Electronic Journal of Mathematical Analysis and Applications
Vol. 9(2) July 2021, pp. 288-294.
ISSN: 2090-729X(online)
http://math-frac.org/Journals/EJMAA/

(1,N)-ARITHMETIC LABELLING OF CHAIN OF EVEN CYCLES, SPLITTING GRAPH OF PATHS AND SPLITTING GRAPH OF CYCLES C_{4m}

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ABSTRACT. A (p,q) - graph G is said to have $(1, N)$ - Arithmetic labelling if there is a one-one function ϕ from the vertex set $V(G)$ to $\{0, 1, N, (N+1), 2N, (2N+1), ..., (q-1)N, (q-1)(N+1)\}$ so that the values of the edges, obtained as the sums of the labelling assigned to their end vertices can be arranged in the arithmetic progression $1, (N+1), (2N+1), ..., (q-1)N+1$. In this paper we prove that certain chain of even cycles, splitting graph of paths and splitting graph of cycles C_{4m} have (1,N) - Arithmetic Labelling for every positive integer $N > 1$.

1. Introduction

B.D Acharya and S.M. Hedge [1],[2] introduced (k, d) - arithmetic graphs and certain vertex valuations of a graph. A (p, q) -graph is said to be (k, d) - arithmetic if its vertices can be assigned distinct non -negative integers so that the values of the edges, obtained as the sums of the numbers assigned to their end vertices, can be arranged in the arithmetic progression $k, k+d, k+2d, ...k+(q-1)d$.

Joseph A.Gallian [3] surveyed numerous graph labelling methods.
V. Ramachandran and C.Sekar [4] introduced $(1, N)$- Arithmetic labelling. They proved that stars, paths, complete bibartite graph $K_{m,n}$, highly irregular graph $Hi(m, m)$,Cycle C_{4k}, ladder and subdivision of ladder have (1,N) - Arithmetic Labelling. They also proved that C_{4k+2} does not have (1,N) - Arithmetic Labelling and no graph G containing an odd cycle has (1,N) - Arithmetic Labelling for any integer N.

In this paper we prove that certain chain of even cycles, splitting graph of paths and splitting graph of cycles C_{4m} have (1,N) - Arithmetic Labelling.

2. Preliminaries

2.1 Definition: [5]Let C_{2k} be an even cycle. Consider n copies of C_{2k}. A chain of even cycles C_{2k} denoted by $C_{2k,n}$ has vertex set $\{v_i, u_j, w_h / 1 \leq i \leq n+1, 1 \leq j, h \leq k-1\}$

2010 *Mathematics Subject Classification.* 05C78.
Key words and phrases. Chain of even cycles, Splitting graph of paths, Splitting graph of cycles C_{4m}.
Submitted April 18, 2020. Revised Feb. 1, 2021.

and edge set $\{v_iu_{3i-2}, v_iw_{3i-2}/1 \le i \le n\} \cup \{v_iu_{3i-3}, v_iw_{3i-3}/2 \le i \le n+1\} \cup \{u_ju_{j+1}, w_hw_{h+1}/1 \le j, h \le k-2\}$
$C_{2k,n}$ has $(2k-1)n+1$ vertices and $2kn$ edges.
$C_{2k,n}$ has $(k-1)n$ upper vertices $u_1, u_2, ...u_{(k-1)n}$,$(k-1)n$ lower vertices $w_1, w_2, .., .w_{(k-1)n}$ and $(n+1)$ middle vertices $v_1, v_2, ..., v_{n+1}$.
Illustration: $C_{8,4}$

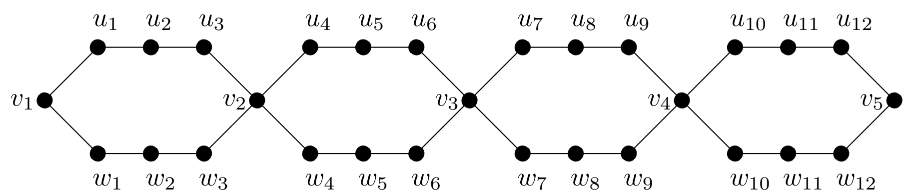

Fig 2.1

2.2 Definition: Let G be a graph, For each vertex v of a graph G, take a new vertex v'. Join v' to those vertices of G adjacent to v. The graph thus obtained is called the splitting graph of G. We denote it by $S^{'}(G)$.

3. Main Results

3.1 Theorem: $C_{4,n}$ is (1,N)- Arithmetic for all $N > 1$ and for integer, $n \ge 2$.
Proof: $C_{4,n}$ has $3n+1$ vertices and $4n$ edges.
Define $f(u_i) = N(i-1)+1$, for $i = 1, 2, ..., n$
$f(w_i) = 2Nn + N(i-1)+1$, for $i = 1, 2, ..., n$.
$f(v_i) = N(i-1)$, for $i = 1, 2, ..., n+1$.
Clearly f is one-one.
The edges have the labels $1, N+1, 2N+1, ..., (4n-1)N+1$.
Therefore $C_{4,n}$ is (1,N)-Arithmetic.
Example:A $(1,7)$- Arithmetic labelling of $C_{4,5}$.

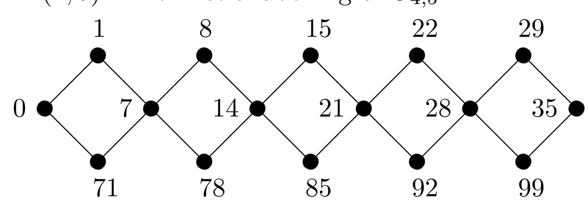

Fig 3.1

3.2 Theorem:$C_{6,2m}$ is (1,N) - Arithmetic for all $N > 1$ and for integer $m \ge 1$.
Proof:$C_{6,2m}$ has $10m+1$ vertices and $12m$ edges.
For $i = 1, 5, ..., 4m-3$,
define $f(u_i) = 7N\frac{(i-1)}{4} + 1$ and $f(w_i) = 7N\frac{(i-1)}{4} + N + 1$
For $i = 2, 6, 10, ..., 4m-2$,
define $f(u_i) = 5N\frac{(i-2)}{4} + 3N$ and $f(w_i) = 5N\frac{(i-2)}{4} + N$.
For $i = 3, 7, 11, ..., (4m-1)$,
define $f(u_i) = 5N\frac{(i-3)}{4} + 4N$ and $f(w_i) = 5N\frac{(i-3)}{4} + 2N$
For $i = 4, 8, 12, ..., 4m$,
define $f(u_i) = 7N\frac{(i-4)}{4} + 5N + 1$ and $f(w_i) = 7N\frac{(i-4)}{4} + 6N + 1$
For $i = 1, 3, 5, ..., (2m+1)$, define $f(v_i) = 5N\frac{(i-1)}{2}$

For $i = 2, 4, 6, ..., 2m$, define $f(v_i) = 7N\frac{(i-2)}{2} + 3N + 1$
Clearly f is one-one.
The edge labels are $1, N+1, 2N+1, ..., (12m-1)N+1$
Thus $C_{6,2m}$ is (1,N)- Arithmetic.
Example:B (1,5)- Arithmetic labelling of $C_{6,6}$.

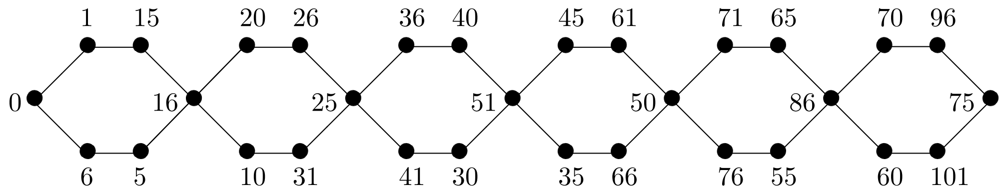

Fig 3.2

3.3.Theorem: $C_{8,n}$ is (1,N) - Arithmetic for all $N > 1$ and for integer $n \geq 2$.
Proof: $C_{8,n}$ has $7n+1$ vertices and $8n$ edges.
For $i = 1, 4, 7, ..., 3n+2$,
define $f(u_i) = 4N\frac{(i-1)}{3} + 1$ and $f(w_i) = 4N\frac{(i-1)}{3} + N + 1$
For $i = 2, 5, 8, ..., 3n-1$,
define $f(u_i) = 4N\frac{(i-2)}{3} + 3N$ and $f(w_i) = 4N\frac{(i-2)}{3} + N$
For $i = 3, 6, 9, ..., 3n$,
define $f(u_i) = 4N\frac{(i-3)}{3} + 2N + 1$ and $f(w_i) = 4N\frac{(i-3)}{3} + 3N + 1$
For $i = 1, 2, .., n+1$,
define $f(v_i) = 4N(i-1)$
Clearly f is one-one.
The edges have the labels $1, N+1, 2N+1, ..., (8n-1)N+1$
Therefore $C_{8,n}$ is (1,N)-Arithmetic.
Example:C (1,9)-Arithmetic labelling of $C_{8,3}$.

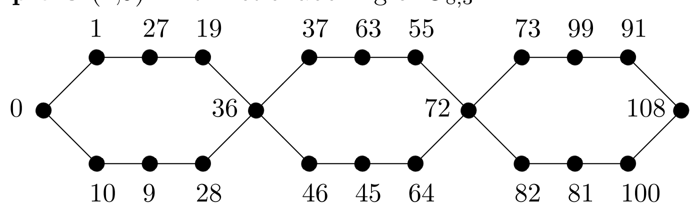

Fig 3.3

3.4 Theorem: Splitting graph of a path of length n is (1,N)-Arithmetic for all integers $N > 1$.
Proof: Let $S^{'}(P_n)$ be the splitting graph of path P_n of length n.
$S^{'}(P_n)$ has $2n+2$ vertices and $3n$ edges.
Let $u_1, u_2, ..., u_{n+1}$ be the vertices of the path P_n and $v_1, v_2, ..., v_{n+1}$ be the new vertices corresponding to $u_1, u_2, ..., u_{n+1}$ respectively.
Case:1 Let $n = 4m+1$, $m \geq 0$.
For $m = 0$, the (1,N)- Arithmetic labelling is as follows.

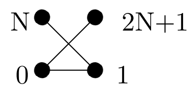

Fig 3.4

Suppose $m \geq 1$.
Define $f(u_i) = N(i-1)$, for $i = 1, 3, 5, 7, ..., 4m+1$.
$f(u_i) = 1 + N(i-2)$, for $i = 2, 4, 6, ..., 4m+2$.
$f(v_1) = (12m+1)N$
$f(v_i) = N(i-2) + N + 1$, for $i = 2, 4, 6, ..., 4m$
$f(v_i) = (12m-2)N - 2N(i-3)$, for $i = 3, 7, 11, ..., 4m-1$
$f(v_i) = (12m-5)N - 2N(i-5)$, for $i = 5, 9, 13, ..., 4m+1$.
$f(v_{4m+2}) = 2N(4m+1) + 1$.
Clearly f is one-one.
The edge labels are $1, N+1, 2N+1, ..., (12m+2)N+1$.
Therefore $S^{'}(P_{4m+1})$ is (1,N)-Arithmetic.
Case:2 Let $n = 4m+3$, $m \geq 0$.
Define $f(u_i) = N(i-1)$, for $i = 1, 3, 5, 7, ..., 4m+3$.
$f(u_i) = 1 + N(i-2)$, for $i = 2, 4, 6, ..., 4m+4$.
$f(v_1) = (12m+6)N$
$f(v_{4m+4}) = (8m+6)N + 1$
$f(v_i) = N(i-2) + N + 1$, for $i = 2, 4, 6, ..., 4m+2$.
$f(v_i) = N(12m+5) + 2N(i-3)$, for $i = 3, 7, 11, ..., 4m+3$.
$f(v_i) = 12mN - 2N(i-5)$, for $i = 5, 9, 13, ..., 4m+1$.
Clearly f is one-one.
The edge labels are $1, N+1, 2N+1, ..., (12m+8)N+1$
Therefore $S^{'}(P_{4m+3})$ is (1,N)-Arithmetic.
Case:3 Let $n = 4m$, $m \geq 1$
For $m = 1$, (1,N)- Arithmetic labelling of $S^{'}(P_4)$is given as follows:

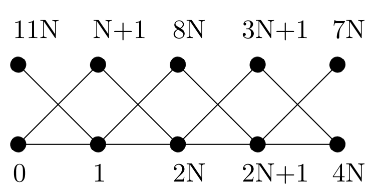

Fig 3.5

Suppose $m \geq 2$.
Define $f(u_i) = N(i-1)$, for $i = 1, 3, 5, 7, ..., 4m+1$.
$f(u_i) = 1 + N(i-2)$, for $i = 2, 4, 6, ..., 4m$.
$f(v_1) = (12m-3)N$
$f(v_{4m+1}) = (8m+1)N$

$f(v_i) = N(i-2) + N + 1,$ for $i = 2, 4, 6, ..., 4m.$
$f(v_i) = N(12m-4) - 2N(i-3),$ for $i = 3, 7, 11, ..., 4m-1.$
$f(v_i) = N(12m-9) - 2N(i-5),$ for $i = 5, 9, 13, ..., 4m-3.$
Clearly f is one-one.
The edge labels are $1, N+1, 2N+1, ..., (12m-1)N+1$
Therefore $S^{'}(P_{4m})$ is (1,N) - Arithmetic.
Case:4 Let $n = 4m+2$, $m \geq 0$
Define $f(u_i) = N(i-1),$ for $i = 1, 3, 5, 7, ..., 4m+3.$
$f(u_i) = 1 + N(i-2),$ for $i = 2, 4, 6, ..., 4m+2.$
$f(v_1) = (12m+5)N$
$f(v_{4m+3}) = (8m+4)N$
$f(v_i) = N(i-2) + N + 1,$ for $i = 2, 4, 6, ..., 4m+2.$
$f(v_i) = N(12m+1) - 2N(i-3),$ for $i = 3, 7, 11, ..., 4m-1.$
$f(v_i) = (12m-2)N - 2N(i-5),$ for $i = 5, 9, 13, ..., 4m+1.$
Example:D (1,6)-Arithmetic labelling of $S^{'}(P_9)$.

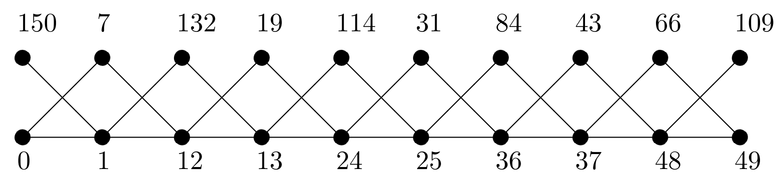

Fig 3.6

Example:E (1,10)-Arithmetic labelling of $S^{'}(P_{11})$.

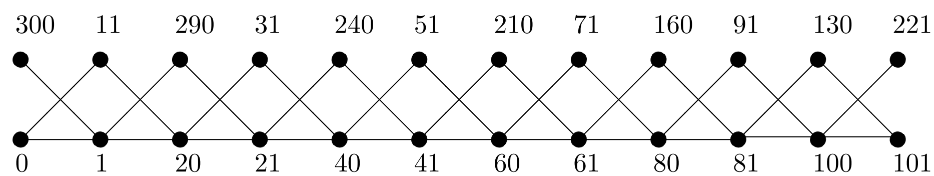

Fig 3.7

Example:F (1,4)- Arithmetic labelling of $S^{'}(P_{12})$.

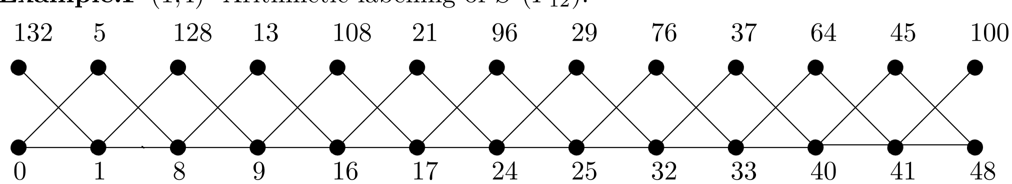

Fig 3.8

Example:G (1,9)- Arithmetic labelling of $S^{'}(P_{10})$.

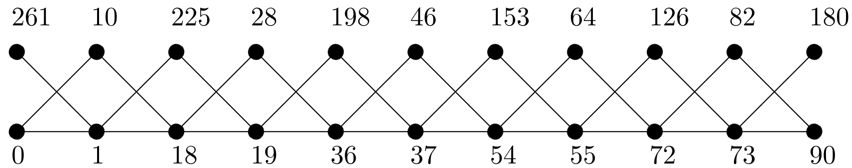

Fig 3.9

3.5 Theorem: $S^{'}(C_{4m})$ is (1,N)-Arithmetic for all $N > 1$ and integer $m \geq 1$.
Proof: $S^{'}(C_{4m})$ has $8m$ vertices and $12m$ edges.
Let $u_1, u_2, ..., u_{4m}$ be the vertices of the cycle C_{4m} and $v_1, v_2, ..., v_{4m}$ be the new vertices corresponding to $u_1, u_2, ..., u_{4m}$ respectively.
Illustration $S^{'}(C_8)$ is given as follows.

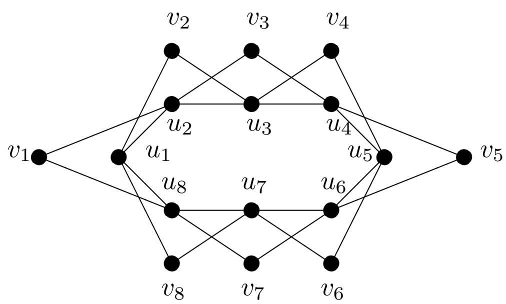

Fig 3.10

Suppose $m = 1$
(1,N)- Arithmetic labelling of $S^{'}(C_4)$ is given as follows:

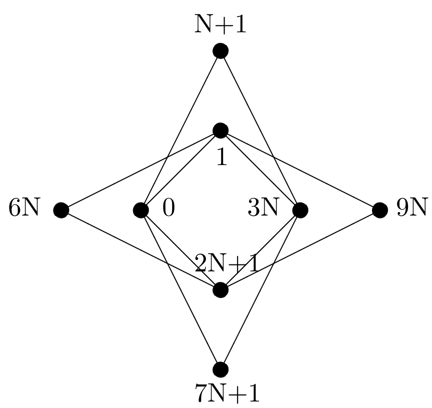

Fig 3.11
Clearly the edge labels are $1, N+1, 2N+1, 3N+1, 4N+1, 5N+1, 6N+1, 7N+1, 8N+1, 9N+1, 10N+1$ and $11N+1$.
Thus $S^{'}(C_4)$ is (1,N)- Arithmetic.

Suppose $m \geq 2$.
Define $f(u_i) = N(i-1)$, for $i = 1, 3, 5, ..., 2m-1$.
$f(u_i) = Ni$, for $i = 2m+1, 2m+3, ..., 4m-1$
$f(u_i) = N(i-2)+1$, for $i = 2, 4, ..., 4m$.
$f(v_1) = 2N(4m-1)$
$f(v_i) = N(i-2)+N+1$, for $i = 2, 4, 6, ..., 4m-2$.
$f(v_3) = 3N(4m-1)$
$f(v_i) = 3N(4m-3) - 2N(i-5)$, for $i = 5, 9, 13, ..., 4m-3$.
$f(v_i) = 3N(4m-4) - 2N(i-7)$, for $i = 7, 11, ..., 4m-1$.
$f(v_{4m}) = N(8m-1)+1$
Clearly f is one -one..
The edges have the labels $1, N+1, 2N+1, ..., (12m-1)N+1$.
Therefore $S^{'}(C_{4m})$ is (1,N) - Arithmetic.
Example:H (1,6)- Arithmetic Labelling of $S^{'}(C_{12})$.

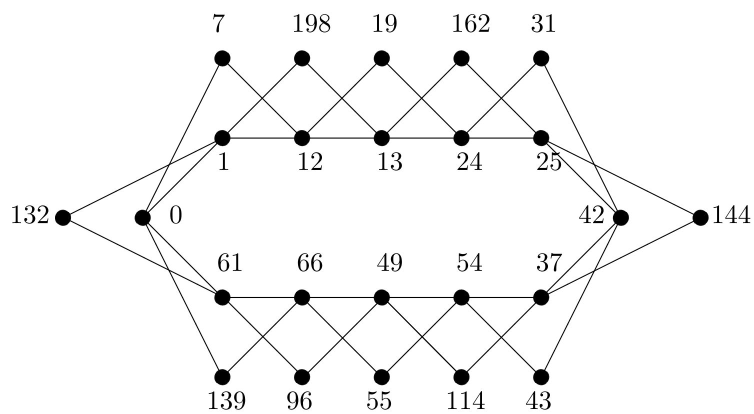

Fig 3.12

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