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# EXACT SOLUTION OF BISWAS-MILOVIC EQUATION USING NEW EFFICIENT METHOD

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Abstract. Biswas and Milovic proposed a generalized model for the NLSE that accounts for several imperfections in the fiber during long distance transmission of these pulses. This paper studies the exact solution of the Biswas-Milovic equation with Kerr law nonlinearity by a new efficient method. The results reveal that the method is explicit, effective, and easy to use.

## 1. Introduction

Biswas-Milovic equation is a generalized version of the familiar nonlinear Schrödinger's equation describing the propagation of solitons through optical fibers for trans-continental and trans-oceanic distances. A broad class of analytical solution methods and numerical solution methods were used to handle these problems [1-23]. The nonlinear Schrödinger's equation is studied in various areas of Applied Mathematics, Theoretical Physics and Engineering. In particular, it appears in the study of Nonlinear Optics, Plasma Physics, Fluid Dynamics, Biochemistry and many other areas [1-43].
The Biswas-Milovic equation is given by [22, 23]:

$$i(q^m)_t + a(q^m)_{xx} + bF(|q|^2)q^m = 0,$$

where $q$ is a complex valued function, while $x$ and $t$ are the two independent variables. The coefficient $a$ and $b$ are constants where $ab > 0$, and parameter $1 \leq m < 2$.
In this paper we outline a reliable strategy of the new homotopy perturbation method for solving the case $m = 1$ of Biswas Milovic equation

$$\begin{aligned} &iq_t + aq_{xx} + bF(|q|^2)q = 0, \\ &q(x,0) = g(x). \end{aligned} \tag{1}$$

The Biswas-Milovic equation plays an important role in mathematical physics. It is very well known that the dynamics of these solitons propagate through these fibers for long distances in a matter of a few femto-seconds. This dynamics is governed by the nonlinear Schrdinger's equation (NLSE). The optical fiber industry

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has grown profoundly based on this technology. Recently, the homotopy perturbation method (HPM) introduced by He [24, 25] has been widely used for solving various integral equations arising from real world modelling, for example thin film flow, heat transfer, and many others [26-37]. The idea behind this method is that the solution is considered as the sum of an infinite series, which converges rapidly to the exact solution. In this paper we present a modified version of HPM and call it NHPM, which performs much better than the HPM. The new homotopy perturbation method (NHPM) was applied to linear and nonlinear PDEs and hyperbolic Telegraph equation. The rest of the paper is organized as follows. In section 2 we apply the method on the non-linear Biswas-Milovic equation. The results of numerical experiments are presented in section 3. Section 4 is dedicated to a brief conclusion. Finally some references are introduced at the end. The Biswas-Milovic equation plays an important role in mathematical physics. It is very well known that the dynamics of these solitons propagate through these fibers for long distances in a matter of a few femto-seconds. This dynamics is governed by the nonlinear Schrdinger's equation (NLSE). The optical fiber industry has grown profoundly based on this technology.

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The rest of the paper is organized as follows. In section 2 we apply the method on the non-linear Biswas-Milovic equation. The results of numerical experiments are presented in section 3. Section 4 is dedicated to a brief conclusion. Finally some references are introduced at the end.

## 2. Application of NHPM to Biswas-Milovic equation

For the purpose of applications illustration of the methodology of the proposed method, using new homotopy perturbation method, we consider the following differential equation

$$A(u(X,t) - f(r) = 0,\ r \in \Omega, \tag{2}$$

$$B(u(X,t), \partial u/\partial n) = 0,\ r \in \Gamma, \tag{3}$$

where $A$ is a general differential operator, $f(r)$ is a known analytic function, $B$ is a boundary condition, $\Gamma$ is the boundary of the domain $\Omega$, and $X = (x_1, x_2, \dots x_n)$. The operator $A$ can be generally divided into two operators, $L$ and $N$, where $L$ is a linear, while $N$ is a non-linear operator. Equation (2) can be written as follows:

$$L(u) + N(u) - f(r) = 0. \tag{4}$$

Using the homotopy technique, we construct a homotopy $U(r,p) : \Omega \times [0,1] \to \mathbb{R}$ which satisfies:

$$H(U,p) = (1-p)[L(U) - L(u_0)] + p[A(U) - f(r)] = 0, \quad p \in [0,1], \quad r \in \Omega, \tag{5}$$

or

$$H(U,p) = L(U) - L(u_0) + pL(u_0) + p[N(U) - f(r)] = 0, \tag{6}$$

where $p \in [0,1]$, is called homotopy parameter, and $u_0$ is an initial approximation for the solution of Eq. (1), which satisfies the boundary conditions. Obviously from Eqs. (5) and (6) we will have

$$H(U,0) = L(U) - L(u_0) = 0, \tag{7}$$

$$H(U,1) = A(U) - f(r) = 0. \tag{8}$$

We can assume that the solution of (5) or (6) can be expressed as a series in $p$, as follows:

$$U = U_0 + pU_1 + p^2U_2 + \cdots . \tag{9}$$

Setting $p = 1$, produces the approximate solution of Eq. (2), which could be written in the following form:

$$u = \lim_{p \to 1} U \ = U_0 + U_1 + U_2 + \ldots$$

Now let us write the Eq. (6) in the following form

$$L(U(X,t)) = u_0(X,t) + p[f(r(X,t)) - u_0(X,t) - N(U(X,t))]. \tag{10}$$

By applying the inverse operator, $L^{-1}$ to both sides of Eq. (10), we have

$$U(X,t) = L^{-1}(u_0(X,t)) + p\left(L^{-1}(f(r)) - L^{-1}(u_0(X,t) - L^{-1}(N(U(X,t)))\right). \tag{11}$$

Suppose that the initial approximation of Eq. (2) has the form

$$u_0(X,t) = \sum_{n=0}^{\infty} a_n(X)P_n(t), \tag{12}$$

where $a_1(X), a_2(X), a_3(X), \ldots$are unknown coefficients and $P_0(t), P_1(t), P_2(t), \ldots$are specific functions depending on the problem. Now by substituting (9) and (12) into the Eq. (11), we get

$$\begin{aligned}\sum_{n=0}^{\infty} p^n U_n(X,t) = U(X,t) = L^{-1}\left(\sum_{n=0}^{\infty} a_n(X)P_n(t)\right) \\ +p\left(L^{-1}(f(r)) - L^{-1}\left(\sum_{n=0}^{\infty} a_n(X)P_n(t)\right) - L^{-1}\left(N(\sum_{n=0}^{\infty} p^n U_n\,(X,t)\right)\right).\end{aligned} \tag{13}$$

Comparing coefficients of terms with the identical powers of $p$, leads to

$$\begin{array}{l} p^0 \;:\; U_0(X,t) = L^{-1}(\sum_{n=0}^{\infty} a_n(X)P_n(t)), \\ p^1 \;:\; U_1(X,t) = L^{-1}(f(r)) - L^{-1}(\sum_{n=0}^{\infty} a_n(X)P_n(t)) - L^{-1}N(U_0(X,t)), \\ p^2 \;:\; U_2(X,t) = -L^{-1}N(U_0(X,t), U_1(X,t)), \\ p^3 \;:\; U_3(X,t) = -L^{-1}N(U_0(X,t), U_1(X,t), U_2(X,t)), \\ \vdots \\ p^j \;:\; U_j(X,t) = -L^{-1}N(U_0(X,t), U_1(X,t), U_2(X,t), \ldots, U_{j-1}(X,t)), \\ \vdots \end{array} \tag{14}$$

Now if we solve these equations in such a way that $U_1(X,t) = 0$, then Eqs. (14) results in $U_1(X,t) = U_2(X,t) = \cdots = 0$.
Therefore the exact solution may be obtained as the following.

$$u(X,t) = U_0(X,t) = \sum_{n=0}^{\infty} a_n(X)P_n(t). \tag{15}$$

It is worthwhile to mention that if $f(r)$, and $u_0(X,t)$, are analytic at $t = t_0$, then their Taylor series defined as the following

$$\begin{array}{l} u_0(X,t) = \sum_{n=0}^{\infty} a_n(X)(t-t_0)^n, \\ f(r(X)) = \sum_{n=0}^{\infty} a_n^*(X)(t-t_0)^n, \end{array}$$

can be used in Eq. (13), where $a_1(X), a_2(X), a_3(X), \ldots$ are unknown coefficients and $a_1^*(X), a_2^*(X), a_3^*(X), \ldots$ are known ones, which should be computed.
To show the capability of the method, we apply the NHPM to some examples in the next section.

## 3. Illustrative examples

In this section we present some numerical results of our scheme for the nonlinear Biswas-Milovic [22, 23].
**Example 1.** Consider the non linear B-M equation

$$iq_t + q_{xx} + 2\left|q\right|^2 q = 0, \tag{16}$$

The initial conditions are given by

$$q(x,0) = e^{ix}.$$

To solve Eq. (16) by the NHPM, we construct the following homotopy:

$$\frac{\partial Q}{\partial t} = q_0 - p\left(q_0 - iQ_{xx} - 2i\left|Q\right|^2 Q\right). \tag{17}$$

Applying the inverse operator, $L^{-1} = \int_0^t (.)dt$ to the both sides of Eq. (17), we obtain

$$Q(x,t) = q(x,0) + \int_0^t q_0 dt - p\int_0^t \left(q_0 - iQ_{xx} - 2i\left|Q\right|^2 Q dt\right), \tag{18}$$

Suppose the solution of Eq. (18) to have the following form

$$Q = Q_0 + pQ_1 + p^2Q_2 + \ldots, \tag{19}$$

where $Q_i$are unknown functions which should be determined. Substituting Eq. (19) into Eq. (18), collecting the same powers of $p$, and equating each coefficient of $p$ to zero, results in

$$\begin{aligned}
p^0 &: \mathrm{Q}_0(x,t) = Q(x,0) + \textstyle\int_0^t q_0(x,t)dt,\\
p^1 &: \mathrm{Q}_1(x,t) = \textstyle\int_0^t \left(q_0 + iQ_{0xx} + 2i\left|Q_0\right|^2 Q_0\right)dt,\\
p^2 &: \mathrm{Q}_2(x,t) = \textstyle\int_0^t \left(iQ_{1xx} + 2i\left|Q_0\right|^2 Q_1 + 4i\left|Q_0\right|\left|Q_1\right| Q_0\right)dt,\\
&\vdots
\end{aligned}$$

Setting $p = 1$, results in the approximate solution of Eq. (19)

$$q = \lim_{p\to 1} Q_0 + pQ_1 + p^2 Q_2 + \ldots,$$

Assuming $q_0(x,t) = \sum_{n=0}^{\infty} a_n(x)t^k, Q(x,0) = q(x,0)$, and solving the above equation for $Q_1(x,t)$leads to the result

$$Q_1(x,t) = \left(a_0(x) - \frac{1}{2}ie^{ix}\right)t + \left(-\frac{1}{2}a_1(x) - \frac{1}{12}e^{ix}\right)t^2 + \cdots$$

Vanishing $Q_1(x,t)$ lets the coefficients $a_n(x)(n = 1,2,3,\cdots)$to take the following values

$$a_0(x) = \frac{1}{2}ie^{ix}, a_1(x) = -\frac{1}{6}e^{ix}, a_2(x) = -\frac{1}{24}ie^{ix}, \cdots$$

Therefore, the exact solutions of the Eq. (16) can be expressed as

$$\begin{aligned}
&q(x,t) = Q_0(x,t) = e^{ix} + a_0(x)t + \frac{1}{2}a_1(x)t^2 + \frac{1}{6}a_2(x)t^3\\
&+\frac{1}{12}a_3(x)t^4 + \frac{1}{20}a_4(x)t^5 + \cdots = e^{i(x+t)}.
\end{aligned}$$

**Example 2.** Consider the non-linear B-M equation

$$iq_t + q_{xx} + \left|q\right|^4 q = 0, \tag{20}$$

with the initial condition

$$q(x,0) = e^{ix}.$$

To solve Eq. (20) by the NHPM, we construct the following homotopy

$$\frac{\partial Q}{\partial t} = q_0 - p\left(q_0 - iQ_{xx} - i\left|Q\right|^4 Q\right). \tag{21}$$

Integrating of Eq. (21) leads to

$$Q(x,t) = q(x,0) + \int_0^t q_0 dt - p\int_0^t \left(q_0 - iQ_{xx} - i\left|Q\right|^4 Q dt\right), \tag{22}$$

Suppose the solutions of equation (19) have the form (19), substituting Eq. (19) into Eq. (22), collecting the terms with the same powers of $p$, and equating each coefficient of $p$ to zero, results in

$$\begin{aligned} p^0 &: \mathrm{Q}_0(x,t) = Q(x,0) + \textstyle\int_0^t q_0(x,t)dt, \\ p^1 &: \mathrm{Q}_1(x,t) = \textstyle\int_0^t \left( q_0 + iQ_{0xx} + i\left|Q_0\right|^4 Q_0 \right) dt, \\ p^2 &: \mathrm{Q}_2(x,t) = \textstyle\int_0^t \left( iQ_{1xx} + i\left|Q_0\right|^4 Q_1 + 4i\left|Q_0\right|^3 \left|Q_1\right| Q_0 \right) dt, \\ &\vdots \end{aligned}$$

Assuming $q_0(x,t) = \sum_{n=0}^{\infty} a_n(x)t^k, Q(x,0) = q(x,0)$, and we set the Taylor series of $Q_1(x,t)$ at $t = 0$, equal to zero, we have

$$Q_1(x,t) = (a_0(x))\, t + \left( -\frac{1}{2}a_1(x) - \frac{1}{12}a_0(x)e^{ix} \right) t^2 + \cdots$$

It can be easily shown that

$$a_0(x) = 0, a_1(x) = 0, a_2(x) = \frac{1}{2}x, a_3(x) = 0, \cdots.$$

Therefore we gain the solution of Eq. (20) as

$$\begin{aligned} q(x,t) = Q_0(x,t) &= e^{ix} + a_0(x)t + \frac{1}{2}a_1(x)t^2 + \frac{1}{6}a_2(x)t^3 \\ &+ \frac{1}{12}a_3(x)t^4 + \frac{1}{20}a_4(x)t^5 + \cdots = e^{ix}. \end{aligned}$$

**Example 3.** In this example we will apply the new method to handle the BME with parabolic law nonlinearity

$$iq_t + 4q_{xx} + 2\left( |q|^2 + |q|^4 \right) q = 0, \tag{23}$$

with the initial condition

$$q(x,0) = e^{-\frac{1}{2}ix}.$$

To solve Eq. (23) by the NHPM, we construct the following homotopy

$$\frac{\partial Q}{\partial t} = q_0 - p\left( q_0 - 4iQ_{xx} - 2i(|Q|^4 + |Q|^2)Q \right). \tag{24}$$

Integrating of Eq. (24) leads to

$$Q(x,t) = q(x,0) + \int_0^t q_0 dt + p\int_0^t \left( q_0 + 4iQ_{xx} + 2i(|Q|^4 + |Q|^2)Qdt \right), \tag{25}$$

Suppose the solutions of equation (23) have the form (19), substituting Eq. (19) into Eq. (25), collecting the terms with the same powers of $p$, and equating each coefficient of $p$ to zero, results in

$$\begin{aligned} p^0 &: \mathrm{Q}_0(x,t) = Q(x,0) + \textstyle\int_0^t q_0(x,t)dt, \\ p^1 &: \mathrm{Q}_1(x,t) = \textstyle\int_0^t \left( q_0 + 4iQ_{0xx} + 2i(\left|Q_0\right|^4 + \left|Q_0\right|^2)Q_0 \right) dt, \\ p^2 &: \mathrm{Q}_2(x,t) = \textstyle\int_0^t \left( 4iQ_{0xx} + 2i(\left|Q_0\right|^4 + \left|Q_0\right|^2)Q_1 + 2i(4\left|Q_0\right|^3\left|Q_1\right| + 2\left|Q_0\right|\left|Q_1\right|)Q_0 \right) dt, \\ &\vdots \end{aligned}$$

Assuming $q_0(x,t) = \sum_{n=0}^{\infty} a_n(x)t^k, Q(x,0) = q(x,0)$, and we set the Taylor series of $Q_1(x,t)$at $t = 0$,equal to zero, we have

$$Q_1(x,t) = \left( a_0(x) - \frac{1}{2}ie^{-\frac{1}{2}ix} \right) t + \left( -\frac{1}{2}a_1(x) - \frac{1}{12}e^{-\frac{1}{2}ix} \right) t^2 + \cdots$$

It can be easily shown that

$$a_0(x) = \frac{1}{2}ie^{-\frac{1}{2}ix}, a_1(x) = -\frac{1}{6}e^{-\frac{1}{2}ix}, a_2(x) = -\frac{1}{24}ie^{-\frac{1}{2}ix}, \cdots .$$

Therefore we gain the solution of Eq. (23) as

$$q(x,t) = Q_0(x,t) = e^{-\frac{1}{2}ix} + a_0(x)t + \frac{1}{2}a_1(x)t^2 + \frac{1}{6}a_2(x)t^3$$
$$+\frac{1}{12}a_3(x)t^4 + \frac{1}{20}a_4(x)t^5 + \cdots = e^{-\frac{1}{2}ix+it}.$$

## 4. Conclusion

In this work, we have used a new homotopy perturbation method for finding solutions of the Biswas-Milovic equation. All the examples show that the NHPM is a powerful mathematical tool for solving Biswas-Milovic equation. It is also a promising method for solving other nonlinear equations. It should be mentioned that we have successfully applied the NHPM to a wide variety of initial-type differential equations of hyperbolic and system of differential equations and we obtained the exact solutions of these systems. Therefore, we believe that new method is a promising technique in finding the exact solutions for a wide variety of mathematical problems. The computations associated with the examples in this article were performed using Maple 13.

## References


[1] Qin Zhou, Qiuping Zhu, Luminita Moraru , Anjan Biswas, Optical solitons in photonic cryatal fibers with spatially inhomogeneous nonlinearities, Optoelectronics and Advanced Materials – Rapid Communications. 16, 995-997, 2014.

[2] Q. Zhou, Q. Zhu, Y. Liu, P. Yao, A. H. Bhrawy, L. Moraru , A. Biswas, Bright-Dark combo optical solitons with non-local nonlinearity in parabolic law medium, Optoelectronics and Advanced Materials-Rapid Communications. 8, 837-839, 2014.

[3] G. Ebadi, Aida Mojavir, Jose-Vega Guzman, Kaisar R. Khan, Mohammad F. Mahmood, Luminita Moraru, Anjan Biswas, Milivoj Belic, Solitons in optical metamaterials by F-expansion scheme, Optoelectronics and Advanced Materials-Rapid Communications, 8, 828-832, 2014.

[4] Q. Zhou, Q. Zhu, A.H. Bhrawy, L. Moraru, A. Biswas, Optical solitons with spatially-dependent coefficients by Lie symmetry, Optoelectronics and Advanced Materials-Rapid Communications. 8, 800-803, 2014.

[5] Qin Zhou, Qiuping Zhu, Yaxian Liu, Anjan Biswas, Ali H. Bhrawy, Kaisar R. Khan, Mohammad F. Mahmood, Milivoj Belic, Solitons in optical metamaterials with parabolic law nonlinearity and spatio-temporal dispersion, Journal of Optoelectronics and Advanced Materials. 16, 1221-1225, (2014).

[6] Jose Vega-Guzman, A. A. Alshaery, E. M. Hilal, A. H. Bhrawy, M. F. Mahmood, Luminita Moraru, Anjan Biswas, Optical soliton perturbation in magneto-opticwaveguides with spatio dispersion, Journal of Optoelectronics and Advanced Materials. 16, 1063-1070, 2014.

[7] A. A. Alshaery, E. M. Hilal, M. A. Banaja, Sadah A. Alkhateeb, Luminita Moraru, Anjan Biswas, Optical solitons in multiple-core couplers, Journal of Optoelectronics and Advanced Materials. 16, 750-758. 2014.

[8] ] A. Biswas, Temporal 1-soliton solution of the complex Ginzburg-Landau equation with power law nonlinearity, Progress in Electromagnetics Research, 96, 1–7, 2009.

[9] M. Savescu, K. R. Khan, R. W. Kohl, L. Moraru, A. Yildirim, A. Biswas, Optical soliton perturbation with improved nonlinaer schrodinger's equation in nanofibers, J. Nanoelectronics and Optoelectronics, 8, 208–220, 2013.

[10] Q. Zhou, Q. Zhu, A. Biswas, Optical solitons in birefringent fibers with parabolic law nonlinearity, Optica Applicata, 41, 399–409, 2014.

[11] Q. Zhou, Q. Zhu, Y. Liu, H. Yu, P. Yao, A. Biswas, Thirring optical solitons in birefringent fibers with spatio-temporal dispersion and Kerr law nonlinearity, Laser Physics, 25 (1): 015402, 2015.

[12] A. Biswas, D. Milovic, M. Savescu, M. F. Mahmood, K. R. Khan, R. Kohl, Optical solitons perturbation in nanofibers with improved nonlinear schrodinger's equation by semi-inverse variational principle, Journal of Nonlinear Optical Physics and Materials, 12, 1250054, 2012.

[13] H. Triki, S. Crutcher, A. Yildirim, T. Hayat, Omar. M. Aldossary, A. Biswas, Bright and dark solitons of the modified complex Ginzburg-Landau equation with parabolic and dual-power law nonlinearity, Romanian Reports in Physics, 64, 367–370, 2012.

[14] A. L. Fabian, R. Kohl, A. Biswas, Perturbation of topological solitons due to sine-gordon equation and its type, Commun. Nonlinear Sci. Numer. Simul., 14, 1227–1244, 2009.

[15] Anjan Biswas, Chenwi Zony, Essaid Zerrad, Soliton perturbation theory for the quadratic nonlinear Klein-Gordon equation, 203, 153-156. 2008.

[16] Houria Triki, Ahmet Yildirim, T. Hayat, Omar M. Aldossary, Anjan Biswas, Shock wave solution of the Benney-Luke equation. Romanian Journal of Physics. 57, 1029-1034. 2012.

[17] Engin Topkara, Daniela Milovic, Amarendra K. Sarma, Essaid Zerrad, Anjan Biswas, Optical solitons with nonkerr law nonlinearity and inter-modal dispersion with time-dependent coefficients, Communications in Nonlinear Science and Numerical Simulation, 15, 2320-2330, 2010.

[18] Anjan Biswas & Daniela Milovic, Bright and dark solitons of the generalized nonlinear Schrdinger's equation, Communications in Nonlinear Science and Numerical Simulation. 15, 1473-1484. 2010.

[19] Anjan Biswas, Topological 1-soliton solution of the nonlinear s nonlinear Schrdinger's equation with kerr law nonlinearity in 1+2 dimensions, Communications in Nonlinear Science and Numerical Simulation. 14, 2845-2847, 2009.

[20] Engin Topkara, Daniela Milovic, Amarendra K. Sarma, Fayequa Majid & Anjan Biswas, A study of optical solitons with kerr and power law nonlinearities by he's variational principle, Journal of the European Optical Society. Volume 4, 09050. (2009).

[21] Ryan Sassaman & Anjan Biswas, Topological and non-topological solutions of the Klein-gordon equations in 1+2 dimensions, Nonlinear Dynamics. 61, 23-28, 2010.

[22] Russell Kohl, Anjan Biswas, Daniela Milovic & Essaid Zerrad , Optical Soliton perturbation in a nonlinear law media. Optics and Laser Technology. 40, 4,(2008)647-662

[23] Russell Kohl, Ramazan Tinaztepe, Abhinandan Chowdhury , Soliton perturbation theory of Biswas-Milovic equation, Optik - International Journal for Light and Electron Optics125, 8, (2014)1926-1936.

[24] J. H. He, Homotopy perturbation technique, Computer Methods in Applied Mechanics and Engineering 178 (1999) 257–262.

[25] J.H. He. A coupling method of homotopy technique and perturbation technique for nonlinear problems, International Journal of Non-Linear Mechanics, 2000, 35 (1): 37-43.

[26] M. Dehghan and F. Shakeri, Use of He's homotpy perturbation method for solving a partial differential equation arising in modeling of flow in porous media, J. Porous Media 11 (2008), 765–778.

[27] J. Biazar, H. Ghazvini, M. Eslami, He's homotopy perturbation method for systems of integro-differential equations, Chaos, Solitons and Fractals, 2009, 39: 1253-1258.

[28] Z. Odibat and S. Momani, Modified homotopy perturbation method: application to quadratic Riccati differential equation of fractional order, Chaos Solitons Fractals, in press.

[29] D.D. Ganji, The application of He's homotopy perturbation method to nonlinear equations arising in heat transfer,Phys. Lett. A 355 (2006), pp. 337–341.

[30] L. Cveticanin, Homotopy–perturbation method for pure nonlinear differential equation, Chaos Solitons Fractals 30(2006), pp. 1221–1230.

[31] A. Belndez, T. Belndez, A. Mrquez, and C. Neipp, Application of He's homotopy perturbation method to conservative truly nonlinear oscillators, Chaos Solitons Fractals 37 (3) (2008), pp. 770–780.

[32] S. Abbasbandy, Numerical solutions of the integral equations: Homotopy perturbation method and Adomian's decomposition method, Appl. Math. Comput. 173 (2006), pp. 493–500.

[33] A. Golbabai and M. Javidi, Application of homotopy perturbation method for solving eighth-order boundary value problems, Appl. Math. Comput. 191 (1) (2007), pp. 334–346.

[34] M.A. Rana, A.M. Siddiqui, Q.K. Ghori. Application of He's homotopy perturbation method to Sumudu transform, International Journal of Nonlinear Science and Numerical Simulation, 2007, 8 (2): 185-190.

[35] T. Ozis, A. Yildirim. Traveling wave solution of Korteweg-de Vries equation using He's homotopy perturbation method, International Journal of Nonlinear Science and Numerical Simulation, 2007, 8 (2): 239-242.

[36] Yildirim A. Application of He's homotopy perturbation method for solving the Cauchy reaction–diffusion problem. Computers and Mathematics with Applications 2009; 57 (4):612–618.

[37] M.Eslami, J.Biazar, Analytical solution Klein–Gordon equation by new homotopy perturbation method, Computational Mathematics and Modeling 25 (1) 2014 124-134.

[38] M Mirzazadeh, M Eslami, Exact solutions for nonlinear variants of Kadomtsev–Petviashvili (n, n) equation using functional variable method, Pramana, 81 (6), 911-924, 2013.

[39] N. Taghizadeh, M. Mirzazadeh, A.S. Paghaleh, Exact solutions for the nonlinear Schrodinger equation with power law nonlinearity, Math. Sci. Lett. 1 (1), 7-16, 2012.

[40] M. Mirzazadeh, M. Eslami, A.H. Arnous, Dark optical solitons of Biswas-Milovic equation with dual-power law nonlinearity, The European Physical Journal Plus, 130, 1-7, 2015.

[41] M Eslami, M Mirzazadeh, Exact solutions for fifth-order KdV-type equations with time-dependent coefficients using the Kudryashov method, The European Physical Journal Plus, 129, 1-6, 2014.

[42] N. Taghizadeh, M. Mirzazadeh, M. Eslami, M. Moradi, Exact travelling wave solutions for some complex nonlinear partial differential equations, Computational Methods for Differential Equations, 1, 169-177, 2013.

[43] M. Mirzazadeh, The extended homogeneous balance method and exact 1-soliton solutions of the Maccari system, Computational Methods for Differential Equations, 2, 83-90, 2014.

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