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SHEHU TRANSFORM ADOMIAN DECOMPOSITION METHOD FOR THE SOLUTION OF SYSTEMS OF INTEGER AND FRACTIONAL ORDER DIFFERENTIAL EQUATIONS

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Abstract. This paper is concerned with the solution of system of nonlinear fractional and integer order ordinary and partial differential equations. To achieve that aim, a method of solution is proposed which is developed from an integral transform and the well-known Adomian decomposition method. The Shehu transform Adomian decomposition method (STADM) proposed leverage on the unique advantage that Shehu transform, unlike Laplace transform, is applicable to both constant and variable coefficient problems. The nonlinearity in all its forms is handled by developing corresponding Adomian polynomials, while the fractional order derivatives are interpreted in Caputo sense. The proposed method is applied to some problems from the literature and in most cases gives the exact solutions. The results are equally presented in 3D graphs for ease of visualization.

1. Literature Review

Most phenomena in nature are described by nonlinear differential equations, majority of which defy analytical methods of solution. Scientists therefore came up with a class of methods called semi analytical methods. This family of methods produces exact solutions whenever such exist in closed form. In the event that the problem lacks exact solution, the truncated series obtained through the methods gives better numerical approximation than the most accurate numerical methods. One of such methods is Adomian Decomposition Method (ADM) that was first introduced by an American mathematician Gorge Adomian ([1], [2]). The method has been used to handle linear and nonlinear algebraic, differential, integral, integro-differential, delay-differential and partial differential equations ([4],[18],[19]). The modifications of ADM have acquired a lot of remarkable results and have been

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applied to various kinds of higher order ordinary and partial differential equations and integral equations ([20], [21]). The method has been widely used for a class of deterministic and stochastic problems in scientific research fields [21].
Integral transforms are commonly used to convert a function to another in expectation to simplify computations. Integral transforms are used in solving ordinary differential equations (ODEs), partial differential equations (PDEs), and fractional differential equations (FDEs), one of the well known integral transforms is the Laplace transform. The limitations of this transform necessitated the development of some new transforms such as Shehu transform, Sumudu transform and so on ([3],[14],[17]).
In spite of the tremendous advantages derived in the results obtained when physical phenomena are modelled into integer order differential equations, there still exist a lot of real life situations that can hardly be effectively represented mathematically except through fractional order differential and integral equations. In recent years, mathematicians have used the Adomian decomposition method with various integral transforms such as Laplace transform, Shehu transform, Samudu transform and so on ([11],[12],[16]). For instance, [18] developed Shehu transform Adomian decomposition method (STADM) algorithm and applied it to solve some linear and nonlinear integral and integro-differential problems. ([7],[8],[9],[13]) solved system of fractional order differential equations using Laplace Adomian decomposition method (LDM) and modified Laplace Adomian decomposition method (MLDM).
The present work presents a method of solution that leveraged on the advantage that Shehu transform is applicable to both constant and variable coefficients differential equations, unlike Laplace transform. Thus, Shehu transform is combined with Adomian decomposition method for the solutions of systems of both integer and fractional order partial and ordinary differential equations.

2. Fractional calculus

The fractional calculus unifies and generalizes the notions of classical calculus ([5],[6],[10],[16]). Fractional calculus is almost as old as calculus itself, and has attracted the attention of researchers in the field of mathematical physics, mathematical biology and mathematical analysis because of the precision it brings when physical problems are modelled with it. There are several definitions associated with fractional calculus. The definitions adopted in this research are discussed in the sequel.

2.1. **The Riemann-Liouville Integral and Derivative.**
The Riemann-Liouville integral of order $\xi \geq 0$ for continuous function g on $[a, b]$ is defined by

$$J^{\xi} g(t) = \frac{1}{\Gamma(\xi)} \int_0^t (t-\tau)^{\xi-1} g(\tau) d\tau, \quad \xi > 0, \quad a < t < b, \tag{1}$$

where Gamma function of ξ is

$$\Gamma(\xi) = \int_0^{\infty} e^{-t} t^{\xi-1} dt. \tag{2}$$

And the result in (1) can equivalently be written as

$$J^{\xi} t^{\eta} = \frac{\Gamma(\eta+1)}{\Gamma(\eta+\xi+1)} t^{\eta+\xi}. \tag{3}$$

The Riemann-Liouville fractional derivative is given as

$$D_t^{\xi} g(t) = \frac{1}{\Gamma(n-\xi)} \frac{d^n}{dt^n} \int_a^t (t-\tau)^{n-\alpha-1} f(\tau) d\tau \tag{4}$$

where n is the smallest integer greater than or equal to ξ, Γ is the usual gamma function, and a is a constant [5].

2.2. **The Caputo Fractional Derivative.** The Caputo fractional derivative operator D^{ξ} of order ξ is defined as follows:

$$D^{\xi} g(t) = \frac{1}{\Gamma(n-\xi)} \int_0^t (t-\tau)^{n-\xi-1} g^{(n)}(\tau) d\tau, \quad \xi > 0 \tag{5}$$

where $r-1 < \xi < r$, $r \in N$ and $t > 0$.
This can as well be written as

$$D^{\xi} t^{\eta} = \frac{\Gamma(\eta+1)}{\Gamma(\eta-\xi+1)} t^{\eta-\xi}. \tag{6}$$

3. Brief Review of Adomian Decomposition Method

In this section, we shall look into a concise review of ADM because the method is combined with the Shehu transform later in what follows.
Consider the initial value problem of the form

$$L(u) + R(u) + N(u) = f(x) \tag{7}$$

with the initial conditions

$$u^{(k)}(0) = c_k \quad k = 0, 1, 2, ..., n-1 \tag{8}$$

where L is the highest order linear operator, N is the nonlinear operator, R is the remaining linear term and $f(x)$ is the inhomogeneous source term.
ADM algorithm requires that L, as a differential operator, has an inverse L^{-1} which is its integral equivalence.
Therefore

$$L(u) = f(x) - R(u) - N(u) \tag{9}$$

Applying L^{-1} to both sides of (2.9) gives

$$u(x) = L^{-1}[f(x)] - L^{-1}[R(u)] - L^{-1}[N(u)] \tag{10}$$

Let the series solution of $u(x)$ be given as

$$u(x) = \sum_{i=o}^{n} u_i(x) \tag{11}$$

Then

$$\sum_{i=o}^{n} u_i(x) = L^{-1}[f(x)] - L^{-1}[R(\sum_{i=o}^{n} u_i(x))] - L^{-1}[\sum_{i=o}^{n} A_n(x))] \tag{12}$$

Thus

$$u_0(x) = \Psi_0 + L^{-1}[f(x)], \tag{13}$$

where

$$\Psi_0 = u(0) + xu^{'}(0) + \frac{x^2}{2!} u^{''}(0) + ... \tag{14}$$

and

$$u_{n+1}(x) = -L^{-1}[R(u_n)] - L^{-1}[A_n(x)], \tag{15}$$

where $A_n(x)$ are the Adomian polynomials derived for the nonlinear terms in the IVP. The Adomian polynomials are obtained from the formula

$$A_n = \frac{1}{n!}\left[\frac{d^n}{d\lambda^n}N\left(\sum_{i=0}^{n}\lambda^i u_i\right)\right]_{\lambda=0}, \quad n = 0, 1, 2, ... \tag{16}$$

And the final solution of the IVP is given as

$$u(x) = \sum_{n=o}^{\infty} u_n(x) \tag{17}$$

See ([1],[2],[15]).

4. Brief Review of Shehu Transform

This section will briefly review the definition and the derivative of Shehu transform.

4.1. **Definition.** The Shehu transform of function $v(x)$ of exponential order is defined over the set of functions

$$P = \left\{v(x) : \exists N, \quad \xi_1, \quad \xi_2 > 0, \quad |v(x)| < N\exp\left(\frac{|x|}{\xi_i}\right), \quad \text{if} \quad x \in (-1)^i[0, \infty)\right\} \tag{18}$$

by the integral

$$S\{v(x)\} = V(s, u) = \int_0^{\infty} e^{-(\frac{s}{u})x} v(x)dx, \tag{19}$$

$$S\{v(x)\} = \lim_{\alpha \longrightarrow 0} \int_0^{\alpha} e^{-(\frac{s}{u})x} v(x)dx, \quad s > 0, u > 0 \tag{20}$$

The integral in (19) converges provided that the limit of the integral exists and diverges otherwise.
The inverse of Shehu transform is given by
$S^{-1}\{V(s, u)\} = v(x), \quad x \geq 0,$
which can still be stated as

$$v(x) = S^{-1}\{V(s, u)\} = \frac{1}{2\pi i}\int_{\alpha - i\infty}^{\alpha + i\infty} \frac{1}{u}\exp^{(\frac{s}{u})x} V(s, u)ds, \tag{21}$$

where s and u are the Shehu transform variables, and α is a real constant and integral in (21) is taken along $s = \alpha$ in the complex plane $s = x + iy$.

4.2. **Shehu Transform of Derivatives.** The Shehu transform of derivatives of a given function $f(x)$ with Shehu transform $F(s, u)$ is defined as
$S\{f^{'}(x)\} = (\frac{s}{u})F(s, u) - f(0).$
The n^{th} order derivative is given as

$$S\{f^{(n)}(x)\} = \left(\frac{s^n}{u^n}\right)F(s, u) - \sum_{i=0}^{n-1}\left(\frac{s}{u}\right)^{n-(i+1)} f^{(i)}(0), \tag{22}$$

where $F(s, u)$ is the Shehu transform of the function $f(x)$. (See [12] for additional information).

5. Statement of the Problem

In this chapter, the algorithm for Shehu transform Adomian decomposition method is developed and applied to some integer order partial differential equations.

6. Methodology: Shehu Transform Adomian Decomposition Method

Consider the nonlinear system of fractional differential equations

$$D^{\alpha_j} u_j(x) = L_j(u_1, u_2, u_3, ..., u_r) + N_j(u_1, u_2, u_3, ..., u_r) \tag{23}$$

with associated initial conditions

$$u_j^{(i)}(0) = c_i^j, \quad j = 1, 2, 3, ..., n. \quad i = 0, 1, 2, ..., n-1 \quad and \quad i_{j-1} \leq \alpha \leq i_j \tag{24}$$

where L_j is the linear operator, N_j is the nonlinear operator and D^{α} is the fractional differential operator.
To solve the system using Shehu transform Adomian decomposition method, we first apply Shehu transform to both sides . Thus we have,

$$S\{D^{\alpha_j} u_j(x)\} = S\{L_j(u_1, u_2, u_3, ..., u_r)\} + S\{N_j(u_1, u_2, u_3, ..., u_r)\} \tag{25}$$

Applying the Shehu transform derivative, we have

$$S\{D^{\alpha_j} u_j(x)\} = \left(\frac{s}{v}\right)^{\alpha_j} U_j(s, v) - \sum_{k=0}^{n-1} \left(\frac{s}{v}\right)^{\alpha_j-(k+1)} u_j^{(k)}(0) \tag{26}$$

Substituting equation (26) into (25), we have

$$\begin{aligned}\left(\frac{s}{v}\right)^{\alpha_j} U_j(s, v) - \sum_{k=0}^{n-1} \left(\frac{s}{v}\right)^{\alpha_j-(k+1)} u_j^{(k)}(0) &= S\{L_j(u_1, u_2, u_3, ..., u_r)\} \\ &+ S\{N_j(u_1, u_2, u_3, ..., u_r)\}\end{aligned} \tag{27}$$

which implies

$$\begin{aligned}\left(\frac{s}{v}\right)^{\alpha_j} U_j(s, v) = \sum_{k=0}^{n-1} \left(\frac{s}{v}\right)^{\alpha_j-(k+1)} u_j^{(k)}(0) \quad &+ \quad S\{L_j(u_1, u_2, u_3, ..., u_r)\} \\ &+ \quad S\{N_j(u_1, u_2, u_3, ..., u_r)\}\end{aligned} \tag{28}$$

Dividing through by $\left(\frac{s}{v}\right)^{\alpha_j}$, we have

$$\begin{aligned}U_j(s, v) = \left(\frac{v}{s}\right)^{\alpha_j} \sum_{k=0}^{n-1} \left(\frac{s}{v}\right)^{\alpha_j-(k+1)} u_j^{(k)}(0) \quad &+ \quad \left(\frac{v}{s}\right)^{\alpha_j} S\{L_j(u_1, u_2, u_3, ..., u_r)\} \\ &+ \quad \left(\frac{v}{s}\right)^{\alpha_j} S\{N_j(u_1, u_2, u_3, ..., u_r)\}\end{aligned} \tag{29}$$

Applying the Adomian decomposition method to the system, we have

$$u_j(x) = \sum_{k=0}^{\infty} u_{j,k}(x), \quad j = 1, 2, 3, ..., n \tag{30}$$

and the nonlinearity is decomposed as

$$N_j(u_1, u_2, u_3, ..., u_r) = \sum_{k=0}^{\infty} A_{j,k}, \quad j = 1, 2, 3, ..., n \tag{31}$$

where $A_{j,m}$ are the Adomian polynomials defined by

$$A_{j,m} = \frac{1}{m!}\left[\frac{d^m}{d\lambda^m} N_j\left(\sum_{k=0}^{m}\lambda^k u_{1,k}, \sum_{k=0}^{m}\lambda^k u_{2,k}, \sum_{k=0}^{m}\lambda^k u_{3,k}, ..., \sum_{k=0}^{m}\lambda^k u_{r,k}\right)\right]_{\lambda=0}, \quad m = 0, 1, 2, ... \tag{32}$$

Substituting equation (31) and (32) into equation (29), we have

$$\begin{aligned}\sum_{k=0}^{\infty} U_{j,k}(s,v) = &\left(\frac{v}{s}\right)^{\alpha_j} \sum_{k=0}^{n-1}\left(\frac{s}{v}\right)^{\alpha_j-(k+1)} u_j^{(k)}(0) \\ &+ \left(\frac{v}{s}\right)^{\alpha_j} S\{L_j(\sum_{k=0}^{\infty} u_{1,k}, \sum_{k=0}^{\infty} u_{2,k}, \sum_{k=0}^{\infty} u_{3,k}, ..., \sum_{k=0}^{\infty} u_{r,k})\} \\ &+ \left(\frac{v}{s}\right)^{\alpha_j} S\{N_j(\sum_{k=0}^{\infty} A_{j,k})\}\end{aligned} \tag{33}$$

We now apply the linearity of Shehu transform and generate the recursive formula for the system as follows

$$U_{j,0}(s,v) = \left(\frac{v}{s}\right)^{\alpha_j} \sum_{k=0}^{n-1}\left(\frac{s}{v}\right)^{\alpha_j-(k+1)} u_j^{(k)}(0) \tag{34}$$

$$U_{j,1}(s,v) = \left(\frac{v}{s}\right)^{\alpha_j} S\{L_j(u_{1,0}, u_{2,0}, u_{3,0}, ..., u_{r,0})\} + \left(\frac{v}{s}\right)^{\alpha_j} S\{N_j(A_{j,0})\} \tag{35}$$

$$U_{j,2}(s,v) = \left(\frac{v}{s}\right)^{\alpha_j} S\{L_j(u_{1,1}, u_{2,1}, u_{3,1}, ..., u_{r,1})\} + \left(\frac{v}{s}\right)^{\alpha_j} S\{N_j(A_{j,1})\} \tag{36}$$

and

$$U_{j,k+1}(s,v) = \left(\frac{v}{s}\right)^{\alpha_j} S\{L_j(u_{1,k}, u_{2,k}, u_{3,k}, ..., u_{r,k})\} + \left(\frac{v}{s}\right)^{\alpha_j} S\{N_j(A_{j,k})\} \tag{37}$$

Taking the inverse Shehu transform of both sides of the system, we have

$$u_{j,0}(x) = S^{-1}\left[\left(\frac{v}{s}\right)^{\alpha_j} \sum_{k=0}^{n-1}\left(\frac{s}{v}\right)^{\alpha_j-(k+1)} u_j^{(k)}(0)\right] \tag{38}$$

$$u_{j,1}(x) = S^{-1}\left[\left(\frac{v}{s}\right)^{\alpha_j} S\{L_j(u_{1,0}, u_{2,0}, u_{3,0}, ..., u_{r,0})\}\right] + S^{-1}\left[\left(\frac{v}{s}\right)^{\alpha_j} S\{N_j(A_{j,0})\}\right] \tag{39}$$

$$u_{j,2}(x) = S^{-1}\left[\left(\frac{v}{s}\right)^{\alpha_j} S\{L_j(u_{1,1}, u_{2,1}, u_{3,1}, ..., u_{r,1})\}\right] + S^{-1}\left[\left(\frac{v}{s}\right)^{\alpha_j} S\{N_j(A_{j,1})\}\right], \tag{40}$$

$$j = 1, 2, 3, ...n$$

and

$$u_{j,k+1}(x) = S^{-1}\left[\left(\frac{v}{s}\right)^{\alpha_j} S\{L_j(u_{1,k}, u_{2,k}, u_{3,k}, ..., u_{r,k})\}\right] + S^{-1}\left[\left(\frac{v}{s}\right)^{\alpha_j} S\{N_j(A_{j,k})\}\right] \tag{41}$$

7. Numerical Examples on Integer Order Partial Differential Equations

7.1. **List of Problems.** The algorithm is applied to some selected nonlinear partial differential equation. The problems considered here are sourced from [8] and are as presented below:

Problem 1: Consider the system of nonlinear integer order PDE

$$u_t - u_{xx} - 2uu_x + (uv)_x = 0, \quad u(x,0) = sinx$$
$$v_t - v_{xx} - 2vv_x + (uv)_x = 0, \quad v(x,0) = sinx$$

Problem 2: Consider the system of nonlinear integer order PDE

$$u_t + (\frac{1}{2}(\frac{1}{2}u^2 + v^2) + v)_x = 0, \qquad u(x,0) = \frac{2x}{10}$$
$$v_t + (uv)_x = 0, \qquad v(x,0) = -\frac{110}{100}$$

Problem 3: Consider the system of nonlinear integer order PDE

$$u_t + (v^2)_x = 1 - 2t + 2x, \qquad u(x,0) = x$$
$$v_t - v_{xxx} + (uv)_x = 1 - 2x, \qquad v(x,0) = -x$$

7.2. **Solutions to the Listed Problems.** Here, we present the complete solution of problem 1, while the solutions to problems 2 and 3 are presented in abridged forms.

Solution to Problem 1

$$u_t - u_{xx} - 2uu_x + (uv)_x = 0 \tag{42}$$
$$v_t - v_{xx} - 2vv_x + (uv)_x = 0 \tag{43}$$

Taking the Shehu transform of both sides of (42) and (43) gives

$$S\{u_t\} - S\Big\{\frac{\partial^2 u}{\partial x^2}\Big\} - S\{2uu_x\} + S\{(uv)_x\} = S\{0\} \tag{44}$$
$$S\{v_t\} - \Big\{\frac{\partial^2 v}{\partial x^2}\Big\} - S\{2vv_x\} + S\{(uv)_x\} = S\{0\} \tag{45}$$

which are the same as

$$S\Big\{\frac{\partial u}{\partial t}\Big\} - S\Big\{\frac{\partial^2 u}{\partial x^2}\Big\} - S\{2uu_x\} + S\{(uv)_x\} = 0 \tag{46}$$
$$S\Big\{\frac{\partial v}{\partial t}\Big\} - \Big\{\frac{\partial^2 v}{\partial x^2}\Big\} - S\{2vv_x\} + S\{(uv)_x\} = 0 \tag{47}$$

But

$$S\Big\{\frac{\partial u}{\partial t}\Big\} = \Big(\frac{s}{u}\Big)U(x,(s,v)) - u(x,0) \tag{48}$$
$$S\Big\{\frac{\partial v}{\partial t}\Big\} = \Big(\frac{s}{u}\Big)V(x,(s,v)) - v(x,0) \tag{49}$$

Substituting equations (48) and (49) in (46) and (47) respectively, we have

$$\frac{s}{u}U(x,(s,v)) - u(s,v) - S\{u_{xx}\} - S\{2uu_{xx}\} - S\{2uu_x\} + S\{(uv)_x\} = 0 \tag{50}$$
$$\frac{s}{u}V(x,(s,v)) - v(s,v) - S\{v_{xx}\} - S\{2vv_{xx}\} - S\{2vv_x\} + S\{(uv)_x\} = 0 \tag{51}$$

Thus

$$U(x,(s,v)) = \frac{u}{s}u(x,0) + \frac{u}{s}S\{u_{xx}\} + \frac{u}{s}S\{2uu_x\} - \frac{u}{s}S\{(uv)_x\} \tag{52}$$

$$V(x,(s,v)) = \frac{u}{s}v(x,0) + \frac{u}{s}S\{v_{xx}\} + \frac{u}{s}S\{2vv_x\} - \frac{u}{s}S\{(uv)_x\} \tag{53}$$

The series solutions are:
$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t)$ and $v(x,t) = \sum_{n=0}^{\infty} v_n(x,t)$.
And also let
$uu_x = \sum_{n=0}^{\infty} A_n$, $(uv)_x = \sum_{n=0}^{\infty} B_n$ and $vv_x = \sum_{n=0}^{\infty} C_n$
$A_0 = u_0u_{0x}$, $A_1 = u_0u_{1x} + u_1u_{0x}$, $A_2 = u_0u_{2x} + u_1u_{1x} + u_2u_{0x}$
$B_0 = u_0v_{0x} + u_{0x}v_0$
$B_1 = u_0v_{1x} + u_1v_{0x} + u_{1x}v_{0x} + u_{0x}v_1$
$B_2 = u_0v_{2x} + u_1v_{1x} + u_2v_{0x} + u_{2x}v_2 + u_{0x}v_1$

$$U(x,(s,u)) = \frac{u}{s}\sin x + \frac{u}{s}(u_{xx}) + \frac{u}{s}S\Big\{2\sum_{n=0}^{\infty} A_n\Big\} - S\Big\{\sum_{n=0}^{\infty} B_n\Big\} \tag{54}$$

$$V(x,(s,u)) = \frac{u}{s}\sin x + \frac{u}{s}(v_{xx}) + \frac{u}{s}S\Big\{2\sum_{n=0}^{\infty} C_n\Big\} - S\Big\{\sum_{n=0}^{\infty} B_n\Big\} \tag{55}$$

The initial approximations for $u(x,t)$ and $v(x,t)$ are obtained as:
$U_0(x,(s,v)) = \frac{u}{s}\sin x$, $V_0(x,(s,v)) = \frac{u}{s}\sin x$.
Taking the inverse Shehu transform of both sides, we have
$u_0(x,t) = \sin x$, $v_0(x,t) = \sin x$.
Also

$$U_{n+1}(x,(s,v)) = \frac{u}{s}S\{u_{nxx}\} + \frac{u}{s}S\Big\{2\sum_{n=0}^{\infty} A_n\Big\} - \frac{u}{s}S\Big\{\sum_{n=0}^{\infty} B_n\Big\} \tag{56}$$

$$\begin{aligned} V_{n+1}(x,(s,v)) &= \frac{u}{s}S\{v_{nxx}\} + \frac{u}{s}S\Big\{2\sum_{n=0}^{\infty} C_n\Big\} - \frac{u}{s}S\Big\{\sum_{n=0}^{\infty} B_n\Big\}, \\ n &= 0,1,2,... \end{aligned} \tag{57}$$

When $n = 0$

$$U_1(x,(s,v)) = \frac{u}{s}S\{u_{0xx}\} + \frac{u}{s}S\{2A_0\} - \frac{u}{s}S\{B_0\}$$
$$V_1(x,(s,v)) = \frac{u}{s}S\{v_{0xx}\} + \frac{u}{s}S\{2C_0\} - \frac{u}{s}S\{B_0\}$$

$$\begin{aligned} U_1(x,(s,v)) &= \frac{u}{s}S\{-\sin x\} + \frac{u}{s}S\{2u_0.u_{0x}\} - \frac{u}{s}S\{u_0v_{0x} + u_{0x}v_0\} \\ U_1(x,(s,v)) &= \frac{u}{s}S\{-\sin x\} + \frac{u}{s}S\{2(\sin x)(\cos x)\} \\ &\quad - \frac{u}{s}S\{\sin x.\cos x + \cos x.\sin x\} \\ U_1(x,(s,v)) &= \frac{u}{s}S\{-\sin x\} + \frac{u}{s}S\{2\sin x\cos x\} - \frac{u}{s}S\{2\sin x\cos x\} \\ U_1(x,(s,v)) &= -\frac{u}{s}S\{\sin x\}. \end{aligned}$$

Taking the inverse Shehu transform, we have

$$u_1(x,t) = S^{-1}\Big[-\frac{u^2}{s^2}\sin x\Big]$$
$$u_1(x,t) = -t\sin x$$

$$\begin{aligned}
V_1(x,(s,v)) &= \frac{u}{s}S\{v_{0xx}\} + \frac{u}{s}S\{2v_0v_{0x}\} - \frac{u}{s}S\{u_0v_{0x} + u_{0x}v_0\}\\
V_1(x,(s,v)) &= \frac{u}{s}S\{-\sin x\} + \frac{u}{s}S\{2(\sin x)(\cos x)\}\\
&\quad - \frac{u}{s}S\{sinx.cosx + cosx.sinx\}\\
V_1(x,(s,v)) &= \frac{u}{s}S\{-\sin x\} + \frac{u}{s}S\{2\sin x\cos x\} - \frac{u}{s}S\{2\sin x\cos x\}\\
V_1(x,(s,v)) &= \frac{u}{s}S\{-\sin x\}
\end{aligned}$$

Taking the inverse Shehu transform

$$v_1(x,t) = S^{-1}\Big[-\frac{u^2}{s^2}\sin x\Big]$$
$$u_1(x,t) = -t\sin x$$

When $n = 1$

$$\begin{aligned}
U_2(x,(s,v)) &= \frac{u}{s}S\{u_{1xx}\} + \frac{u}{s}S\{2A_1\} - \frac{u}{s}S\{B_1\}\\
U_2(x,(s,v)) &= \frac{u}{s}S\{t\sin x\} + \frac{u}{s}S\{2(u_0u_{1x} + u_1u_{0x})\}\\
&\quad - \frac{u}{s}S\{u_0v_{1x} + v_0u_{1x} + u_1v_{0x} + u_{0x}v_1\}\\
U_2(x,(s,v)) &= \frac{u}{s}S\{t\sin x\} + \frac{u}{s}S\{2(u_0u_{1x} + u_1u_{0x})\}\\
&\quad - \frac{u}{s}S\{\sin x(-\cos x) + (\sin x)(-\sin x)\\
&\quad + (-\sin x)(\cos x) + (\cos x)(-\sin x)\}\\
U_2(x,(s,v)) &= \frac{u}{s}S\{t\sin x\} + \frac{u}{s}S\{2(\sin x)(-\cos x)\\
&\quad + 2(-\sin x)(\cos x)\} - \frac{u}{s}S\{4(-\sin x\cos x)\}\\
u_2(x,(s,v)) &= \frac{u}{s}S\{t\sin x\}
\end{aligned}$$

Taking the inverse Shehu transform gives

$$\begin{aligned}
u_2(x,t) &= S^{-1}[\frac{u^3}{s^3}S\{\sin x\}]\\
&= \frac{t^2}{2!}\sin x
\end{aligned}$$

Similarly

$$\begin{aligned}
V_2(x,(s,v)) &= \frac{u}{s}S\{v_{1xx}\} + \frac{u}{s}S\{2C_1\} - \frac{u}{s}S\{B_1\}\\
V_2(x,(s,v)) &= \frac{u}{s}S\{t\sin x\} + \frac{u}{s}S\{2(v_0v_{1x} + v_1v_{0x})\}\\
&\quad - \frac{u}{s}S\{u_0v_{1x} + v_0u_{1x} + u_1v_{0x} + u_{0x}v_1\}
\end{aligned}$$

$$
\begin{aligned}
V_2(x,(s,v)) &= \frac{u}{s}S\{t\sin x\} + \frac{u}{s}S\{2(\sin x)(-\cos x) + 2(-\sin x)(\cos x)\} \\
&\quad - \frac{u}{s}S\{(\sin x)(-\cos x) + (\sin x)(-cosx) + (\cos x)(-\sin x)\} \\
V_2(x,(s,v)) &= \frac{u}{s}S\{t\sin x\} + \frac{u}{s}S\{4(-\sin x\cos x)\} \\
&\quad - \frac{u}{s}S\{4(-\sin x\cos x)\} \\
V_2(x,(s,u)) &= \frac{u}{s}\Big[\frac{u^2}{s^2}\sin x\Big]
\end{aligned}
$$

Taking the inverse Shehu transform of both sides

$$
\begin{aligned}
v_2(x,t) &= S^{-1}\Big[\frac{u}{s}(\frac{u^2}{s^2}\sin x)\Big] \\
v_2(x,t) &= \frac{t^2}{2}\sin x
\end{aligned}
$$

Following the same procedure for $n = 2$, we get

$$
\begin{aligned}
u_3(x,t) &= -\frac{t^3}{3!}\sin x \\
v_3(x,t) &= \frac{t^3}{3!}\sin x.
\end{aligned}
$$

Therefore, the series solutions of the system are

$$
\begin{aligned}
u(x,t) &= u_0(x,t) + u_1(x,t) + u_2(x,t) + u_3(x,t) + u_4(x,t) + \dots \\
&= \sin x - t\sin x + \frac{t^2}{2!}\sin x - \frac{t^3}{3!}\sin x + \frac{t^4}{4!}\sin x + \dots \\
&= \sin x\left[1 - t + \frac{t^2}{2!} - \frac{t^3}{3!} + \frac{t^4}{4!} + \dots\right] \\
&= e^{-t}\sin x
\end{aligned}
$$

$$
\begin{aligned}
v(x,t) &= v_0(x,t) + v_1(x,t) + v_2(x,t) + v_3(x,t) + v_4(x,t) + \dots \\
&= \sin x - t\sin x + \frac{t^2}{2!}\sin x - \frac{t^3}{3!} + \frac{t^4}{4!}sinx + \dots \\
&= \sin x\left[1 - t + \frac{t^2}{2!} - \frac{t^3}{3!} + \frac{t^4}{4!} + \dots\right] \\
&= e^{-t}\sin x
\end{aligned}
$$

Solution to Problem 2

Taking the Shehu transform of both sides of the system, we have

$$
\begin{aligned}
S\Big\{\frac{\partial u}{\partial t}\Big\} + \frac{1}{4}S\Big\{\frac{\partial}{\partial x}u^2\Big\} + \frac{1}{2}S\Big\{\frac{\partial}{\partial x}v^2\Big\} + S\Big\{\frac{\partial v}{\partial x}\Big\} &= 0 \\
S\Big\{\frac{\partial v}{\partial t}\Big\} + S\Big\{\frac{\partial}{\partial x}(uv)\Big\} &= 0
\end{aligned}
$$

$$
\begin{aligned}
\Big(\frac{s}{v}\Big)U(x,(s,v)) - u(x,0) + \frac{1}{4}S\Big\{\frac{\partial}{\partial x}u^2\Big\} + \frac{1}{2}S\Big\{\frac{\partial}{\partial x}v^2\Big\} + S\Big\{\frac{\partial v}{\partial x}\Big\} &= 0 \\
\Big(\frac{s}{v}\Big)V(x,(s,v)) - v(x,0) + S\Big\{\frac{\partial}{\partial x}(uv)\Big\} &= 0
\end{aligned}
$$

Applying the given initial conditions, and dividing through by $(\frac{s}{v})$, we have

$$U(x,(s,v)) = \frac{2x}{10}\Big(\frac{v}{s}\Big) - \frac{1}{4}\Big(\frac{v}{s}\Big)S\Big\{\frac{\partial}{\partial x}u^2\Big\} - \frac{1}{2}\Big(\frac{v}{s}\Big)S\Big\{\frac{\partial}{\partial x}v^2\Big\} - (\frac{v}{s})S\Big\{\frac{\partial v}{\partial x}\Big\}$$

$$V(x,(s,v)) = -\frac{110}{100}\Big(\frac{v}{s}\Big) - \Big(\frac{v}{s}\Big)S\Big\{\frac{\partial}{\partial x}(uv)\Big\}$$

The series solutions are
$U(x,(s,v)) = \sum_{n=0}^{\infty} U_n(x,(s,v))$, $V(x,(s,v)) = \sum_{n=0}^{\infty} V_n(x,(s,v))$
and the nonlinearities are decomposed by Adomian method as
$u^2 = \sum_{n=0}^{\infty} A_n$, $v^2 = \sum_{n=0}^{\infty} B_n$ and $uv = \sum_{n=0}^{\infty} C_n$.
We therefore obtained the final pair of series solutions as:

$$u(x,t) = \frac{2x}{10}\Big[1 - \frac{t}{10} + \Big(\frac{t}{10}\Big)^2 - \Big(\frac{t}{10}\Big)^3 + \dots\Big]$$
$$v(x,t) = -\frac{110}{10^2}\Big[1 - \frac{2t}{10} + \frac{3t^2}{10^2} - \frac{4t^3}{10^3} + \dots\Big].$$

Solution to problem 3
The pair of series solutions to the problem are obtained through similar procedure as presented in Problems 1 and 2 as:

$$\begin{aligned} u(x,t) &= u_0(x,t) + u_1(x,t) + \dots \\ &= x + t + -t^2 + 2xt - 2xt + t^2 - 4xt^2 + \frac{4t^3}{3} - \frac{8xt^3}{3} \\ v(x,t) &= v_0(x,t) + v_1(x,t) + \dots \\ &= -x + x - 2xt + 2xt - \frac{t^3}{3} + xt^2 - \frac{t^4}{2} + \frac{sint^3}{3} \end{aligned}$$

8. Numerical Examples on Fractional Order Ordinary Differential Equations

In this section, some problems from [13] are solved using the developed algorithm (STADM)

8.1. List of Problems Solved Using STADM.

Problem 1: Consider the system of nonlinear fractional ODE

$$D^{\alpha}y_1(t) = y_1(t) + [y_2(t)]^2, \quad 1 < \alpha \leq 2, \quad y_1(0) = 0, \quad y_1'(0) = 1 \tag{58}$$
$$D^{\beta}y_2(t) = y_1(t) + 5y_2(t), \quad 2 < \beta \leq 3, \quad y_2(0) = 0, \quad y_2'(0) = 1, \quad y_2''(0) = 1. \tag{59}$$

Problem 2: Consider the system of nonlinear fractional ODE

$$D^{\alpha}y_1(t) = \frac{1}{2}y_1(t), \quad y_1(0) = 1,$$
$$D^{\beta}y_2(t) = y_2(t) + y_1^2(t), \quad y_2(0) = 0, \quad 0 < \alpha, \beta \geq 1.$$

Problem 3: Consider the system of nonlinear fractional ODE

$$D^{\alpha}x(t) = t^2 + \frac{y^2}{4}, \quad x(0) = 0, \quad \alpha = \beta = \frac{1}{2}.$$
$$D^{\beta}y(t) = t^2 + \frac{x^2}{4}, \quad y(0) = 1.$$

8.2. **Solution to the listed problems.** **Solution to Problem I**
We first take Shehu transform of both sides of the system

$$S\{D^\alpha y_1(t)\} = S\{y_1(t)\} + S\{y_2^2(t)\}$$
$$S\{D^\beta y_2(t)\} = S\{y_1(t)\} + 5S\{y_2(t)\}.$$

$$\left(\frac{s}{v}\right)^\alpha Y_1(s,v) - \left(\frac{s}{v}\right)^{\alpha-1} y_1(0) - \left(\frac{s}{v}\right)^{\alpha-2} y_1'(0) = Y_1(s,v) + S\{y_2^2(t)\}$$

$$\left(\frac{s}{v}\right)^\beta Y_2(s,v) - \left(\frac{s}{v}\right)^{\beta-1} y_2(0) - \left(\frac{s}{v}\right)^{\beta-2} y_2'(0) - \left(\frac{s}{v}\right)^{\beta-3} y_2''(0) = Y_1(s,v) + 5Y_2(s,v)$$

Applying the initial conditions and dividing through by $(\frac{s}{v})^\alpha$ and $(\frac{s}{v})^\beta$ respectively, we have

$$Y_1(s,v) = (\frac{v}{s})^2 + (\frac{v}{s})^\alpha Y_1(s,v) + (\frac{v}{s})^\alpha S\{y_2^2(t)\}.$$
$$Y_2(s,v) = (\frac{v}{s})^2 + (\frac{v}{s})^3 + (\frac{v}{s})^\beta Y_1(s,v) + 5(\frac{v}{s})^\beta Y_2(s,v).$$

We have $Y_1 = \sum_{n=0}^\infty Y_{1,n}$, $Y_2 = \sum_{n=0}^\infty Y_{2,n}$ and $y_2^2 = \sum_{n=0}^\infty A_n$ where A_n is the set Adomian polynomials.
Therefore,

$$\begin{aligned} y_1 = t &+ \frac{t^{\alpha+1}}{\Gamma(\alpha+2)} + \frac{2!t^{\alpha+2}}{\Gamma(\alpha+3)} + \frac{3!t^{\alpha+3}}{\Gamma(\alpha+4)} + \frac{4!t^{\alpha+4}}{4\Gamma(\alpha+5)} + \frac{t^{2\alpha+1}}{\Gamma(2\alpha+2)} \\ &+ \frac{2!t^{2\alpha+2}}{\Gamma(2\alpha+3)} + \frac{3!t^{2\alpha+3}}{\Gamma(2\alpha+4)} + \frac{4!t^{2\alpha+4}}{4\Gamma(2\alpha+5)} \\ &+ \frac{12\Gamma(\beta+3)t^{\alpha+\beta+2}}{\Gamma(\beta+2)\Gamma(\beta+\alpha+3)} + \frac{6\Gamma(\beta+4)t^{\alpha+\beta+3}}{\Gamma(\beta+2)\Gamma(\beta+\alpha+4)} \\ &+ \frac{5\Gamma(\beta+5)t^{\alpha+\beta+4}}{\Gamma(\beta+3)\Gamma(\beta+\alpha+5)} \end{aligned}$$

$$\begin{aligned} y_2 = t &+ \frac{t^2}{2!} + \frac{6t^{\beta+1}}{\Gamma(\beta+2)} + \frac{5t^{\beta+2}}{\Gamma(\beta+3)} + \frac{t^{\alpha+\beta+1}}{\Gamma(\alpha+\beta+2)} \\ &+ \frac{2!t^{\alpha+\beta+2}}{\Gamma(\alpha+\beta+3)} + \frac{3!t^{\alpha+\beta+3}}{\Gamma(\alpha+\beta+4)} \\ &+ \frac{4!t^{\alpha+\beta+4}}{4\Gamma(\alpha+\beta+5)} + \frac{30t^{2\beta+1}}{\Gamma(2\beta+2)} + \frac{25t^{2\beta+2}}{\Gamma(2\beta+3)} \end{aligned}$$

Solution to Problem 2
Taking the Shehu transform of the system

$$S\{D^\alpha y_1(t)\} = \frac{1}{2}S\{y_1(t)\}$$
$$S\{D^\beta y_2(t)\} = S\{y_2(t)\} + S\{y_1^2(t)\}.$$

We have

$$\left(\frac{s}{v}\right)^{\alpha} Y_1(s,v) - \left(\frac{s}{v}\right)^{\alpha-1} y_1(0) = \frac{1}{2} Y_1(s,v)$$
$$\left(\frac{s}{v}\right)^{\beta} Y_2(s,v) - \left(\frac{s}{v}\right)^{\beta-1} y_2(0) = Y_2(s,v) + S\{y_1^2(t)\}$$

Applying the initial conditions, re-arranging and dividing through by $(\frac{s}{v})^{\alpha}$ and $(\frac{s}{v})^{\beta}$ respectively, we have

$$Y_1(s,v) = \frac{v}{s} + \frac{1}{2}(\frac{v}{s})^{\alpha} Y_1(s,v)$$
$$Y_2(s,v) = (\frac{v}{s})^{\beta}[Y_2(s,v) + S\{y_1^2(t)\}]$$

Let $Y = \sum_{n=0}^{\infty} Y_n$ and $y_1^2 = \sum_{n=0}^{\infty} A_{1,n}$, we have

$$\sum_{n=0}^{\infty} Y_{1,n} = (\frac{v}{s}) + \frac{1}{2}(\frac{v}{s})^{\alpha}[\sum_{n=0}^{\infty} A_{1,n}]$$
$$\sum_{n=0}^{\infty} Y_{2,n} = (\frac{v}{s})^{\beta}[\sum_{n=0}^{\infty} Y_{2,n} + \sum_{n=0}^{\infty} A_{1,n}]$$

Following the same procedure as in Problem 1, we arrived at the following solutions for $n = 0, 1, 2, ...$

$$y_{1,0} = 1, \quad y_{2,0} = 0.$$
$$y_{1,2} = \frac{1}{4}\frac{t^{2\alpha}}{\Gamma(2\alpha+1)}, \quad y_{2,2} = \frac{t^{2\beta}}{\Gamma(2\beta+1)} + \frac{t^{\beta+\alpha}}{\Gamma(\beta+\alpha+1)}.$$
$$y_{1,3} = \frac{1}{8}\frac{t^{3\alpha}}{\Gamma(3\alpha+1)}, \quad y_{2,3} = \frac{t^{3\beta}}{\Gamma(3\beta+1)} + \frac{t^{\alpha+2\beta}}{\Gamma(\alpha+2\beta+1)} + \frac{\Gamma(2\alpha+1)t^{2\alpha+\beta}}{\Gamma^2(\alpha+1)\Gamma(2\alpha+\beta+1)}.$$

Solution to Problem 3

The following results are obtained for the problem for $n = 0, 1, 2, ...$ as follows

$$x_0(t) = \frac{2!t^{\alpha+2}}{\Gamma(\alpha+3)}, \quad y_0(t) = 1 + \frac{2!t^{\beta+2}}{\Gamma(\beta+3)}$$

$$x_1 = \frac{1}{4}\frac{t^{\alpha}}{\Gamma(\alpha+1)} + \frac{t^{\alpha+\beta+2}}{\Gamma(\alpha+\beta+3)} + \frac{\Gamma(2\alpha+5)t^{\alpha+2\beta+4}}{\Gamma^2(\beta+3)\Gamma(\alpha+2\beta+5)},$$
$$y_1 = \frac{\Gamma(2\alpha+5)t^{2\alpha+\beta+4}}{\Gamma^2(\alpha+1).\Gamma(2\alpha+\beta+5)}.$$

Thus, the solutions of $x(t)$ and $y(t)$ are,

$$\begin{aligned} x(t) &= x_0(t) + x_1(t) + \dots \\ &= \frac{t^{\alpha}}{4}\Gamma(\alpha+1) + \frac{2!t^{\alpha+2}}{\Gamma(\alpha+3)} + \frac{t^{\alpha+\beta+2}}{\Gamma(\alpha+\beta+3)} + \frac{\Gamma(2\alpha+5)t^{\alpha+2\beta+4}}{\Gamma^2(\alpha+3)\Gamma(\alpha+2\beta+5)} + \dots \\ y(t) &= y_0(t) + y_1(t) + \dots \\ &= 1 + \frac{2!t^{\beta+2}}{\Gamma(\beta+3)} + \frac{\Gamma(2\alpha+5)t^{2\alpha+\beta+4}}{\Gamma^2(\alpha+3)\Gamma(2\alpha+\beta+5)} + \dots \end{aligned}$$

9. Results

9.1. **3D Graphs for PDEs.**

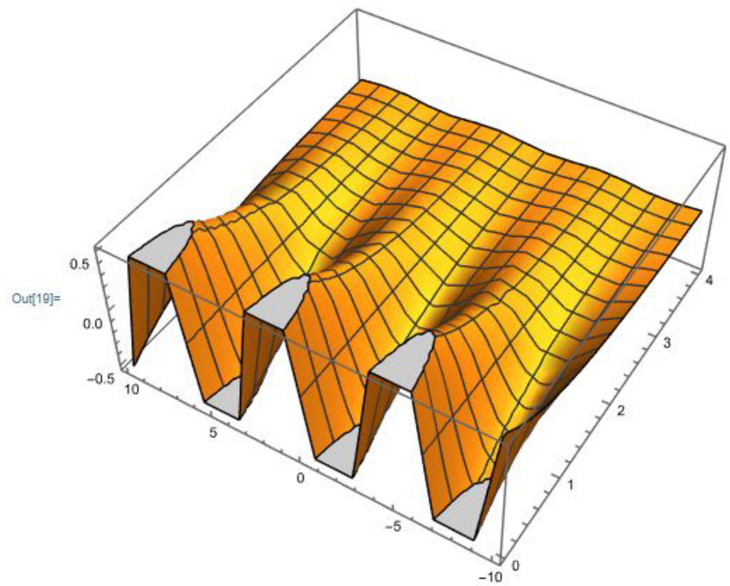


Figure 1. 3D Approximate solution to problem 1 by STADM

Problem 1: Comparison between STADM and the method in the literature				
{t }	LADM $\{y_1(t)\}$	STADM$\{y_1(t)\}$	LADM$\{y_2(t)\}$	STADM $\{y_2(t)\}$
0.00	0.000	0.00000	0.000	0.00000
0.10	0.100	0.10018	0.105	0.10503
0.20	0.201	0.20149	0.220	0.22041
0.30	0.305	0.30533	0.347	0.34714
0.40	0.413	0.41347	0.487	0.48688
0.50	0.528	0.52815	0.642	0.64215
0.60	0.653	0.65226	0.817	0.81635
0.70	0.791	0.78950	1.015	1.01393
0.80	0.949	0.94464	1.247	1.24056
0.90	1.130	1.12381	1.519	1.50328
1.00	1.316	1.33492	1.844	1.81086

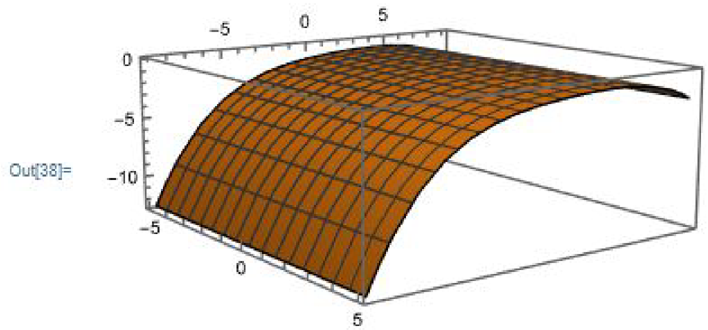


FIGURE 2. 3D Approximate solution to problem 2 by STADM

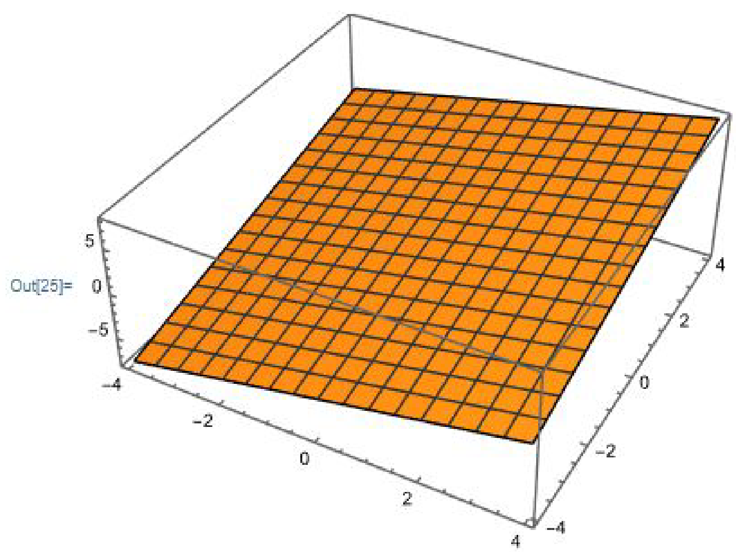
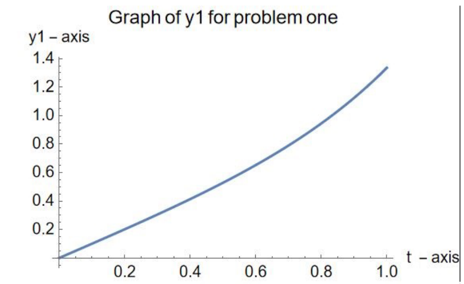


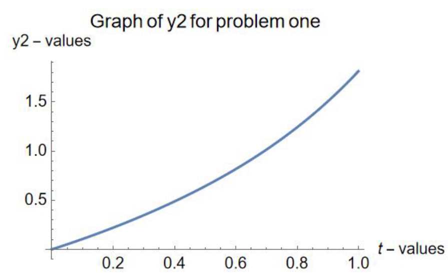
FIGURE 3. 3D Approximate solution to problem 3 by STADM

10. Discussion of Results and Summary

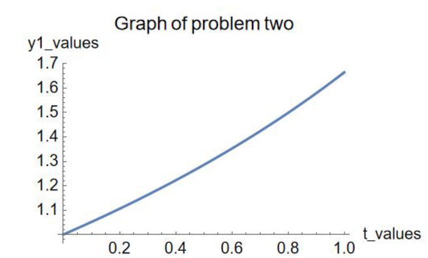
Six numerical examples have been presented for both system of nonlinear fractional order ordinary differential equations and system of nonlinear partial differential equations, three for each. The six problems are taken from the literature to ascertain the reliability of this method. Problems 1, 2 and 3 under integer order have been solved by [8] using Laplace Adomian decomposition method. The problems are solved using our proposed method, STADM, and the results produced are the same. However for problem 3 under integer order, [8] used MLDM, but here, the proposed method is applied directly but with more computational volume. However with the emergence of "noise terms" from the first and second iterations, we arrived at almost the same results. For fractional order, the problems attempted

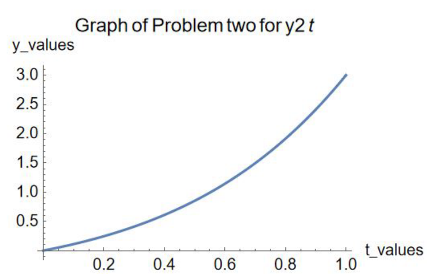


9.2. **2D Graphs for Fractional ODEs.**

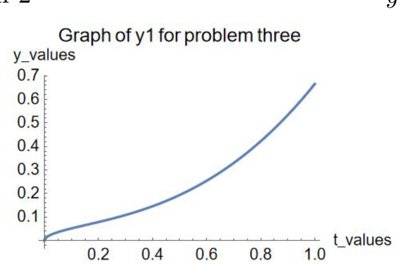


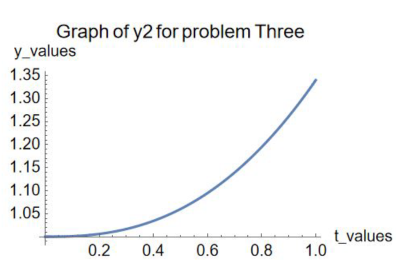
y_1 graph for problem 1 y_2 graph for problem 1





y_1 graph for problem 2 y_2 graph for problem 2





using our method were earlier solved by [13] using Laplace Adomian decomposition method and modified Laplace Adomian decomposition method. Using the proposed method, STADM, we were able to produce results that compete well with that in the literature. Meanwhile, the results are depicted in 3D graphs for PDEs and 2D graphs for fractional ODEs. Therefore, STADM is a useful mathematical tool for solving system of nonlinear fractional order ordinary differential equations and system of nonlinear partial differential equations.
In conclusion, this paper developed a new approach to the solution of system of nonlinear fractional order ordinary differential equations as well as that of system of nonlinear partial differential equations. To achieve that, a Laplace-type integral transform; Shehu Transform which has a unique advantage of being able to solve both constant coefficient as well as variable coefficient problems was integrated into the much celebrated Adomian Decomposition Method (ADM). The nonlinearities encountered were handled seamlessly by the ADM. The new method developed was subsequently applied to selected problems in the literature. The results obtained compared well with those in the literature. In fact, the results are equally presented in 3D graphs where applicable and in 2D graphs otherwise.

11. Conclusion

Shehu transform Adomian decomposition method has proven to be an effective method of solution to nonlinear system of integer and fractional order differential eqautions. The method is therefore a useful mathematical tool for obtaining reliable solution of nonlinear systems of both integer and fractional order differential equations. In the future research, we shall extend the methods reported in this paper to solutions of multi order fractional differential equations.

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