

\mathcal{I} -CESÀRO SUMMABILITY OF SEQUENCES OF SETS

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ABSTRACT. In this paper, we defined concept of Wijsman \mathcal{I} -Cesàro summability for sequences of sets and investigate the relationships between the concepts of Wijsman strongly \mathcal{I} -Cesàro summability, Wijsman strongly \mathcal{I} -lacunary summability, Wijsman p -strongly \mathcal{I} -Cesàro summability and Wijsman \mathcal{I} -statistical convergence.

1. INTRODUCTION AND BACKGROUND

The concept of convergence of sequences of real numbers has been extended to statistical convergence independently by Fast [7] and Schoenberg [16]. The idea of \mathcal{I} -convergence was introduced by Kostyrko et al. [13] as a generalization of statistical convergence which is based on the structure of the ideal \mathcal{I} of subset of the set of natural numbers. Recently, Das et al. [5] introduced new notions, namely \mathcal{I} -statistical convergence and \mathcal{I} -lacunary statistical convergence by using ideal.

Freedman et al. [6] established the connection between the strongly Cesàro summable sequences space and the strongly lacunary summable sequences space. Connor [9] gave the relationships between the concepts of strongly p -Cesàro summability and statistical convergence of sequences.

The concept of convergence of sequences of numbers has been extended by several authors to convergence of sequences of sets. The one of these such extensions considered in this paper is the concept of Wijsman convergence (see, [2, 3, 4, 14, 20, 21]). Nuray and Rhoades [14] extended the notion of convergence of set sequences to statistical convergence and gave some basic theorems. Furthermore, the concept of strongly summable set sequences was given by Nuray and Rhoades [14]. Ulusu and Nuray [18] defined the concept of Wijsman lacunary statistical convergence of sequences of sets and considered its relation with Wijsman statistical convergence, which was defined by Nuray and Rhoades. Also, Ulusu and Nuray [19] introduced the concept of Wijsman strongly lacunary summability of sequences of sets. Kışi and Nuray [11] introduced a new convergence notion, for sequences of sets, which

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is called Wijsman \mathcal{I} -convergence by using ideal. Recently, the concepts of Wijsman \mathcal{I} -statistical convergence and Wijsman strongly \mathcal{I} -lacunary convergence for sequences of sets was given by Kişı et al. [12].

In this paper, we defined concept of Wijsman \mathcal{I} -Cesàro summability for sequences of sets and investigate the relationships between the concepts of Wijsman strongly \mathcal{I} -Cesàro summability, Wijsman strongly \mathcal{I} -lacunary summability, Wijsman p -strongly \mathcal{I} -Cesàro summability and Wijsman \mathcal{I} -statistical convergence.

Now, we recall the basic definitions and concepts (See [1, 2, 8, 10, 11, 12, 13, 14, 15, 17, 18, 19, 22]).

A sequence $x = (x_k)$ is said to be statistical convergent to the number L if for every $\varepsilon > 0$,

$$\lim_{n \rightarrow \infty} \frac{1}{n} |\{k \leq n : |x_k - L| \geq \varepsilon\}| = 0,$$

where the vertical bars denote the number of elements in the enclosed set.

A family of sets $\mathcal{I} \subseteq 2^{\mathbb{N}}$ is called an ideal if and only if

- (i) $\emptyset \in \mathcal{I}$,
- (ii) For each $A, B \in \mathcal{I}$ we have $A \cup B \in \mathcal{I}$,
- (iii) For each $A \in \mathcal{I}$ and each $B \subseteq A$ we have $B \in \mathcal{I}$.

An ideal is called non-trivial if $\mathbb{N} \notin \mathcal{I}$ and non-trivial ideal is called admissible if $\{n\} \in \mathcal{I}$ for each $n \in \mathbb{N}$.

A family of sets $\mathcal{F} \subseteq 2^{\mathbb{N}}$ is a filter if and only if

- (i) $\emptyset \notin \mathcal{F}$,
- (ii) For each $A, B \in \mathcal{F}$ we have $A \cap B \in \mathcal{F}$,
- (iii) For each $A \in \mathcal{F}$ and each $B \supseteq A$ we have $B \in \mathcal{F}$.

Proposition 1.1. ([13]) \mathcal{I} is a non-trivial ideal in \mathbb{N} if and only if

$$\mathcal{F}(\mathcal{I}) = \{M \subset \mathbb{N} : (\exists A \in \mathcal{I})(M = \mathbb{N} \setminus A)\}$$

is a filter in \mathbb{N} .

An admissible ideal $\mathcal{I} \subset 2^{\mathbb{N}}$ satisfies the property (AP), if for every countable family of mutually disjoint sets $\{A_1, A_2, \dots\}$ belonging to \mathcal{I} , there exists a countable family of sets $\{B_1, B_2, \dots\}$ such that $A_j \Delta B_j$ is a finite set for $j \in \mathbb{N}$ and $B = \bigcup_{j=1}^{\infty} B_j \in \mathcal{I}$ (hence $B_j \in \mathcal{I}$ for each $j \in \mathbb{N}$).

Let $\mathcal{I} \subset 2^{\mathbb{N}}$ be an admissible ideal. A sequence $x = (x_k)$ of elements of \mathbb{R} is said to be \mathcal{I} -convergent to $L \in \mathbb{R}$ if for every $\varepsilon > 0$ the set

$$A(\varepsilon) = \{k \in \mathbb{N} : |x_k - L| \geq \varepsilon\}$$

belongs to \mathcal{I} .

Let (X, ρ) be a metric space. For any point $x \in X$ and any non-empty subset A of X , we define the distance from x to A by

$$d(x, A) = \inf_{a \in A} \rho(x, a).$$

The following definitions, we let (X, ρ) be a metric space and A, A_k be any non-empty closed subsets of X .

The sequence $\{A_k\}$ is bounded if $\sup_k \{d(x, A_k)\} < \infty$ for each $x \in X$. The set of all bounded set sequences is denoted by L_∞ .

The sequence $\{A_k\}$ is Wijsman convergent to A if for each $x \in X$,

$$\lim_{k \rightarrow \infty} d(x, A_k) = d(x, A).$$

The sequence $\{A_k\}$ is Wijsman statistical convergent to A if $(d(x, A_k))$ is statistical convergent to $d(x, A)$; i.e., for every $\varepsilon > 0$ and for each $x \in X$,

$$\lim_{n \rightarrow \infty} \frac{1}{n} \left| \{k \leq n : |d(x, A_k) - d(x, A)| \geq \varepsilon\} \right| = 0.$$

The sequence $\{A_k\}$ is Wijsman Cesàro summable to A if for each $x \in X$,

$$\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n d(x, A_k) = d(x, A),$$

the sequence $\{A_k\}$ is Wijsman strongly Cesàro summable to A if for each $x \in X$,

$$\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n |d(x, A_k) - d(x, A)| = 0,$$

and the sequence $\{A_k\}$ is Wijsman strongly p -Cesàro summable to A if for each p positive real number and for each $x \in X$,

$$\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n |d(x, A_k) - d(x, A)|^p = 0.$$

By a lacunary sequence we mean an increasing integer sequence $\theta = \{k_r\}$ such that $k_0 = 0$ and $h_r = k_r - k_{r-1} \rightarrow \infty$ as $r \rightarrow \infty$. Throughout this paper the intervals determined by θ will be denoted by $I_r = (k_{r-1}, k_r]$, and ratio $\frac{k_r}{k_{r-1}}$ will be abbreviated by q_r .

Let θ be a lacunary sequence. The sequence $\{A_k\}$ is Wijsman strongly lacunary convergent to A if

$$\lim_{r \rightarrow \infty} \frac{1}{h_r} \sum_{k \in I_r} |d(x, A_k) - d(x, A)| = 0.$$

Throughout the paper, we let (X, ρ) be a separable metric space, $\mathcal{I} \subseteq 2^{\mathbb{N}}$ be an admissible ideal and A, A_k be any non-empty closed subsets of X .

The sequence $\{A_k\}$ is Wijsman \mathcal{I} -convergent to A , if for every $\varepsilon > 0$ and for each $x \in X$,

$$A(x, \varepsilon) = \{k \in \mathbb{N} : |d(x, A_k) - d(x, A)| \geq \varepsilon\} \in \mathcal{I}.$$

The sequence $\{A_k\}$ is Wijsman \mathcal{I} -statistical convergent to A , if for every $\varepsilon > 0$, $\delta > 0$ and for each $x \in X$,

$$\left\{ n \in \mathbb{N} : \frac{1}{n} \left| \{k \leq n : |d(x, A_k) - d(x, A)| \geq \varepsilon\} \right| \geq \delta \right\} \in \mathcal{I}.$$

In this case, we write $A_k \xrightarrow{S(\mathcal{I}_W)} A$.

Let θ be a lacunary sequence. The sequence $\{A_k\}$ is Wijsman strongly \mathcal{I} -lacunary summable to A , if for every $\varepsilon > 0$ and for each $x \in X$,

$$\left\{ r \in \mathbb{N} : \frac{1}{h_r} \sum_{k \in I_r} |d(x, A_k) - d(x, A)| \geq \varepsilon \right\} \in \mathcal{I}.$$

In this case, we write $A_k \xrightarrow{N_\theta[\mathcal{I}_W]} A$.

2. MAIN RESULTS

In this section, we defined concepts of Wijsman \mathcal{I} -Cesàro summability, Wijsman strongly \mathcal{I} -Cesàro summability and Wijsman p -strongly \mathcal{I} -Cesàro summability for sequences of sets. Also, we investigate the relationships between the concepts of Wijsman strongly \mathcal{I} -Cesàro summability, Wijsman strongly \mathcal{I} -lacunary summability, Wijsman p -strongly \mathcal{I} -Cesàro summability and Wijsman \mathcal{I} -statistical convergence.

Definition 2.1. *The sequence $\{A_k\}$ is Wijsman \mathcal{I} -Cesàro summable to A if for every $\varepsilon > 0$ and for each $x \in X$,*

$$\left\{ n \in \mathbb{N} : \left| \frac{1}{n} \sum_{k=1}^n d(x, A_k) - d(x, A) \right| \geq \varepsilon \right\} \in \mathcal{I}.$$

In this case, we write $A_k \xrightarrow{C_1(\mathcal{I}_W)} A$.

Definition 2.2. *The sequence $\{A_k\}$ is Wijsman strongly \mathcal{I} -Cesàro summable to A if for every $\varepsilon > 0$ and for each $x \in X$,*

$$\left\{ n \in \mathbb{N} : \frac{1}{n} \sum_{k=1}^n |d(x, A_k) - d(x, A)| \geq \varepsilon \right\} \in \mathcal{I}.$$

In this case, we write $A_k \xrightarrow{C_1[\mathcal{I}_W]} A$.

Theorem 2.3. *Let θ be a lacunary sequence. If $\liminf_r q_r > 1$ then,*

$$A_k \xrightarrow{C_1[\mathcal{I}_W]} A \Rightarrow A_k \xrightarrow{N_\theta[\mathcal{I}_W]} A.$$

Proof. If $\liminf_r q_r > 1$, then there exists $\delta > 0$ such that $q_r \geq 1 + \delta$ for all $r \geq 1$. Since $h_r = k_r - k_{r-1}$, we have

$$\frac{k_r}{h_r} \leq \frac{1 + \delta}{\delta} \quad \text{and} \quad \frac{k_{r-1}}{h_r} \leq \frac{1}{\delta}.$$

Let $\varepsilon > 0$ and we define the set

$$S = \left\{ k_r \in \mathbb{N} : \frac{1}{k_r} \sum_{k=1}^{k_r} |d(x, A_k) - d(x, A)| < \varepsilon \right\},$$

for each $x \in X$. We can easily say that $S \in \mathcal{F}(\mathcal{I})$, which is a filter of the ideal \mathcal{I} , so we have

$$\begin{aligned} \frac{1}{h_r} \sum_{k \in I_r} |d(x, A_k) - d(x, A)| &= \frac{1}{h_r} \sum_{k=1}^{k_r} |d(x, A_k) - d(x, A)| \\ &\quad - \frac{1}{h_r} \sum_{k=1}^{k_{r-1}} |d(x, A_k) - d(x, A)| \\ &= \frac{k_r}{h_r} \cdot \frac{1}{k_r} \sum_{k=1}^{k_r} |d(x, A_k) - d(x, A)| \\ &\quad - \frac{k_{r-1}}{h_r} \cdot \frac{1}{k_{r-1}} \sum_{k=1}^{k_{r-1}} |d(x, A_k) - d(x, A)| \\ &\leq \left(\frac{1 + \delta}{\delta} \right) \varepsilon - \frac{1}{\delta} \varepsilon' \end{aligned}$$

for each $x \in X$ and $k_r \in S$. Choose $\eta = \left(\frac{1 + \delta}{\delta} \right) \varepsilon + \frac{1}{\delta} \varepsilon'$. Therefore, for each $x \in X$

$$\left\{ r \in \mathbb{N} : \frac{1}{h_r} \sum_{k \in I_r} |d(x, A_k) - d(x, A)| < \eta \right\} \in \mathcal{F}(\mathcal{I})$$

and this completes the proof. \square

Theorem 2.4. *Let θ be a lacunary sequence. If $\limsup_r q_r < \infty$ then,*

$$A_k \xrightarrow{N_\theta[\mathcal{I}_W]} A \Rightarrow A_k \xrightarrow{C_1[\mathcal{I}_W]} A.$$

Proof. If $\limsup_r q_r < \infty$, then there exists $M > 0$ such that $q_r < M$, for all $r \geq 1$. Let $A_k \xrightarrow{N_\theta[\mathcal{I}_W]} A$ and we define the sets T and R such that

$$T = \left\{ r \in \mathbb{N} : \frac{1}{h_r} \sum_{k \in I_r} |d(x, A_k) - d(x, A)| < \varepsilon_1 \right\}$$

and

$$R = \left\{ n \in \mathbb{N} : \frac{1}{n} \sum_{k=1}^n |d(x, A_k) - d(x, A)| < \varepsilon_2 \right\},$$

for every $\varepsilon_1, \varepsilon_2 > 0$ and for each $x \in X$. Let

$$a_j = \frac{1}{h_j} \sum_{k \in I_j} |d(x, A_k) - d(x, A)| < \varepsilon_1$$

for each $x \in X$ and for all $j \in T$. It is obvious that $T \in \mathcal{F}(\mathcal{I})$.

Choose n is any integer with $k_{r-1} < n < k_r$, where $r \in T$. Then, we have

$$\begin{aligned}
 \frac{1}{n} \sum_{k=1}^n |d(x, A_k) - d(x, A)| &\leq \frac{1}{k_{r-1}} \sum_{k=1}^{k_r} |d(x, A_k) - d(x, A)| \\
 &= \frac{1}{k_{r-1}} \left(\sum_{k \in I_1} |d(x, A_k) - d(x, A)| \right. \\
 &\quad \left. + \sum_{k \in I_2} |d(x, A_k) - d(x, A)| \right. \\
 &\quad \left. + \cdots + \sum_{k \in I_r} |d(x, A_k) - d(x, A)| \right) \\
 &= \frac{k_1}{k_{r-1}} \left(\frac{1}{h_1} \sum_{k \in I_1} |d(x, A_k) - d(x, A)| \right) \\
 &\quad + \frac{k_2 - k_1}{k_{r-1}} \left(\frac{1}{h_2} \sum_{k \in I_2} |d(x, A_k) - d(x, A)| \right) \\
 &\quad + \cdots + \frac{k_r - k_{r-1}}{k_{r-1}} \left(\frac{1}{h_r} \sum_{k \in I_r} |d(x, A_k) - d(x, A)| \right) \\
 &= \frac{k_1}{k_{r-1}} a_1 + \frac{k_2 - k_1}{k_{r-1}} a_2 + \cdots + \frac{k_r - k_{r-1}}{k_{r-1}} a_r \\
 &\leq \left(\sup_{j \in T} a_j \right) \cdot \frac{k_1}{k_{r-1}} \\
 &< \varepsilon_1 \cdot M.
 \end{aligned}$$

for each $x \in X$. Choose $\varepsilon_2 = \frac{\varepsilon_1}{M}$ and in view of the fact that

$$\bigcup \{n : k_{r-1} < n < k_r, r \in T\} \subset R,$$

where $T \in \mathcal{F}(\mathcal{I})$. It follows from our assumption on θ that the set R also belongs to $\mathcal{F}(\mathcal{I})$ and this completes the proof. \square

We have the following theorem by Theorem 2.3 and Theorem 2.4.

Theorem 2.5. *Let θ be a lacunary sequence. If $1 < \liminf_r q_r < \limsup_r q_r < \infty$ then,*

$$A_k \xrightarrow{C_1[\mathcal{I}_W]} A \Leftrightarrow A_k \xrightarrow{N_\theta[\mathcal{I}_W]} A.$$

Definition 2.6. *The sequence $\{A_k\}$ is Wijsman p -strongly \mathcal{I} -Cesàro summable to A if for each p positive real number, for every $\varepsilon > 0$ and for each $x \in X$,*

$$\left\{ n \in \mathbb{N} : \frac{1}{n} \sum_{k=1}^n |d(x, A_k) - d(x, A)|^p \geq \varepsilon \right\} \in \mathcal{I}.$$

In this case, we write $A_k \xrightarrow{C_p[\mathcal{I}_W]} A$.

Theorem 2.7. *If $\{A_k\}$ is Wijsman p -strongly \mathcal{I} -Cesàro summable to A then, $\{A_k\}$ is Wijsman \mathcal{I} -statistical convergent to A .*

Proof. Let $A_k \xrightarrow{C_p[\mathcal{I}_W]} A$ and given $\varepsilon > 0$. Then, we have

$$\begin{aligned} \sum_{k=1}^n |d(x, A_k) - d(x, A)|^p &\geq \sum_{\substack{k=1 \\ |d(x, A_k) - d(x, A)| \geq \varepsilon}}^n |d(x, A_k) - d(x, A)|^p \\ &\geq \varepsilon^p \cdot \left| \{k \leq n : |d(x, A_k) - d(x, A)| \geq \varepsilon\} \right| \end{aligned}$$

for each $x \in X$ and so

$$\frac{1}{\varepsilon^p \cdot n} \sum_{k=1}^n |d(x, A_k) - d(x, A)|^p \geq \frac{1}{n} \left| \{k \leq n : |d(x, A_k) - d(x, A)| \geq \varepsilon\} \right|.$$

Hence, for given $\delta > 0$

$$\begin{aligned} \left\{ n \in \mathbb{N} : \frac{1}{n} \left| \{k \leq n : |d(x, A_k) - d(x, A)| \geq \varepsilon\} \right| \geq \delta \right\} \\ \subseteq \left\{ n \in \mathbb{N} : \frac{1}{n} \sum_{k=1}^n |d(x, A_k) - d(x, A)|^p \geq \varepsilon^p \cdot \delta \right\} \in \mathcal{I}, \end{aligned}$$

for each $x \in X$. Therefore, $A_k \xrightarrow{S(\mathcal{I}_W)} A$. \square

Theorem 2.8. *Let $\{A_k\} \in L_\infty$. If $\{A_k\}$ is Wijsman \mathcal{I} -statistical convergent to A then, $\{A_k\}$ is Wijsman p -strongly \mathcal{I} -Cesàro summable to A .*

Proof. Suppose that $\{A_k\}$ is bounded and $A_k \xrightarrow{S(\mathcal{I}_W)} A$. Then, there is an $M > 0$ such that

$$|d(x, A_k) - d(x, A)| \leq M,$$

for each $x \in X$ and for all k . Given $\varepsilon > 0$, we have

$$\begin{aligned} \frac{1}{n} \sum_{k=1}^n |d(x, A_k) - d(x, A)|^p &= \frac{1}{n} \sum_{\substack{k=1 \\ |d(x, A_k) - d(x, A)| \geq \varepsilon}}^n |d(x, A_k) - d(x, A)|^p \\ &\quad + \frac{1}{n} \sum_{\substack{k=1 \\ |d(x, A_k) - d(x, A)| < \varepsilon}}^n |d(x, A_k) - d(x, A)|^p \\ &\leq \frac{1}{n} \cdot M^p \cdot \left| \{k \leq n : |d(x, A_k) - d(x, A)| \geq \varepsilon\} \right| \\ &\quad + \frac{1}{n} \cdot \varepsilon^p \cdot \left| \{k \leq n : |d(x, A_k) - d(x, A)| < \varepsilon\} \right| \\ &\leq \frac{M^p}{n} \cdot \left| \{k \leq n : |d(x, A_k) - d(x, A)| \geq \varepsilon\} \right| + \varepsilon^p. \end{aligned}$$

Then, for any $\delta > 0$

$$\left\{ n \in \mathbb{N} : \frac{1}{n} \sum_{k=1}^n |d(x, A_k) - d(x, A)|^p \geq \delta \right\} \\ \subseteq \left\{ n \in \mathbb{N} : \frac{1}{n} \left| \left\{ k \leq n : |d(x, A_k) - d(x, A)| \geq \varepsilon \right\} \right| \geq \frac{\delta^p}{M^p} \right\} \in \mathcal{I},$$

for each $x \in X$. Therefore, $A_k \xrightarrow{C_p[\mathcal{I}_w]} A$. \square

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