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NEAR SOFT CONTINUOUS AND NEAR SOFT JP-CONTINUOUS FUNCTIONS

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Abstract. Most real life situations need some sort of approximation to fit mathematical models. Pawlak introduced approximations as a means of approximating one set of object with another set of objects using an indiscernibility relation that is based on a comparison between the feature values of objects. Near sets were introduced by Peters where objects with affinities were considered perceptually near to others. Soft set theory was proposed by Molodtsov as a general framework for reasoning about vague concepts. We obtained near soft set by combining two concepts soft set and near sets. Based on a near set topology and near soft open sets, the approximation of a near soft set is proposed to obtain a function called near soft continuous . In this paper, we defined near soft continuous and near soft JP- continuous functions and give some examples about this functions.

1. Introduction

The other notion of near sets have been given by Peters [1, 2] where objects, affinities are considered perceptually near to each other, i.e., objects with similar descriptions to some degrees.

The concept of soft theory as a mathematical tool for dealing with uncertainties, which is initiated by Molodtsov [3], has been studied by many scientists and has proposed a new approach for uncertainty [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13]. Feng and Li [5] have investigated the problem of combining soft sets with rough sets, and introduced the notion of rough soft sets. Afterwards, Tasbozan [14] combine near sets approach with soft set theory.

2. Soft Sets

Let U be an initial universe set and E be a collection of all possible parameter with respect to U, where parameters are the characteristics or properties of objects in U. Then we will call E the universe set of parameters with respect to U.

Definition 1 [3] A pair (F, A) is called soft set over U if $A \subseteq E$ and $F : A \to \mathcal{P}(U)$, where $\mathcal{P}(U)$ is the set of all subsets of U.

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Example 1 Let $\mathcal{U} = \{x_1, x_2, x_3, x_4\}$, $A = \{h_3\} \subseteq E = \{h_1, h_2, h_3, h_4\}$ denote a initial universe set and a set of parameters, respectively. Sample values of the $h_i, i = 1, 2, 3, 4$. Let (F, A) be a soft set is defined by $A = \{h_3\}$, $F(h_3) = \{x_2, x_3, x_4\}$. $(F, A) = \{h_3, (x_2, x_3, x_4)\}$.

Definition 2 [4] Let's assume that two soft sets (F, A) and (G, B) are defined over U. Then (G, B) is called a soft subset of (F, A) denoted by $(F, A) \subseteq (G, B)$, if $B \subseteq A$ and $G(\phi) \subset F(\phi)$ for all $\phi \in B$. Two soft sets (F, A) and (G, B) over U are said to be equal, denoted by $(F, A) = (G, B)$ if $(F, A) \subseteq (G, B)$ and $(G, B) \subseteq (F, A)$.

Definition 3 [5] Let (F, A) and (G, B) be two soft sets over a common universe U.

(1) The restricted intersection of (F, A) and (G, B) denoted by (F, A) (G, B) is defined as the soft set (H, C), where $C = A \cap B$ and $H(c) = F(c) \cap G(c)$ for all $c \in C$.
(2) The extended union of (F, A) and (G, B) denoted by $(F, A)\ \widetilde{\cup}(G, B)$ is defined as the soft set (H, C), where $C = A \cup B$ and $\forall\ e \in C$

$$H(e) = \begin{cases} F(e), & e \in A - B \\ G(e), & e \in B - A \\ F(e) \cup G(e), & e \in A \cap B \end{cases} \tag{1}$$

Definition 4 [6] Let τ be the collection of soft sets over U and let A be the nonempty set of parameters. Then τ is said to be a soft topology on U if the following conditions are satisfied:

(i): $(\emptyset, A)$, $(U, A) \in \tau$, where $\emptyset(a) = \emptyset$ and $U(a) = U$, for all $a \in A$.
(ii): The intersection of any two soft sets in τ belongs to τ.
(iii): The union of any number of soft sets in τ belongs to τ.

The triplet (U, A, τ) is called a soft topological space over U.

3. Near Soft Set and Near Soft Topology

In this section, we will obtain near soft set by combining two concepts soft set and near sets. We introduce the concept of a near soft sets and near soft topology and give some of their properties.

Definition 5 [14] Let $NAS = (\mathcal{O}, \mathcal{F}, \sim_{Br}, N_r, \upsilon_{N_r})$ be a nearness approximation space and $\sigma = (F, B)$ be a soft set over $\mathcal{O}$. The lower and upper near approximation of $\sigma = (F, B)$ with respect to NAS are denoted by $N_r{*}(\sigma) = (F_*, B)$ and $N_r^*(\sigma) = (F^*, B)$, which are soft sets over with the set-valued mappings given by

$F_*(\phi) = N_r{*}(F(\phi))$=$\cup\{x \in \mathcal{O} : [x]_{Br} \subseteq F(\phi)\}$ and

$F^*(\phi) = N_r^*(F(\phi))$=$\cup\{x \in \mathcal{O} : [x]_{Br} \cap F(\phi) \neq \emptyset\}$ where all $\phi \in B$. The operators $N_r{*}$ and N_r^* are called the lower and upper near approximation operators on soft sets, respectively. If $Bnd_{N_r(B)}(\sigma)0$, then the soft set σ is called a near soft set.

Definition 6 [14] Let $\mathcal{O}$ be an initial universe set, E be the universe set of parameters and $A, B \subseteq E$

(1) (K, A) is called a relative null near soft set (with respect to the parameters of A) if $K(\phi) = \emptyset$, for all $\phi \in A$.
(2) (W, B) is called a relative whole near soft set (with respect to the parameters of B) if $W(\phi) = \mathcal{O}$, for all $\phi \in B$.

Definition 7 [14] The relative complement of a near soft set (F, A) denoted by $(F, A)^c$, is defined as the near soft set (F^c, A) where $F^c(\phi) = \mathcal{O} - F(\phi)$ for all $\phi \in A$.

Definition 8 [14] (F,B) and (G,B) two near soft sets on $(\mathcal{O},B)$.(F,B) is defined to be near soft subset of (G,B) if $N_*(F,B) \subseteq N_*(G,B)$ for all $\phi \in B$,i.e., $N_*(F(\phi),B) \subseteq N_*(G(\phi),B)$ for all $\phi \in B$ and is denoted by $(F,B) \sqsubseteq (G,B)$.

Theorem 1 [14] Let (F,B) and (G,B) be two near soft sets on $(\mathcal{O},B)$. Then the following holds:

(1) $(F,B)^c \sqcap (G,B)^c = [(F,A) \sqcup (G,B)]^c$
(2) $((F,B) \sqcap (G,B))^c = (F,A)^c \sqcup (G,B)^c$.

Theorem 2 [14] Let (F_i,B) be a family of near soft sets on $(\mathcal{O},B)$.Then the following holds:

(1) $\sqcap_i (F_i,B)^c = (\sqcup_i (F_i,B))^c$
(2) $\sqcup_i (F_i,B)^c = (\sqcap_i (F_i,B))^c$.

Definition 9 [14] Let $\sigma = (F,B)$ be a near soft set over $\mathcal{O}$, τ be the collection of near soft subsets of σ, B is the nonempty set of parameters, and then $(\mathcal{O},B)$ is said to be a near soft topology on σ if the following conditions are met:

i): $(\emptyset,B)$, $(\mathcal{O},B) \in \tau$ where $\emptyset(\phi) = \emptyset$ and $F(\phi) = F$, for all $\phi \in B$.
ii): The intersection of any two near soft sets in τ belongs to τ.
iii): The union of any number of near soft sets in τ belongs to τ.

$(\mathcal{O},\tau,B)$ is called a near soft topological space.

Definition 10 [14] Let $(\mathcal{O},\tau,B)$ be a near soft topological space over $\mathcal{O}$. A near soft subset of $(\mathcal{O},B)$ is called near soft closed if its complement is open and a member of τ.

4. Near Soft İnterior, Near Soft Closure, Near Soft Open Set, Near Soft JP-Open Set

Definition 11 Let $(\mathcal{O},\tau,B)$ be a near soft topological space over $\mathcal{O}$ and (F,B) be a near soft set over $\mathcal{O}$. Then the near soft closure of (F,B), denoted by $cl(F,B)$ is the intersection of all near soft closed sets of (F,B). Clearly $cl(F,B)$ is the smallest near soft closed set over $\mathcal{O}$ which contains (F,B).

Definition 12 Let $(\mathcal{O},\tau,B)$ be a near soft topological space over $\mathcal{O}$ and (F,B) be a near soft set over $\mathcal{O}$. Then the near soft interior of (F,B), denoted by $int(F,B)$ is the collection of all near soft open super sets of (F,B). Clearly $int(F,B)$ is the biggest near soft open set over $\mathcal{O}$ which remains with in (F,B).

Proposition 1 Let $(\mathcal{O},\tau,B)$ be a near soft topological space over $\mathcal{O}$ and $(F,B),(G,B)$ be near soft sets over $\mathcal{O}$. Then

(1) $int(\emptyset) = \emptyset$ and $int(cl(\mathcal{O})) = cl(\mathcal{O})$
(2) $int(F,B) \widetilde{\subseteq} (F,B)$
(3) $int(int(F,B)) = (F,B)$
(4) (F,B) is a near soft open set if and only if $int(F,B) = (F,B)$
(5) $(F,B) \widetilde{\subseteq} (G,B)$ implies $int(F,B) \widetilde{\subseteq} int(G,B)$
(6) $cl(cl(F,B)) = cl(F,B)$
(7) $(F,B) \widetilde{\subseteq} (G,B)$ implies $cl(F,B) \widetilde{\subseteq} cl(G,B)$.

Proof. (1) and (2) are obvious.

(3) Since $int(F,B)$ is near soft open and $int(int(F,B))$ is the union of all near soft open subsets in X contained in $int(F,B)$. $int(F,B) \widetilde{\subseteq} int(int(F,B))$. Also $int(int(F,B)) \widetilde{\subseteq} int(F,B)$. Hence $int(int(F,B)) = (F,B)$.

(4) If (F,B) is a near soft open sets over $\mathcal{O}$ then (F,B) is a is a near soft open sets over X which contains (F,B). Thus $int(F,B)$ is the largest near soft open set in (F,B). Then $int(F,B)=(F,B)$.

(5) Let $(F,B)\tilde{\subseteq}(G,B)$. Since $int(F,B)\tilde{\subseteq}(F,B)\tilde{\subseteq}(G,B)$. $int(F,B)$ is a near soft open subset of (G,B). So $int(F,B)\tilde{\subseteq}int(G,B)$.

(6) and (7) can be made similarly.

Definition 13 A near soft set (F,B) in a near soft topological space $(\mathcal{O},\tau,B)$ is said to be

(1) near soft semi-open if $(F,B)\tilde{\subseteq}cl(int(F,B))$
(2) near soft semi-closed(scl) if $int(cl(F,B))\tilde{\subseteq}(F,B)$
(3) near soft pre-open if $(F,B)\tilde{\subseteq}int(cl(F,B))$
(4) near soft α-open if $(F,B)\tilde{\subseteq}int(cl(int(F,B)))$
(5) near soft β-open if $(F,B)\tilde{\subseteq}cl(int(cl(F,B)))$
(6) near soft β-closed if $int(cl(int(F,B)))\tilde{\subseteq}(F,B)$.
(7) near soft generalized closed(g-closed) set if $cl(F,B)\tilde{\subseteq}(U,B)$. whenever $(F,B)\tilde{\subseteq}(U,B)$ and (U,B) near soft open in $(\mathcal{O},\tau,B)$. The complement of a near soft g-closed set is called a near soft g-open set.
(8) near soft $\hat{g}$ closed set if $cl(F,B)\tilde{\subseteq}(U,B)$. whenever $(F,B)\tilde{\subseteq}(U,B)$ and (U,B) near soft semi-open in $(\mathcal{O},\tau,B)$. The complement of a near soft $\hat{g}$ -closed set is called a near soft $\hat{g}$ -open set.
(9) near soft clopen set if it is both near soft closed and near soft open set.
(10) near soft JP-closed set if $scl(F,B)\tilde{\subseteq}int(U,B)$ whenever $(A,B)\tilde{\subseteq}(U,B)$ and (U,B) near soft $\hat{g}-$open in $(\mathcal{O},\tau,B)$. The complement of a near soft JP-closed set is called a near soft JP-open set and their collection is denoted by $NSJPO(\mathcal{O},\tau,B)$.

The family of all near soft pre-open sets(resp. near soft β-open sets) in a near soft topological space $(\mathcal{O},\tau,B)$ will be denote by $NSPO(resp.\ NSBO)$

Example 2 Let $\mathcal{O}=\{x_1,x_2,x_3,x_4,x_5\}, B=\{\phi_1,\phi_2\}$ be denote a set of perceptual objects and a set of functions as in $[x_1]_{\phi_1}=\{x_1,x_4\},[x_2]_{\phi_1}=\{x_2,x_3,x_5\}$, $[x_1]_{\phi_2}=\{x_1,x_4\}$, $[x_2]_{\phi_2}=\{x_2,x_3\},[x_5]_{\phi_2}=\{x_5\}$, respectively.

$$\tau=\{(\emptyset,B),(\mathcal{O},B),(F_1,B),(F_2,B),(F_3,B),(F_4,B),(F_5,B)\} \tag{2}$$

where $(F_1,B),(F_2,B),(F_3,B),(F_4,B),(F_5,B)$ are near soft sets over $\mathcal{O}$ defined as follows:

$$\begin{aligned}
(F_1,B) &= \{(\phi_2,\{x_5\})\}\\
(F_2,B) &= \{(\phi_1,\{x_1\}),(\phi_2,\{x_2,x_3\})\}\\
(F_3,B) &= \{(\phi_1,\{x_1\}),(\phi_2,\{x_2,x_3,x_4\})\}\\
(F_4,B) &= \{(\phi_1,\{x_1\}),(\phi_2,\{x_2,x_3,x_5\})\}\\
(F_5,B) &= \{(\phi_1,\{x_1\}),(\phi_2,\{x_2,x_3,x_4,x_5\})\}
\end{aligned}$$

Then τ defines a near soft topology on $\mathcal{O}$ and hence $(\mathcal{O},\tau,B)$ is a near soft topological space over $\mathcal{O}$. Now we give $(H,B)=\{(\phi_1,\{x_1,x_4\})\}$ is a near soft pre-open set because $int(cl(H,B))=int(\mathcal{O},B)=(\mathcal{O},B)$ and $(H,B)\subset(\mathcal{O},B)$ but not a near soft α-open set because $int(H,B) = (\emptyset, B)$ and$(H,B)\sim int(cl(int(H,B)))$ also it is a near soft β-open set but not a near soft semi-open set can be viewed as similar.

5. Near Soft Continuous, Near Soft JP-Continuous

Definition 14 Let $(\mathcal{O}_X, \tau, B)$ and $(\mathcal{O}_Y, \tau^{'}, B)$ is a near soft topological spaces. A function $f : (\mathcal{O}_X, \tau, B) \to (\mathcal{O}_Y, \tau^{'}, B)$ is said to be

- ***:** near soft continuous if $f^{-1}((G,B))$ is near soft set in $(\mathcal{O}_X, \tau, B)$ for every near soft open set (G,B) of $(\mathcal{O}_Y, \tau^{'}, B)$.
- ***:** near soft semi continuous if $f^{-1}((G,B))$ is near soft semi-open in $(\mathcal{O}_X, \tau, B)$ for every near soft open set (G,B) of $(\mathcal{O}_Y, \tau^{'}, B)$.
- ***:** near soft pre-continuous if $f^{-1}((G,B))$ is near soft pre-open in $(\mathcal{O}_X, \tau, B)$ for every near soft open set (G,B) of $(\mathcal{O}_Y, \tau^{'}, B)$.
- ***:** near soft $\alpha-$continuous if $f^{-1}((G,B))$ is near soft α-open in $(\mathcal{O}_X, \tau, B)$ for every near soft open set (G,B) of $(\mathcal{O}_Y, \tau^{'}, B)$.
- ***:** near soft $\beta-$continuous if $f^{-1}((G,B))$ is near soft β-open in $(\mathcal{O}_X, \tau, B)$ for every near soft open set (G,B) of $(\mathcal{O}_Y, \tau^{'}, B)$.
- ***:** near soft strongly continuous if $f^{-1}((G,B))$ is near soft clopen set in $(\mathcal{O}_X, \tau, B)$ for every near soft open subset (G,B) of $(\mathcal{O}_Y, \tau^{'}, B)$.
- ***:** near soft perfectly continuous if $f^{-1}((G,B))$ is near soft clopen set in $(\mathcal{O}_X, \tau, B)$ for every near soft open set (G,B) of $(\mathcal{O}_Y, \tau^{'}, B)$.
- ***:** near soft JP continuous if $f^{-1}((G,B))$ is near soft JP open set in $(\mathcal{O}_X, \tau, B)$ for every near soft open set (G,B) of $(\mathcal{O}_Y, \tau^{'}, B)$.
- ***:** near soft strongly JP continuous if $f^{-1}((G,B))$ is near soft open set in $(\mathcal{O}_X, \tau, B)$ for every near soft JP open set (G,B) of $(\mathcal{O}_Y, \tau^{'}, B)$.

Example 3 Let $\mathcal{O}_X = \mathcal{O}_Y = \{x_1, x_2, x_3\}$, $B = \{\phi_1, \phi_2\}$ and let the near soft topology on $\mathcal{O}_X$ and $\mathcal{O}_Y$ respectively $\tau = \{(\emptyset, B), (\mathcal{O}_X, B), (F,B) = \{(\phi_1, \{x_1\}), (\phi_2, \{x_2\})\}\}$, $\tau^{'} = \{(\emptyset, B), (\mathcal{O}_X, B), (G,B) = \{(\phi_1, \{x_1\}), (\phi_2, \{x_3\})\}\}$. (F,B) and (G,B) are near soft open sets in $\mathcal{O}_X$ and $\mathcal{O}_Y$ respectively with

$$\begin{aligned} [x_1]_{\phi_1} &= \{x_1\}, \quad [x_2]_{\phi_1} = \{x_2, x_3\} \\ [x_1]_{\phi_2} &= \{x_1, x_2\}, \quad [x_3]_{\phi_2} = \{x_3\} \end{aligned}$$

If the mapping $f : (\mathcal{O}_X, \tau, B) \to (\mathcal{O}_Y, \tau^{'}, B)$ defined as $f(x_1) = x_1$, $f(x_2) = x_3$, $f(x_3) = x_2$ then

$$\begin{aligned} f^{-1}((G,B)) &= (F,B) \\ f^{-1}(\{(\phi_1, \{x_1\}), (\phi_2, \{x_3\})\}) &= \{(\phi_1, \{x_1\}), (\phi_2, \{x_2\})\} \end{aligned}$$

is a near soft open set in $\mathcal{O}_X$. Therefore f is a near soft continuous function.

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