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(G'/G) AND EXTENDED (G'/G) METHODS FOR SOLVING THE NONLINEAR REACTION-DIFFUSION EQUATION AND KDVB EQUATION

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ABSTRACT. In this paper, we construct the travelling wave solutions to two nonlinear models of physical phenomenas. The first is the nonlinear reaction-diffusion equation with a reaction term while the second is the Korteweg de Vries Burger equation (KdVB). Based on (G'/G)-expansion method, we obtain abundant exact travelling wave solutions of the two equations with arbitrary parameters. The travelling wave solutions are expressed by the hyperbolic functions, trigonometric functions and rational forms. It is shown that the extended (G'/G)-expansion method is a powerful and concise mathematical tool for solving nonlinear partial differential equations. The crucial advantage of extended method over the basic (G'/G)-expansion method is that extended method provide more general and abundant new exact traveling wave solutions with many real parameters. The exact solutions of PDEs have its vital significant to disclose the internal mechanism of the complex physical phenomena.

1. Introduction

Nonlinear evolution equations (NLEEs) are widely used to describe many important phenomenas and dynamic processes in physics, mechanics, chemistry, biology, etc. The investigation of exact solutions of NLEEs plays an important role in the study of nonlinear physical phenomena. There has been a great amount of activity aiming to find methods for solutions of NLEEs [1]- [10]. Many approaches to NLEEs have been proposed, for example, the variational iteration method [11], the homotopy perturbation method [12], tanh function method [13], the F-expansion method [14], the sine-cosine method [15], Hirota method [16] and Jacobi elliptic function method [17]. Recently, Wang et al. [18] proposed the (G'/G)-expansion method and showed that it is powerful for finding analytic solutions of PDEs. Next, Bekir [19] applied the method to some nonlinear evolution equations gaining traveling wave solutions. Later, Zhang et al. [20] have generalized the method to obtain non-traveling wave solutions and coefficient function solutions and Zhang et al.

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[21] further extended the method to solve an evolution equation with variable coefficients. A. Biswas et al apply the method to some nonlinear evolution equations [22, 23].

Guo and Zhou [24] first proposed the extended (G'/G) expansion method based on new ansatz. They applied the method to the Whitham-Broer-Kaup-Like equations and couple Hirota-Satsuma KdV equations. Zhang et al. [25] presented an improved (G'/G) expansion method for solving nonlinear evolution equations NLEEs. Li et al. [26] introduced $(G'/G, 1/G)$-expansion method to obtain traveling wave solutions of the Zakharov equations. Hayek [27] proposed extended (G'/G)-expansion method for constructing exact solutions to the KdV and Burgers equations with power-law nonlinearity.

In this paper, our aim is to investigate abundant new exact traveling wave solutions of the nonlinear reaction-diffusions equation and KdVB by equation using the (G'/G) expansion method and extended (G'/G) expansion method.

2. Description of the (G'/G)-expansion method

We suppose that the given nonlinear partial differential equation for u(x,t) to be in the form

$$N(u, u_t, u_x, u_{tt}, u_{xx}, u_{tx}, ...) = 0 \tag{1}$$

The essence of the (G'/G) expansion method can be presented in the following steps:

1. Seek traveling wave solutions of (1) by taking $u(x, t) = u(\xi), \xi = x - ct$ and transform (1) to the corresponding ordinary differential equation of the form,

$$N(u, -cu', u', c^2u'', u'', -cu'',) = 0 \tag{2}$$

2. If possible, integrate (2) term by term one or more times. This yields constant(s) of integration. For simplicity, the integration constant(s) can be set to zero.

3. Introduce the solution $u(\xi)$ of (2) in the finite series form

$$u(\xi) = \sum_{i=0}^{M} a_i (\frac{G'}{G})^i \tag{3}$$

where a_i are real constants with $a_M \neq 0, M$ is a positive integer to be determined. In the extended (G'/G) method, the finite series (3) extend to the following form,

$$u(\xi) = \sum_{i=-M}^{M} a_i (\frac{G'}{G})^i \tag{4}$$

The function $G(\xi)$ is the solution of the auxiliary linear ordinary differential equation

$$G'' + \lambda G' + \mu G = 0 \tag{5}$$

where λ and μ are real constants to be determined.

4. Determine M. This, usually, can be accomplished by balancing the linear term(s) of highest order with the highest order nonlinear term(s) in (2).

5. Substituting (3) or (4) together with (5) into (2) yields an algebraic equation involving powers of (G'/G). Equating the coefficients of each power of (G'/G) to zero gives a system of algebraic equations for a_i, λ, μ and c. Then, we solve the system with the aid of a computer algebra system (CAS), such as Mathematica,

to determine these constants. On the other hand, depending on the sign of the discriminant$\Delta = \lambda^2 - 4\mu$, the solutions of (5) have been well known. So, we have,

$$(\frac{G'}{G}) = \begin{cases} \frac{-\lambda}{2} + (\frac{\sqrt{\lambda^2-4\mu}}{2})\frac{k_1 sinh(\frac{(\sqrt{\lambda^2-4\mu})\xi}{2})+k_2 cosh(\frac{(\sqrt{\lambda^2-4\mu})\xi}{2})}{k_1 cosh(\frac{(\sqrt{\lambda^2-4\mu})\xi}{2})+k_2 sinh(\frac{(\sqrt{\lambda^2-4\mu})\xi}{2})}, & \Delta > 0; \\ \frac{-\lambda}{2} + (\frac{\sqrt{\lambda^2-4\mu}}{2})\frac{-k_1 sin(\frac{(\sqrt{\lambda^2-4\mu})\xi}{2})+k_2 cos(\frac{(\sqrt{\lambda^2-4\mu})\xi}{2})}{k_1 cos(\frac{(\sqrt{\lambda^2-4\mu})\xi}{2})+k_2 sin(\frac{(\sqrt{\lambda^2-4\mu})\xi}{2})}, & \Delta < 0; \\ \frac{-\lambda}{2} + \frac{k_1}{k_1\xi+k_2}, & \Delta = 0. \end{cases} \quad (6)$$

3. Applications

In this section, we will demonstrate the (G'/G)-expansion method and extended (G'/G)-expansion method on two of the well-known nonlinear evolution equations, namely, the nonlinear reaction-diffusion equation with reaction term and 1D - KdVB equation.

3.1. **The nonlinear reaction-diffusion equation with reaction term.** An example of practical interest is known as the nonlinear reactions-diffusions equation with a reaction term [14, 28]. In general one can assume for such classes of equations the form of a conservation equation in three space dimensions so that

$$(\frac{\partial u}{\partial t}) = -\nabla \overrightarrow{F} + f(\overrightarrow{x}, t, u) \quad (7)$$

where $\overrightarrow{F}$ is a general flux transport due to diffusion and $f(\overrightarrow{x}, t, u)$ is the source or reaction term. For the case of general diffusion problems we can take $\overrightarrow{F} = -k\nabla u$, so that(7) becomes

$$\frac{\partial u}{\partial t} = \nabla(k\nabla u) + f(\overrightarrow{x}, t, u) \quad (8)$$

and $k = k(\overrightarrow{x}, u)$ is a function of x and u. For the case of several chemicals or interacting species, the vector $u_i(\overrightarrow{x}, t), i = 1, \cdots, n$ represents concentrations or densities each diffusing with its own diffusion coefficient k_i and interacting according to the vector source term $(\overrightarrow{x}, t, u)$. In the one dimensional case with f is a function of u only, we can write (8) as

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}(k(u)\frac{\partial u}{\partial x}) + f(u) \quad (9)$$

If we set $k(u) = 2u$ and the function $f(u) = pu - qu^2$, where p and q are constants, we obtain the following evolution equation

$$u_t - (u^2)_{xx} = pu - qu^2, u = u(x,t). \quad (10)$$

Now we apply the solution steps to (10), setting $u(x,t) = f(\xi), \xi = x - ct$, we obtain

$$-cf' - (f^2)'' - pf + qf^2 = 0 \quad (11)$$

The balancing number, $M = -1$, and hence a transformation is necessary. Let $f = V^{-1}$, then (11) transforms to

$$2VV'' - pV^3 + V^2(cV' + q) - 6(V')^2 = 0 \quad (12)$$

3.1.1. *Solution of reaction-diffusion equation using (G'/G) method.* Balancing VV'' and V^2V', we have, $2M+2=3M+1$, then $M=1$, in this case we have

$$V(\xi)=a_0+a_1(\frac{G'}{G}) \tag{13}$$

$$V'=-a_1(\mu+\lambda(\frac{G'}{G})+(\frac{G'}{G})^2), \tag{14}$$

$$V''=a_1(\lambda\mu+(2\mu+\lambda^2)(\frac{G'}{G})+3\lambda(\frac{G'}{G})^2+2(\frac{G'}{G})^3), \tag{15}$$

Substituting (13) - (15) into (12), setting the coefficients of $(G'/G)^i, i=0,1,\cdots,4$ to zero, we obtain the following system of algebraic equations for a_0, a_1, λ, μ , c:

$$(\frac{G'}{G})^0=-6\mu^2a_1^2+2\lambda\mu a_0a_1-pa_0^3+a_0^2(q-c\mu a_1)=0,$$
$$(\frac{G'}{G})^1=(2q+2\lambda^2+4\mu)a_0a_1-a_1a_0^2(3p+c\lambda)-10\lambda\mu a_1^2-2c\mu a_0a_1^2=0,$$
$$(\frac{G'}{G})^2=6\lambda a_0a_1-ca_1a_0^2+qa_1^2-a_1^2(4\lambda^2+8\mu+3pa_0+2c\lambda a_0)-c\mu a_1^3=0,$$
$$(\frac{G'}{G})^3=4a_1a_0-a_1^2(6\lambda+2ca_0)-a_1^3(p+\lambda c)=0,$$
$$(\frac{G'}{G})^4=-2a_1^2-ca_1^3=0.$$

Solving the above system with the aid of Mathematica, we have the following two sets of solutions:

First set:

$$a_1=-2\frac{\sqrt{q}}{p}, c=\frac{p}{\sqrt{q}}, \lambda=\frac{q-2pa_0}{2\sqrt{q}}, \mu=\frac{-pqa_0+p^2a_0^2}{4q} \tag{16}$$

From (16), it is clear that a_0 is free parameter and $\lambda^2-4\mu=\frac{q}{4}, q>0$ substituting in (6), we obtain

$$\frac{G'}{G}=\frac{2pa_0-q}{4\sqrt{q}}+(\frac{\sqrt{q}}{4})\frac{k_1\sinh(\frac{\sqrt{q}}{4})(x-\frac{pt}{\sqrt{q}})+k_2\cosh(\frac{\sqrt{q}}{4})(x-\frac{pt}{\sqrt{q}})}{k_1\cosh(\frac{\sqrt{q}}{4})(x-\frac{pt}{\sqrt{q}})+k_2\sinh(\frac{\sqrt{q}}{4})(x-\frac{pt}{\sqrt{q}})}$$

substitute in (13), we have

$$V=a_0-(2\frac{\sqrt{q}}{p})(\frac{2pa_0-q}{4\sqrt{q}}+(\frac{\sqrt{q}}{4})\frac{k_1\sinh(\frac{\sqrt{q}}{4})(x-\frac{pt}{\sqrt{q}})+k_2\cosh(\frac{\sqrt{q}}{4})(x-\frac{pt}{\sqrt{q}})}{k_1\cosh(\frac{\sqrt{q}}{4})(x-\frac{pt}{\sqrt{q}})+k_2\sinh(\frac{\sqrt{q}}{4})(x-\frac{pt}{\sqrt{q}})}) \tag{17}$$

If we set $p=q=1$, this leads to $c=1, a_1=-2$ and choosing $a_0=2$, then we have

$$V=(\frac{1}{2})(1-\frac{k_1\sinh\frac{x-t}{4}+k_2\cosh\frac{x-t}{4}}{k_1\cosh\frac{x-t}{4}+k_2\sinh\frac{x-t}{4}}),$$

finally, the solution, $u(\xi)=1/V(\xi)$ then,

$$u=2\frac{k_1\cosh\frac{x-t}{4}+k_2\sinh\frac{x-t}{4}}{(k_1-k_2)(\cosh\frac{x-t}{4}-\sinh\frac{x-t}{4})}$$
$$=2\frac{e^{\frac{x-t}{4}}}{k_1-k_2}(k_1\cosh\frac{x-t}{4}+k_2\sinh\frac{x-t}{4})$$

where k_1 and k_2 are arbitrary constants, $k_1 \neq k_2$.
Second set:

$$a_1 = 2\frac{\sqrt{q}}{p}, c = -\frac{p}{\sqrt{q}}, \lambda = \frac{-q + 2pa_0}{2\sqrt{q}}, \mu = \frac{-pqa_0 + p^2a_0^2}{4q} \tag{18}$$

from (18) substitute in (6), we obtain

$$\frac{G'}{G} = \frac{q - 2pa_0}{4\sqrt{q}} + (\frac{\sqrt{q}}{4})\frac{k_1 \sinh(\frac{\sqrt{q}}{4})(x + \frac{pt}{\sqrt{q}}) + k_2 \cosh(\frac{\sqrt{q}}{4})(x + \frac{pt}{\sqrt{q}})}{k_1 \cosh(\frac{\sqrt{q}}{4})(x + \frac{pt}{\sqrt{q}}) + k_2 \sinh(\frac{\sqrt{q}}{4})(x + \frac{pt}{\sqrt{q}})}$$

From (13), we have

$$V = a_0 + 2\frac{\sqrt{q}}{p}(\frac{q - 2pa_0}{4\sqrt{q}} + (\frac{\sqrt{q}}{4})\frac{k_1 \sinh(\frac{\sqrt{q}}{4})(x + \frac{pt}{\sqrt{q}}) + k_2 \cosh(\frac{\sqrt{q}}{4})(x + \frac{pt}{\sqrt{q}})}{k_1 \cosh(\frac{\sqrt{q}}{4})(x + \frac{pt}{\sqrt{q}}) + k_2 \sinh(\frac{\sqrt{q}}{4})(x + \frac{pt}{\sqrt{q}})}) \tag{19}$$

Setting in (19), $p = q = 1$, this leads to $c = -1, a_1 = 2$ and choosing$a_0 = 2$, then we have

$$V = (\frac{1}{2})(1 + \frac{k_1 \sinh \frac{x+t}{4} + k_2 \cosh \frac{x+t}{4}}{k_1 \cosh \frac{x+t}{4} + k_2 \sinh \frac{x+t}{4}}),$$

finally, the solution is,

$$u = 2(e^{-\frac{x+t}{4}})\frac{k_1 \cosh \frac{x+t}{4} + k_2 \sinh \frac{x+t}{4}}{(k_1 + k_2)}, k_1 + k_2 \neq 0.$$

3.1.2. *Solution of reaction-diffusion equation using extended (G'/G) method.* In this subsection we will use the extended (G'/G) method to solve (13), we have

$$V = a_{-1}(\frac{G'}{G})^{-1} + a_0 + a_1(\frac{G'}{G}) \tag{20}$$

$$V' = a_{-1}(1 + \lambda(\frac{G'}{G})^{-1} + \mu(\frac{G'}{G})^{-2}) - a_1(\mu + \lambda(\frac{G'}{G}) + (\frac{G'}{G})^2), \tag{21}$$

$$\begin{aligned} V'' = a_{-1}(2\mu^2(\frac{G'}{G})^{-3} + 3\lambda\mu(\frac{G'}{G})^{-2} + (2\mu + \lambda^2)(\frac{G'}{G})^{-1}) + (\lambda\mu a_1 + \lambda a_{-1}) \\ + a_1((2\mu + \lambda^2)(\frac{G'}{G}) + 3\lambda(\frac{G'}{G})^2 + 2(\frac{G'}{G})^3), \end{aligned} \tag{22}$$

Substituting (20) - (22) into (12), setting the coefficients of $(G'/G)^i, i = -4, -3 \cdots, 4$ to zero, we obtain the following system of algebraic equations for $a_0, a_1, a_{-1}, \lambda, \mu$

and c:

$$(\frac{G'}{G})^{-4} = -2\mu^2 a_{-1}^2 + c\mu a_{-1}^3 = 0,$$

$$(\frac{G'}{G})^{-3} = (-6\lambda\mu + 2c\mu a_0)a_{-1}^2 + (-p + c\lambda)a_{-1}^3 + 4\mu^2 a_{-1}a_0 = 0,$$

$$(\frac{G'}{G})^{-2} = (2q - 4\lambda^2 - 8\mu + ca_{-1})a_{-1}^2 + ((2c\lambda - 3p)a_{-1} + 6\lambda\mu + c\mu a_0)a_{-1}a_0$$
$$- c\mu a_1 a_{-1}^2 + 16\mu^2 a_{-1}a_1 = 0,$$

$$(\frac{G'}{G})^{-1} = -10\lambda a_{-1}^2 + (2q + 2\lambda^2 + 4\mu + 2ca_{-1} + (c\lambda - 3p)a_0)a_{-1}a_0 - c\mu a_1 a_{-1}^2$$
$$+ 16\mu^2 a_1 a_{-1} = 0,$$

$$(\frac{G'}{G})^{0} = -6\mu^2 a_1^2 + 2\lambda\mu a_0 a_1 - pa_0^3 + a_0^2(q - c\mu a_1) = 0,$$

$$(\frac{G'}{G})^{1} = 32\lambda a_1 a_{-1} + (2q + 2\lambda^2 + 4\mu - 3pa_0 - c\lambda a_0)a_1 a_0 - 10\lambda\mu$$
$$a_1^2 - (3p + c\lambda)a_{-1}a_1^2 - 2c\mu a_0 a_1^2 = 0,$$

$$(\frac{G'}{G})^{2} = 16a_1 a_{-1} + 6\lambda a_1 a_0 - ca_1 a_0^2 - a_1^2(4\lambda^2 + ca_{-1} + 8\mu + 3pa_0 + 2c\lambda a_0 - q) - c\mu a_1^3 = 0,$$

$$(\frac{G'}{G})^{3} = 4a_1 a_0 - a_1^2(6\lambda + 2ca_0) - a_1^3(p + \lambda c) = 0,$$

$$(\frac{G'}{G})^{4} = -2a_1^2 - ca_1^3 = 0.$$

Solving the above system of equations, we have a huge number of solution sets, we select of them the following sets of solutions:
Set a:

$$\mu = \frac{-q}{64}, c = \frac{-p}{\sqrt{q}}, a_0 = \frac{q}{2p}, a_1 = 2\frac{\sqrt{q}}{p}, a{-}1 = \frac{\sqrt{q^3}}{32p}, \lambda = 0. \tag{23}$$

From (23), we deduce that $\lambda^2 - 4\mu = \frac{q}{64}, q > 0$, substatuite in (6), we obtain:

$$(\frac{G'}{G}) = \frac{\sqrt{q}}{8}\frac{k_1 \sinh\frac{\sqrt{q}\xi}{8} + k_2 \cosh\frac{\sqrt{q}\xi}{8}}{k_1 \cosh\frac{\sqrt{q}\xi}{8} + k_2 \sinh\frac{\sqrt{q}\xi}{8}},$$

and,

$$V = \frac{q}{2p} + \frac{q}{4p}(\frac{k_1 \sinh\frac{\sqrt{q}\xi}{8} + k_2 \cosh\frac{\sqrt{q}\xi}{8}}{k_1 \cosh\frac{\sqrt{q}\xi}{8} + k_2 \sinh\frac{\sqrt{q}\xi}{8}} + \frac{k_1 \cosh\frac{\sqrt{q}\xi}{8} + k_2 \sinh\frac{\sqrt{q}\xi}{8}}{k_1 \sinh\frac{\sqrt{q}\xi}{8} + k_2 \cosh\frac{\sqrt{q}\xi}{8}}) \tag{24}$$

For simplicity we set $p = q = 1$, then we have, $c = -1$ and (24) transformed to

$$V = \frac{1}{2} + \frac{1}{4}(\frac{k_1 \sinh\frac{x+t}{8} + k_2 \cosh\frac{x+t}{8}}{k_1 \cosh\frac{x+t}{8} + k_2 \sinh\frac{x+t}{8}} + \frac{k_1 \cosh\frac{x+t}{8} + k_2 \sinh\frac{x+t}{8}}{k_1 \sinh\frac{x+t}{8} + k_2 \cosh\frac{x+t}{8}})$$
$$= \frac{1}{2} + (\frac{(k_1^2 + k_2^2)\cosh\frac{x+t}{4} + 2k_1 k_2 \sinh\frac{x+t}{4}}{2(k_1^2 + k_2^2)\sinh\frac{x+t}{4} + 4k_1 k_2 \cosh\frac{x+t}{4}})$$
$$= \frac{1}{2}(\frac{(k_1 + k_2)^2\cosh\frac{x+t}{4} + \sinh\frac{x+t}{4}}{(k_1^2 + k_2^2)\sinh\frac{x+t}{4} + 2k_1 k_2 \cosh\frac{x+t}{4}})$$

$$V = \frac{1}{2}\frac{(k_1+k_2)^2 e^{\frac{x+t}{4}}}{(k_1^2+k_2^2)\sinh\frac{x+t}{4} + 2k_1k_2\cosh\frac{x+t}{4}}$$

thus,

$$u = \frac{2e^{\frac{-(x+t)}{4}}}{(k_1+k_2)^2}(\sinh\frac{x+t}{4} + 2k_1k_2\cosh\frac{x+t}{4})$$

Set b:

$$\mu = \frac{-11q}{338}, c = \frac{p}{\sqrt{q}}, a_0 = \frac{11q}{13p}, a_{-1} = 0, a_1 = -2\frac{\sqrt{q}}{p}, \lambda = \frac{-9\sqrt{q}}{26}. \qquad (25)$$

Equation (25) leads to $\lambda^2 - 4\mu = \frac{q}{4} > 0$, then we have

$$(\frac{G'}{G}) = \frac{9\sqrt{q}}{52} + \frac{\sqrt{q}}{4}\frac{k_1\sinh(\frac{\sqrt{q}}{4}(x-\frac{pt}{\sqrt{q}})) + k_2\cosh(\frac{\sqrt{q}}{4}(x-\frac{pt}{\sqrt{q}}))}{k_1\cosh(\frac{\sqrt{q}}{4}(x-\frac{pt}{\sqrt{q}})) + k_2\sinh(\frac{\sqrt{q}}{4}(x-\frac{pt}{\sqrt{q}}))}$$

thus,

$$\begin{aligned} V &= \frac{q}{2p}(1 - \frac{k_1\sinh(\frac{\sqrt{q}}{4}(x-\frac{pt}{\sqrt{q}})) + k_2\cosh(\frac{\sqrt{q}}{4}(x-\frac{pt}{\sqrt{q}}))}{k_1\cosh(\frac{\sqrt{q}}{4}(x-\frac{pt}{\sqrt{q}})) + k_2\sinh(\frac{\sqrt{q}}{4}(x-\frac{pt}{\sqrt{q}}))}) \\ &= \frac{q}{2p}(\frac{(k_1-k_2)e^{\frac{-\sqrt{q}}{4}(x-\frac{pt}{\sqrt{q}})}}{k_1\cosh(\frac{\sqrt{q}}{4}(x-\frac{pt}{\sqrt{q}})) + k_2\sinh(\frac{\sqrt{q}}{4}(x-\frac{pt}{\sqrt{q}}))}) \end{aligned}$$

The solution takes the form,

$$u = \frac{2pe^{\frac{-\sqrt{q}}{4}(x-\frac{pt}{\sqrt{q}})}}{q(k_1-k_2)}(k_1\cosh(\frac{\sqrt{q}}{4}(x-\frac{pt}{\sqrt{q}})) + k_2\sinh(\frac{\sqrt{q}}{4}(x-\frac{pt}{\sqrt{q}})))$$

Set c:

$$a_1 = 2\frac{\sqrt{q}}{p}, c = -\frac{p}{\sqrt{q}}, \lambda = \frac{-q+2pa_0}{2\sqrt{q}}, \mu = \frac{-pqa_0 + p^2a_0^2}{4q} \qquad (26)$$

Equation (26) is exactly the same as (18) then we have the same solution.

Set d:

$$a_1 = -2\frac{\sqrt{q}}{p}, c = \frac{p}{\sqrt{q}}, \lambda = \frac{q-2pa_0}{2\sqrt{q}}, \mu = \frac{-pqa_0 + p^2a_0^2}{4q} \qquad (27)$$

Equation (27) is exactly the same as (16) then we have the same solution.

Set e:

$$a_1 = 0, c = \frac{2p}{\sqrt{q}}, \lambda = \frac{\sqrt{q}}{2}, \mu = 0, a_0 = \frac{2a_{-1}}{\sqrt{q}}. \qquad (28)$$

Equation (28) implies that,

$$(\frac{G'}{G}) = \frac{-\sqrt{q}}{4} + \frac{\sqrt{q}}{4}\frac{k_1\sinh(\frac{\sqrt{q}}{4}(x-\frac{2pt}{\sqrt{q}})) + k_2\cosh(\frac{\sqrt{q}}{4}(x-\frac{2pt}{\sqrt{q}}))}{k_1\cosh(\frac{\sqrt{q}}{4}(x-\frac{2pt}{\sqrt{q}})) + k_2\sinh(\frac{\sqrt{q}}{4}(x-\frac{2pt}{\sqrt{q}}))}$$

we obtain,

$$V=\frac{2a_{-1}}{\sqrt{q}}+\frac{4a_{-1}e^{\frac{\sqrt{q}}{4}(x-\frac{2pt}{\sqrt{q}})}(k_1-k_2)}{k_1\cosh(\frac{\sqrt{q}}{4}(x-\frac{2pt}{\sqrt{q}}))+k_2\sinh(\frac{\sqrt{q}}{4}(x-\frac{2pt}{\sqrt{q}}))}$$

the traveling wave solution takes the form,

$$u=\frac{1}{2a_{-1}}(\frac{(k_2-k_1)\sqrt{q}}{(k_2-k_1)+2e^{\frac{\sqrt{q}}{4}(x-\frac{2pt}{\sqrt{q}})}(k_1\cosh(\frac{\sqrt{q}}{4}(x-\frac{2pt}{\sqrt{q}}))+k_2\sinh(\frac{\sqrt{q}}{4}(x-\frac{2pt}{\sqrt{q}})))})$$

where $a_{-1}\neq 0, k_2\neq k_1$.

3.2. **Korteweg de Vries Burger equation (KdVB).** A second instructive model is the following 1D-KdVB equation [29] - [32]in the form,

$$u_t+\varepsilon uu_x+\nu u_{xx}+\alpha u_{xxx}=0, \tag{29}$$

where ε,ν and α are parameters. Step1, leads to the following ODE,

$$-cu'+\frac{\varepsilon}{2}(u^2)'+\nu u''+\alpha u'''=0, \tag{30}$$

integrate (30), we have

$$-cu+\frac{\varepsilon}{2}u^2+\nu u'+\alpha u''=0, \tag{31}$$

3.2.1. *Solution of KdVB equation using G'/G method.* Balancing the nonlinear term u^2 with u'', we obtain $M=2$, so we have:

$$u=a_0+a_1(\frac{G'}{G})+a_2(\frac{G'}{G})^2 \tag{32}$$

$$u'=-2a_2(\frac{G'}{G})^3-(a_1+2a_2\lambda)(\frac{G'}{G})^2-(a_1\lambda+2a_2\mu)(\frac{G'}{G})-\mu a_1, \tag{33}$$

$$u''=6a_2(\frac{G'}{G})^4+(2a_1+10a_2\lambda)(\frac{G'}{G})^3+(8a_2\mu+3a_1\lambda+4a_2\lambda^2)(\frac{G'}{G})^2 \\ +(6a_2\mu\lambda+2a_1\mu+a_1\lambda^2)(\frac{G'}{G})+(\lambda\mu a_1+2a_2\mu^2). \tag{34}$$

Substituting (32) - (34)into (31), setting the coefficients of $(G'/G)^i, i=0,1,\cdots,4$ to zero, we obtain the following system of algebraic equations for a_0,a_1,a_2,λ,μ and c:

$$(\frac{G'}{G})^0=(-\mu\nu+\mu\alpha\lambda)a_1+2\alpha\mu^2a_2-ca_0+\frac{\varepsilon}{2}a_0^2=0,$$
$$(\frac{G'}{G})^1=(-c+\alpha\lambda^2+2\alpha\mu-\lambda\nu+\varepsilon a_0)a_1+(6\alpha\lambda-2\nu)\mu a_2=0,$$
$$(\frac{G'}{G})^2=(3\alpha\lambda-\nu+\frac{\varepsilon}{2}a_1)a_1+(-c+4\alpha\lambda^2+8\alpha\mu-2\lambda\nu+\varepsilon a_0)a_2=0,$$
$$(\frac{G'}{G})^3=2\alpha a_1+(10\alpha\lambda-2\nu+\varepsilon a_1)a_2=0,$$
$$(\frac{G'}{G})^4=6\alpha a_2+\frac{\varepsilon}{2}a_2^2=0.$$

Solving the above system with the aid of Mathematica, we have the following two sets of solutions:
Set 1:

$$a_0 = \frac{-\varepsilon a_1^2}{48\alpha}, a_2 = \frac{-12\alpha}{\varepsilon}, c = \frac{-6\nu^2}{25\alpha}, \lambda = \frac{12\nu - 5\varepsilon a_1}{60\alpha}, \mu = \frac{-24\varepsilon\nu a_1 + 5\varepsilon^2 a_1^2}{2880\alpha^2}. \quad (35)$$

Equation (35) shows that a_1 is a free parameter and $\lambda^2 - 4\mu = \frac{\nu^2}{25\alpha^2} > 0$ substituting in (6), we obtain

$$(\frac{G'}{G}) = \frac{-12\nu + 5\varepsilon a_1}{120\alpha} + \frac{\nu}{10\alpha}\frac{k_1\sinh(\frac{\nu}{10\alpha}(x+\frac{6\nu^2 t}{25\alpha})) + k_2\cosh(\frac{\nu}{10\alpha}(x+\frac{6\nu^2 t}{25\alpha}))}{k_1\cosh(\frac{\nu}{10\alpha}(x+\frac{6\nu^2 t}{25\alpha})) + k_2\sinh(\frac{\nu}{10\alpha}(x+\frac{6\nu^2 t}{25\alpha}))}, \quad (36)$$

Choosing $a_1 = 0$, then we have, $a_0 = 0$ and $\lambda = \frac{\nu}{5\alpha}$ and then (36) transformed to

$$(\frac{G'}{G}) = \frac{-\nu}{10\alpha} + \frac{\nu}{10\alpha}\frac{k_1\sinh(\frac{\nu}{10\alpha}(x+\frac{6\nu^2 t}{25\alpha})) + k_2\cosh(\frac{\nu}{10\alpha}(x+\frac{6\nu^2 t}{25\alpha}))}{k_1\cosh(\frac{\nu}{10\alpha}(x+\frac{6\nu^2 t}{25\alpha})) + k_2\sinh(\frac{\nu}{10\alpha}(x+\frac{6\nu^2 t}{25\alpha}))}, \quad (37)$$

$$= \frac{\nu}{10\alpha}\frac{(k_2 - k_1)e^{-(x+\frac{6\nu^2 t}{25\alpha})}}{k_1\cosh(\frac{\nu}{10\alpha}(x+\frac{6\nu^2 t}{25\alpha})) + k_2\sinh(\frac{\nu}{10\alpha}(x+\frac{6\nu^2 t}{25\alpha}))},$$

$$u(x,t) = \frac{-3}{25\varepsilon\alpha}(\frac{\nu(k_2 - k_1)e^{-(x+\frac{6\nu^2 t}{25\alpha})}}{k_1\cosh(\frac{\nu}{10\alpha}(x+\frac{6\nu^2 t}{25\alpha})) + k_2\sinh(\frac{\nu}{10\alpha}(x+\frac{6\nu^2 t}{25\alpha}))})^2, \quad (38)$$

Set 2:

$$a_0 = \frac{576\nu^2 - 25\varepsilon a_1^2}{1200\alpha\varepsilon}, a_2 = \frac{-12\alpha}{\varepsilon}, c = \frac{6\nu^2}{25\alpha}, \lambda = \frac{12\nu - 5\varepsilon a_1}{60\alpha}, \mu = \frac{-24\varepsilon\nu a_1 + 5\varepsilon^2 a_1^2}{2880\alpha^2}. \quad (39)$$

Similarly,

$$(\frac{G'}{G}) = \frac{-12\nu + 5\varepsilon a_1}{120\alpha} + \frac{\nu}{10\alpha}\frac{k_1\sinh(\frac{\nu}{10\alpha}(x-\frac{6\nu^2 t}{25\alpha})) + k_2\cosh(\frac{\nu}{10\alpha}(x-\frac{6\nu^2 t}{25\alpha}))}{k_1\cosh(\frac{\nu}{10\alpha}(x-\frac{6\nu^2 t}{25\alpha})) + k_2\sinh(\frac{\nu}{10\alpha}(x-\frac{6\nu^2 t}{25\alpha}))}, \quad (40)$$

if we choose $a_1 = 0$, then $a_0 = \frac{12\nu^2}{25\alpha\varepsilon}, \lambda = \frac{\nu}{5\alpha}$ so

$$(\frac{G'}{G}) = \frac{\nu}{10\alpha}\frac{(k_2 - k_1)e^{-(x-\frac{6\nu^2 t}{25\alpha})}}{k_1\cosh(\frac{\nu}{10\alpha}(x-\frac{6\nu^2 t}{25\alpha})) + k_2\sinh(\frac{\nu}{10\alpha}(x-\frac{6\nu^2 t}{25\alpha}))}, \quad (41)$$

substituting (39), (41) in (32), we have

$$u(x,t) = \frac{12\nu^2}{25\alpha\varepsilon} - \frac{3\nu^2}{25\varepsilon\alpha}(\frac{(k_2 - k_1)e^{-(x-\frac{6\nu^2 t}{25\alpha})}}{k_1\cosh(\frac{\nu}{10\alpha}(x-\frac{6\nu^2 t}{25\alpha})) + k_2\sinh(\frac{\nu}{10\alpha}(x-\frac{6\nu^2 t}{25\alpha}))})^2, \quad (42)$$

3.2.2. *Extended (G'/G)-expansion method applied to KdVB equation.* The extended (G'/G)-expansion method when applied to KdVB equation transform (4) to

$$u = a_{-2}(\frac{G'}{G})^{-2} + a_{-1}(\frac{G'}{G})^{-1} + a_0 + a_1(\frac{G'}{G}) + a_2(\frac{G'}{G})^2 \tag{43}$$

Substituting (43) and its derivatives into (31), setting the coefficients of $(G'/G)^i, i = -4, -3, \cdots, 4$ to zero, we obtain the following sets of solutions:

Set 1:

$$a_0 = \frac{3\nu^2}{10\alpha\varepsilon}, a_2 = \frac{-12\alpha}{\varepsilon}, c = \frac{6\nu^2}{25\alpha}, a_{-1} = \frac{3\nu^3}{500\alpha^2\varepsilon},$$
$$a_{-2} = \frac{-3\nu^4}{40000\alpha^3\varepsilon}, \lambda = 0, \mu = \frac{-\nu^2}{400\alpha^2}. \tag{44}$$

From (6), we have

$$(\frac{G'}{G}) = \frac{\nu}{20\alpha}\frac{k_1\sinh(\frac{\nu}{20\alpha}(x-\frac{6\nu^2 t}{25\alpha})) + k_2\cosh(\frac{\nu}{20\alpha}(x-\frac{6\nu^2 t}{25\alpha}))}{k_1\cosh(\frac{\nu}{20\alpha}(x-\frac{6\nu^2 t}{25\alpha})) + k_2\sinh(\frac{\nu}{20\alpha}(x-\frac{6\nu^2 t}{25\alpha}))}, \tag{45}$$

The solution can be obtained from the direct substitution in (43) from (44) and (45) as follows:

$$u = \frac{3\nu^2}{5\alpha\varepsilon}(\frac{1}{2} + \frac{H}{5} - \frac{H^2}{20} + \frac{H^{-1}}{5} - \frac{H^{-2}}{20}) \tag{46}$$

where,

$$H = \frac{k_1\sinh(\frac{\nu}{20\alpha}(x-\frac{6\nu^2 t}{25\alpha})) + k_2\cosh(\frac{\nu}{20\alpha}(x-\frac{6\nu^2 t}{25\alpha}))}{k_1\cosh(\frac{\nu}{20\alpha}(x-\frac{6\nu^2 t}{25\alpha})) + k_2\sinh(\frac{\nu}{20\alpha}(x-\frac{6\nu^2 t}{25\alpha}))}.$$

Set 2:

$$a_0 = a_{-1} = a_{-2} = \mu = 0, a_2 = \frac{-12\alpha}{\varepsilon}, a_1 = \frac{24\nu}{5\varepsilon}, c = \frac{6\nu^2}{25\alpha}, \lambda = \frac{-\nu}{5\alpha}. \tag{47}$$

From (6), we have

$$(\frac{G'}{G}) = \frac{\nu}{10\alpha}\frac{(k_2+k_1)e^{\frac{\nu}{10\alpha}(x-\frac{6\nu^2 t}{25\alpha})}}{k_1\cosh(\frac{\nu}{10\alpha}(x-\frac{6\nu^2 t}{25\alpha})) + k_2\sinh(\frac{\nu}{10\alpha}(x-\frac{6\nu^2 t}{25\alpha}))}, \tag{48}$$

Substituting (47) and (48) into (43), we have

$$u = \frac{3\nu^2}{25\alpha\varepsilon}(4D - D^2) \tag{49}$$

where,

$$D = \frac{(k_2+k_1)e^{\frac{\nu}{10\alpha}(x-\frac{6\nu^2 t}{25\alpha})}}{k_1\cosh(\frac{\nu}{10\alpha}(x-\frac{6\nu^2 t}{25\alpha})) + k_2\sinh(\frac{\nu}{10\alpha}(x-\frac{6\nu^2 t}{25\alpha}))}.$$

Set 3:

$$a_0 = \frac{-9\nu^2}{50\alpha\varepsilon}, a_2 = \frac{-12\alpha}{\varepsilon}, c = \frac{-6\nu^2}{25\alpha}, a_{-1} = \frac{3\nu^3}{500\alpha^2\varepsilon},$$
$$a_1 = \frac{12\nu}{5\varepsilon}, a_{-2} = \frac{-3\nu^4}{40000\alpha^3\varepsilon}, \lambda = 0, \mu = \frac{-\nu^2}{400\alpha^2}. \tag{50}$$

Similarly the solution takes the form,

$$u = \frac{3\nu^2}{25\alpha\varepsilon}(\frac{-3}{2} + A - \frac{A^2}{4} + \frac{A^{-1}}{4} - \frac{A^{-2}}{4}) \tag{51}$$

where,

$$A = \frac{k_1 \sinh(\frac{\nu}{20\alpha}(x + \frac{6\nu^2 t}{25\alpha})) + k_2 \cosh(\frac{\nu}{20\alpha}(x + \frac{6\nu^2 t}{25\alpha}))}{k_1 \cosh(\frac{\nu}{20\alpha}(x + \frac{6\nu^2 t}{25\alpha})) + k_2 \sinh(\frac{\nu}{20\alpha}(x + \frac{6\nu^2 t}{25\alpha}))}.$$

Set 4:

$$a_0 = a_{-1} = a_{-2} = a_1 = \mu = 0, a_2 = \frac{-12\alpha}{\varepsilon}, c = \frac{-6\nu^2}{25\alpha}, \lambda = \frac{\nu}{5\alpha}. \tag{52}$$

$$(\frac{G'}{G}) = \frac{\nu}{10\alpha} \frac{(k_2 - k_1)e^{-}(x + \frac{6\nu^2 t}{25\alpha})}{k_1 \cosh(\frac{\nu}{10\alpha}(x + \frac{6\nu^2 t}{25\alpha})) + k_2 \sinh(\frac{\nu}{10\alpha}(x + \frac{6\nu^2 t}{25\alpha}))},$$

The solution is,

$$u = \frac{-3\nu^2}{25\alpha\varepsilon} \frac{(k_2 - k_1)e^{-}(x + \frac{6\nu^2 t}{25\alpha})}{k_1 \cosh(\frac{\nu}{10\alpha}(x + \frac{6\nu^2 t}{25\alpha})) + k_2 \sinh(\frac{\nu}{10\alpha}(x + \frac{6\nu^2 t}{25\alpha}))}$$

Set 5:

$$a_0 = \frac{-3(25\alpha^2\lambda^2 + 10\alpha\lambda\nu - \nu^2)}{25\alpha\varepsilon}, a_{-2} = \frac{-3(25\alpha^2\lambda^2 - \nu^2)}{2500\alpha^3\varepsilon}, a_1 = a_2 = 0,$$
$$a_{-1} = \frac{-3(5\lambda\alpha - \nu)(5\lambda\alpha + \nu)^2}{125\alpha^2\varepsilon}, c = \frac{6\nu^2}{25\alpha}, \mu = \frac{25\alpha^2\lambda^2 - \nu^2}{100\alpha^2}. \tag{53}$$

So,

$$(\frac{G'}{G}) = \frac{-\lambda}{2} + \frac{\nu}{10\alpha} \frac{k_1 \sinh(\frac{\nu}{10\alpha}(x - \frac{6\nu^2 t}{25\alpha})) + k_2 \cosh(\frac{\nu}{10\alpha}(x - \frac{6\nu^2 t}{25\alpha}))}{k_1 \cosh(\frac{\nu}{10\alpha}(x - \frac{6\nu^2 t}{25\alpha})) + k_2 \sinh(\frac{\nu}{10\alpha}(x - \frac{6\nu^2 t}{25\alpha}))} \tag{54}$$

It is clear that λ is a free parameter. The solution takes the form,

$$\frac{u}{-3} = \frac{25\alpha^2\lambda^2 + 10\alpha\lambda\nu - \nu^2}{25\alpha\varepsilon} - \frac{(5\lambda\alpha - \nu)(5\lambda\alpha + \nu)^2}{125\alpha^2\varepsilon}(\frac{G'}{G})^{-1} - \frac{25\alpha^2\lambda^2 - \nu^2}{2500\alpha^3\varepsilon}(\frac{G'}{G})^{-2}$$

Set 6:

$$a_0 = \frac{-3(25\alpha^2\lambda^2 + 10\alpha\lambda\nu - 3\nu^2)}{25\alpha\varepsilon}, a_{-2} = 0 = a_{-1} = 0, a_1 = \frac{-12(5\lambda\alpha - \nu)}{(5\varepsilon)}$$
$$, c = \frac{6\nu^2}{25\alpha}, \mu = \frac{25\alpha^2\lambda^2 - \nu^2}{100\alpha^2}. \tag{55}$$

We have,

$$(\frac{G'}{G}) = \frac{-\lambda}{2} + \frac{\nu}{10\alpha} \frac{k_1 \sinh(\frac{\nu}{10\alpha}(x - \frac{6\nu^2 t}{25\alpha})) + k_2 \cosh(\frac{\nu}{10\alpha}(x - \frac{6\nu^2 t}{25\alpha}))}{k_1 \cosh(\frac{\nu}{10\alpha}(x - \frac{6\nu^2 t}{25\alpha})) + k_2 \sinh(\frac{\nu}{10\alpha}(x - \frac{6\nu^2 t}{25\alpha}))}, \tag{56}$$

$$u = -3\frac{(25\alpha^2\lambda^2 + 10\alpha\lambda\nu - 3\nu^2)}{25\alpha\varepsilon} - 12\frac{(5\lambda\alpha - \nu)}{5\varepsilon}(\frac{G'}{G}) - \frac{12\alpha}{\varepsilon}(\frac{G'}{G})^2$$

Set 7:

$$a_0 = \frac{-3(25\alpha^2\lambda^2 + 10\alpha\lambda\nu - 3\nu^2)}{25\alpha\varepsilon}, a_{-2} = a_{-1} = 0, a_1 = \frac{-12(5\lambda\alpha - \nu)}{(5\varepsilon)}$$
$$, c = \frac{-6\nu^2}{25\alpha}, \mu = \frac{25\alpha^2\lambda^2 - \nu^2}{100\alpha^2}. \quad (57)$$

From (57), we obtain,

$$(\frac{G'}{G}) = \frac{-\lambda}{2} + \frac{\nu}{10\alpha}\frac{k_1\sinh(\frac{\nu}{10\alpha}(x + \frac{6\nu^2 t}{25\alpha})) + k_2\cosh(\frac{\nu}{10\alpha}(x + \frac{6\nu^2 t}{25\alpha}))}{k_1\cosh(\frac{\nu}{10\alpha}(x + \frac{6\nu^2 t}{25\alpha})) + k_2\sinh(\frac{\nu}{10\alpha}(x + \frac{6\nu^2 t}{25\alpha}))}, \quad (58)$$

$$u = -3\frac{(25\alpha^2\lambda^2 + 10\alpha\lambda\nu - 3\nu^2)}{25\alpha\varepsilon} - 12\frac{(5\lambda\alpha - \nu)}{5\varepsilon}(\frac{G'}{G}) - \frac{12\alpha}{\varepsilon}(\frac{G'}{G})^2$$

Set 8:

$$a_0 = \frac{-12\nu^2}{25\alpha\varepsilon}, a_{-2} = a_{-1} = 0, a_1 = \frac{24\nu}{5\varepsilon},$$
$$, c = \frac{-6\nu^2}{25\alpha}, a_2 = \frac{-12\alpha}{\varepsilon}, \mu = 0, \lambda = \frac{-\nu}{5\alpha}. \quad (59)$$

The obtained set of solutions, results to

$$u = \frac{-12\nu^2}{25\alpha\varepsilon} + \frac{12\nu^2}{25\alpha\varepsilon}B - \frac{3\nu^2}{25\alpha\varepsilon}B^2, \quad (60)$$

where,

$$B = \frac{(k_1 + k_2)e^{\frac{\nu}{10\alpha}(x + \frac{6\nu^2 t}{25\alpha})}}{k_1\cosh(\frac{\nu}{10\alpha}(x + \frac{6\nu^2 t}{25\alpha})) + k_2\sinh(\frac{\nu}{10\alpha}(x + \frac{6\nu^2 t}{25\alpha}))} \quad (61)$$

Set 9:

$$a_0 = \frac{12\nu^2}{25\alpha\varepsilon}, a_{-2} = a_{-1} = a_1 = \mu = 0, c = \frac{6\nu^2}{25\alpha}, a_2 = \frac{-12\alpha}{\varepsilon}, \lambda = \frac{\nu}{5\alpha}. \quad (62)$$

we have,

$$\frac{G'}{G} = \frac{\nu}{10\alpha}\frac{(k_2 - k_1)e^{\frac{-\nu}{10\alpha}(x - \frac{6\nu^2 t}{25\alpha})}}{k_1\cosh(\frac{\nu}{10\alpha}(x - \frac{6\nu^2 t}{25\alpha})) + k_2\sinh(\frac{\nu}{10\alpha}(x - \frac{6\nu^2 t}{25\alpha}))} \quad (63)$$

the final solution is,

$$u = \frac{12\nu^2}{25\alpha\varepsilon} - \frac{3\nu^2}{25\alpha\varepsilon}(\frac{(k_2 - k_1)e^{-\frac{\nu}{10\alpha}(x - \frac{6\nu^2 t}{25\alpha})}}{k_1\cosh(\frac{\nu}{10\alpha}(x - \frac{6\nu^2 t}{25\alpha})) + k_2\sinh(\frac{\nu}{10\alpha}(x - \frac{6\nu^2 t}{25\alpha}))})$$

4. Conclusion

The study shows that (G'/G)-expansion method is quite efficient and practically well suited for use in calculating traveling wave solutions for the nonlinear reaction-diffusion equation with a reaction term and the Korteweg de Vries Burger equation (KdVB). The reliability of the method and the reduction in the size of computational domain give this method a wider applicability. The main advantage of the extended (G'/G)-expansion method over the regular (G'/G)-expansion method is that the method offers more general and huge amount of new exact traveling wave solutions with some free parameters. All the solutions obtained by the regular (G'/G)-expansion method are obtained by the extended method, and in addition we obtain some new solutions. Outside the elemental application, the exact solutions of nonlinear evolution equations aid the numerical solvers to assess the correctness of their results and assist them in the stability analysis.

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