x \in T(Q)} \|(I - P_n)(x)\| \\ &= \lim_{n \rightarrow \infty} \sup_{y \in Q} \|(I - P_n)(T(y))\|\end{aligned}$$

for all  $Q \in \mathcal{M}_{X_T}$  where

$$\|(I - P_n)(T(y))\| = \left( \sum_{k \geq n} |y_k - y_{k-1}|^p \right)^{1/p}$$

for  $X = \ell_p$  and

$$\|(I - P_n)(T(y))\| = \sup_{k \geq n} |y_k - y_{k-1}|$$

for  $X = c_0$ .

### 3. INFINITE SYSTEMS OF DIFFERENTIAL EQUATIONS

In this section, we study the solvability of the infinite systems of differential equations in the Banach sequence space  $bv_p$  ( $1 \leq p < \infty$ ).

Consider the ordinary differential equation

$$x' = f(t, x) \tag{8}$$

with the initial condition

$$x(0) = x_0. \tag{9}$$

Then the following result for the existence of the Cauchy problem (8)-(9) was given in [16] which is a slight modification of the result proved in [18].

Assume that  $X$  is a real Banach space with the norm  $\|\cdot\|$ . Let us take an interval  $J = [0, T]$ ,  $T > 0$  and  $\overline{B}(x_0, s)$  the closed ball in  $X$  centered at  $x_0$  with radius  $s$ .

**Theorem 3.1.** [16, Theorem 2] Assume that  $f(t, x)$  is a function defined on  $J \times X$  with values in  $X$  such that

$$\|f(t, x)\| \leq Q + R\|x\|,$$

for any  $x \in X$ , where  $Q$  and  $R$  are nonnegative constants. Further, let  $f$  be uniformly continuous on the set  $J_1 \times \overline{B}(x_0, s)$ , where

$$s = (QT_1 + RT_1\|x_0\|)/(1 - RT_1)$$

and  $J_1 = [0, T_1] \subset J$ ,  $RT_1 < 1$ . Moreover, assume that for any nonempty set  $Y \subset \overline{B}(x_0, s)$  and for almost all  $t \in J$  the following inequality holds:

$$\mu(f(t, Y)) \leq q(t)\mu(Y), \tag{10}$$

with a sublinear measure of noncompactness  $\mu$  such that  $\{x_0\} \in \ker \mu$ . Then problem (8)-(9) has a solution  $x$  such that  $\{x(t)\} \in \ker \mu$  for  $t \in J_1$ ; where  $q(t)$  is an integrable function on  $J$ , and  $\ker \mu = \{E \in \mathcal{M}_X : \mu(E) = 0\}$  is the kernel of the measure  $\mu$ .

**Remark 3.2.** In the case when  $\mu = \chi$  (the Hausdorff measure of noncompactness) the assumption of the uniform continuity on  $f$  can be replaced by the weaker one requiring only the continuity of  $f$ .

We will be interested in the existence of solution  $x(t) = (x_i(t))$  of the infinite systems of differential equations

$$x_i' = f_i(t, x_0, x_1, \dots), \quad (11)$$

with the initial conditions

$$x_i(0) = x_i^0, \quad (12)$$

( $i = 0, 1, 2, \dots$ ) which are defined on the interval  $J = [0, T]$  and such that  $x(t) \in bv_p$  for each  $t \in J$ .

An existence theorem for problem (11)-(12) in the space  $bv_p$  can be formulated by making the following assumptions:

- (i)  $x_0 = (x_i^0) \in bv_p$ ,
- (ii)  $f_i : J \times \mathbb{R}^\infty \rightarrow \mathbb{R}$  ( $i = 0, 1, 2, \dots$ ) maps continuously the set  $J \times bv_p$  into  $bv_p$ ,
- (iii) there exists nonnegative functions  $q_i(t)$  and  $r_i(t)$  defined on  $J$  such that

$$|f_i(t, x) - f_{i-1}(t, x)|^p \leq q_i(t) + r_i(t)|x_i - x_{i-1}|^p,$$

for  $t \in J$ ;  $x = (x_i) \in bv_p$  and  $i = 0, 1, 2, \dots$ ,

(iv) the functions  $q_i(t)$  are continuous on  $J$  and the function series  $\sum_{i=0}^{\infty} q_i(t)$  converges uniformly on  $J$ ,

(v) the sequence  $(r_i(t))$  is equibounded on the interval  $J$  and the function  $r(t) = \limsup_{i \rightarrow \infty} r_i(t)$  is integrable on  $J$ .

Now, we prove the following result.

**Theorem 3.3.** Under the assumptions (i)-(v), problem (11)-(12) has a solution  $x(t) = (x_i(t))$  defined on the interval  $J = [0, T]$  whenever  $RT < 1$ ; where  $R$  is defined as the number

$$R = \sup \{r_i(t) : t \in J, i = 0, 1, 2, \dots\}.$$

Moreover,  $x(t) \in bv_p$  for any  $t \in J$ .

*Proof.* For any  $x(t) \in bv_p$  and  $t \in J$ , under the above assumptions, we have

$$\begin{aligned} \|f(t, x)\|_{bv_p}^p &= \sum_{i=0}^{\infty} |f_i(t, x) - f_{i-1}(t, x)|^p \\ &\leq \sum_{i=0}^{\infty} \left[ q_i(t) + r_i(t)|x_i - x_{i-1}|^p \right] \\ &\leq \sum_{i=0}^{\infty} q_i(t) + \left( \sup_{i \geq 0} r_i(t) \right) \cdot \sum_{i=0}^{\infty} |x_i - x_{i-1}|^p \\ &\leq Q + R\|x\|_{bv_p}^p, \end{aligned}$$

where  $Q = \sup_{t \in J} \left( \sum_{i=0}^{\infty} q_i(t) \right)$ .

Now, choose the number  $s = (QT + RT\|x_0\|_{bv_p}^p)/(1 - RT)$  as defined in Theorem 3.1. Consider the operator  $f = (f_i)$  on the set  $J \times \overline{B}(x_0, s)$ . Let us take a set  $Y \in \mathcal{M}_{bv_p}$ . Then by using Corollary 2.4, we get

$$\begin{aligned}\chi(f(t, Y)) &= \lim_{n \rightarrow \infty} \sup_{x \in Y} \left( \sum_{i \geq n} |f_i(t, x) - f_{i-1}(t, x)|^p \right) \\ &\leq \lim_{n \rightarrow \infty} \left( \sum_{i \geq n} q_i(t) + \left( \sup_{i \geq n} r_i(t) \right) \cdot \sum_{i \geq n} |x_i - x_{i-1}|^p \right).\end{aligned}$$

Hence by assumptions (iv)-(v), we get

$$\chi(f(t, Y)) \leq r(t)\chi(Y),$$

i.e. the operator  $f$  satisfies condition (10) of Theorem 3.1. Hence by Theorem 3.1 and Remark 3.2 we conclude that there exists a solution  $x = x(t)$  of problem (11)-(12) such that  $x(t) \in bv_p$  for any  $t \in J$ .

This completes the proof of the theorem.

**Remark 3.4.** We observe that the above theorem can be applied to the perturbed diagonal infinite system of differential equations of the form

$$x_i' = a_i(t)x_i + g_i(t, x_0, x_1, \dots),$$

with the initial conditions

$$x_i(0) = x_i^0, \quad (i = 0, 1, 2, \dots)$$

where  $t \in J$ .

An existence theorem for problem (11)-(12) in the space  $bv_p$  can be formulated by making the following assumptions:

- (i)  $x_0 = (x_i^0) \in bv_p$ ,
- (ii) the sequence  $(|a_i(t)|)$  is defined and equibounded on the interval  $J = [0, T]$ . Moreover, the function  $a(t) = \limsup_{i \rightarrow \infty} \sup |a_i(t)|$  is integrable on  $J$ ,
- (iii) the mapping  $g = (g_i)$  maps continuously the set  $J \times bv_p$  into  $bv_p$ ,
- (iv) there exists nonnegative functions  $b_i(t)$  such that

$$|f_i(t, x) - f_{i-1}(t, x)|^p \leq b_i(t),$$

for  $t \in J$ ;  $x = (x_i) \in bv_p$  and  $i = 0, 1, 2, \dots$ ,

(v) the functions  $b_i(t)$  are continuous on  $J$  and the function series  $\sum_{i=0}^{\infty} b_i(t)$  converges uniformly on  $I$ .

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