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ANALYSIS ON STABILITY OF FUZZY FRACTIONAL DELAYED PREDATOR PREY SYSTEM

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Abstract. In this paper a fuzzy fractional delay predator prey (FFDPP) system is investigated by adopting fuzzy parameter in delay predator-prey system. Based on the concept of eigenvalue, the linear stability of FFDPP and steady state are discussed. Here, certain conditions were used to find the trivial steady state for all integer value of delays. Meanwhile same conditions assures the locally asymptotic stability of non-trivial steady state for particular values of delay. Sufficient conditions are presented to ensure the stability of FFDPP system. Further Numerical steady state fuzzy numbers are used to verify the results.

1. Introduction

Fractional calculus has started to attract increasing attention of many authors because of its applications in various fields in recent years. Mainly Kilbas[17] and Podulbny [21] studied various fractional differential systems. In the 18th century, Laplace and Condorcet were introduced Delay differential equations, also known as difference differential equations. The basic concepts concerning stability of systems described by equations was developed by Pontryagin in 1942. Delay differential equations are a type of differential equation where the time derivatives at the current time depend on the solution, and possibly its derivatives, at previous times.

In [1] Abbas studied the existence results for the fractional system. In 2014, Eloe and Neugebauer [13] explained the existence of a fractional boundary value problem by using eigenvalues. Also, in [5] Ammi studied the existence of fractional functional differential equations. Many works for the solution of fractional system can be seen in [2, 16, 22, 25, 27] which includes fixed point analysis and Lyapunov inequality and etc. Ahmad et.al [3], Lin et.al [18] and Priyadharsini [24] analyzed the asymptotic behavior of a fractional integro-differential equations. In 2010, El-sayed et.al [12] showed a stability for a fractional order system with delays. In [6] Asl et.al analyzed the linear delay fractional dynamical system and Bhat [8] investigated the controllability of a delay system. In [11] El. Sayed studied the stability of fractional mackey glass equation with chos control. There are many

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contribution from Forde [14] and Ibrahim [15], who studied different fractional delay systems.

In the literature, Predator-prey model is one of the most popular in mathematical ecology, which is used to represent the basis of models in the analysis of population dynamics. This model have been studied heavily during last four decades which includes the contributions of Bellman [7] and Lotka [19]. In real life, most of us accepted the fact of delayed uncertainty, which is very important study in most applications. Chanjin et. al [9] studied the two delayed predator prey model and Elttreby [10] investigated the functional order predator prey model.

The area of fuzzy is a very broad field of study. It is used to model the exact phenomena under the condition uncertainty. The concept of fuzzy set and system was introduced by Zadeh [31] and its development has been growing rapidly to various situation of theory and application including the stability theory of differential equations with uncertainty. Stefaninia et. al [28] introduced the parametric representation of fuzzy number. In [4] fractional differential equation with fuzzy initial condition is basically solved by Arshad and Lupulescu. Maan et.al [20] studied the steady state stability of fuzzy delay predator prey system. In [23, 26] includes the method of finding solution of fuzzy differential equations. It may be noted that Toaha [29] investigated the Stability of Harvesting model with delay and Yuan [30] studied the numerical solution of stability of brusselator chemical reaction system.

This paper is outlined as follows: In section 2, some basic definitions and results were given. In section 3, fractional delay predator-prey system is analyzed generally. Section 4 includes the numerical examples in order to provide the effectiveness of the proposed theory.

2. Preliminaries

Basic definitions regarding the fractional derivatives, fuzzy number,steady states and characteristic equations are presented in this section.

Definition 1. *The Riemann-Liouville fractional integral operator of order $q > 0$ of a function $x \in L^1(\mathcal{R}^+)$ is defined by*

$$ {}_{t_0}I_t^q f(t) = \frac{1}{\Gamma(\alpha)} \int_{t_0}^t (t-s)^{q-1} f(s)\mathrm{d}s, \tag{1}$$

where $\Gamma(.)$ is the Euler's Gamma function.

Definition 2. *The Riemann-Liouville fractional derivative of order $p > 0, n-1 < p < n, n \in N$, is defined as*

$$ {}_{t_0}D^p f(t) = \frac{1}{\Gamma(n-p)} \left(\frac{d}{dt}\right)^n \int_0^t (t-s)^{n-\alpha-1} f(s)\mathrm{d}s, \tag{2}$$

where D^n is the ordinary differential operator and the function $f(t)$ has absolutely continuous derivative up to order $(n-1)$.

Definition 3. *The Caputo fractional derivative of order $p > 0, n-1 < p < n, n \in N$, is defined as*

$$ {}_{t_0}^{C}D^p f(t) = \frac{1}{\Gamma(n-p)} \int_0^t (t-s)^{n-p-1} f^n(s)\mathrm{d}s, \tag{3}$$

where the function $f(t)$ has absolutely continuous derivative up to order $(n-1)$.

Definition 4. *A fuzzy number is a function such as $u : R \to [0,1]$ satisfying the following properties:*

(1) *u is normal, i.e $\exists x_0 \in R$ with $u(x_0) = 1$*
(2) *u is a convex fuzzy set i.e u (λ x + (1- λ)y)$\geq$ minu$\{(x), u(y)\}$, $\forall$ x, y $\in$ R, $\lambda \in$[0, 1].*
(3) *u is upper semi-continuous on R.*
(4) *$\{\overline{x \in R : u(x) > 0}\}$ is compact where $\overline{A}$ denotes the closure of A.*

Definition 5. *A fuzzy number u is completely determined by any pair $u = (\overline{u}, \underline{u})$ of functions $\underline{u}(\alpha), \overline{u}(\alpha) : [0,1] \to R$ satisfying the three conditions:*

(1) *$\underline{u}(\alpha), \overline{u}(\alpha)$ is a bounded, monotonic, (nondecreasing, non increasing) left-continuous function for all $\alpha \in (0,1]$ and right-continuous for $\alpha = 0$.*
(2) *For all $\alpha \in (0,1]$ we have: $\underline{u}(\alpha) \leq \overline{u}(\alpha)$.*
For every $u = (\underline{u}, \overline{u})$, $v = (\underline{v}, \overline{v})$ and $k > 0$, $(\underline{u+v})(\alpha) = \underline{u}(\alpha) + \underline{v}(\alpha)$, $(\overline{u+v})(\alpha) = \overline{u}(\alpha) + \overline{v}(\alpha)$, $(ku)(\alpha) = k\underline{u}(\alpha), (\overline{ku})(\alpha)) = k\overline{u}(\alpha)$

Fuzzy sets is a mapping from a universal set into $[0,1]$. Conversely, every function $\mu : X \to [0,1]$ can be represented as a fuzzy set [31] .

Definition 6. *We can define a set $F_1 = \{x \in \Re$, is about $a_2\}$ with triangular membership function as below*

$$\mu F_1(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, & x \in [a_1, a_2) \\ 1 & x = a_2 \\ \frac{-x+a_3}{a_3-a_2} & x \in (a_2, a_3] \\ 0 & otherwise \end{cases}$$

So the Fuzzy set F can be written as any ordinary function F $= (x, \mu_F(x)) : x \in X$.

Fuzzy Fractional Delay System. Consider the linear fractional fuzzy delay system as follows:

$$ {}_{t_0}^{C}D_t^p \underline{x}_\alpha(t) = A_\alpha \underline{x}_\alpha(t) + B_\alpha \underline{x}_\alpha(t-\tau) \quad (4)$$
$$ {}_{t_0}^{C}D_t^p \overline{x}_\alpha(t) = A_\alpha \overline{x}_\alpha(t) + B_\alpha \overline{x}_\alpha(t-\tau), \quad 0 \leq \alpha \leq 1 \quad (5)$$
$$ \underline{x}_\alpha(t) = \underline{x}_{\alpha 0}$$
$$ \overline{x}_\alpha(t) = \overline{x}_{\alpha 0}$$

Suppose $(a_{ij})_\alpha = [(a_{ij})_\alpha^-, (a_{ij})_\alpha^+]$, $A_\alpha = [A_\alpha^-, A_\alpha^+]$ where $A_\alpha^- = [(a_{ij})_\alpha^-]_{n\times n}$, $A_\alpha^+ = [(a_{ij})_\alpha^+]_{n\times n}$ and $(b_{ij})_\alpha = [(b_{ij})_\alpha^-, (b_{ij})_\alpha^+]$,$B_\alpha = [B_\alpha^-, B_\alpha^+]$ where $B_\alpha^- = [(b_{ij})_\alpha^-]_{n\times n}$, $B_\alpha^+ = [(b_{ij})_\alpha^+]_{n\times n}$.
Let introduce the following definitions :

Definition 7. *Let $A(\mu,\alpha) = [a_{ij}(\mu,\alpha)]_{n\times n} = (1-\mu)A_\alpha^- + \mu A_\alpha^+$, $B(\mu,\alpha) = [b_{ij}(\mu,\alpha)]_{n\times n} = (1-\mu)B_\alpha^- + \mu B_\alpha^+$, for $\mu \in [0,1]$. The solution of (4-5) is $(\underline{x}_\alpha(t), \overline{x}_\alpha(t))$ is also a solution of the problem given below.*
$${}_{t_0}^{C}D_t^p \underline{x}_\alpha(t) = \cup_{\mu=0}^{1} C(\mu,\alpha)\underline{x}_\alpha(t) + \cup_{\mu=0}^{1} D(\mu,\alpha)\underline{x}_\alpha(t-\tau),$$
$${}_{t_0}^{C}D_t^p \overline{x}_\alpha(t) = \cup_{\mu=0}^{1} C(\mu,\alpha)\overline{x}_\alpha(t) + \cup_{\mu=0}^{1} D(\mu,\alpha)\overline{x}_\alpha(t-\tau),$$
$$\underline{x}_\alpha(t) = \underline{x}_{\alpha 0}$$
$$\overline{x}_\alpha(t) = \overline{x}_{\alpha 0}$$
The elements of the matrices C and D are determined from of $A(\mu,\alpha)$ and $B(\mu,\alpha)$as

follows:

$$c_{ij} = \begin{cases} ea_{ij}(\mu,\alpha), & a_{ij} \geq 0 \\ ga_{ij}(\mu,\alpha), & a_{ij} < 0 \end{cases}$$

and

$$d_{ij} = \begin{cases} eb_{ij}(\mu,\alpha), & b_{ij} \geq 0 \\ gb_{ij}(\mu,\alpha), & b_{ij} < 0 \end{cases}$$

where e is the identity operation and g corresponds to negative value in $\Re$ and $\forall$ $z,w \in \Re$,
$e:(z,w) \to (z,w)$,
$g:(z,w) \to (w,z)$.

3. Fractional Fuzzy Delay Predator-Prey System

Very recently, Normah Mann et al. [20] investigated the stability of ordinary fuzzy delay predator prey system. Motivated by the above research paper, this paper will deal with the model of the form

$$ {}^{C}_{t_0}D^p_t x(t) = x(t)(1 - x(t)) - dx(t) + be^{-c\tau} x(t-\tau), \quad 0 \leq p \leq 1 \tag{6}$$

Here $b(x)$ is a continuous, positive, decreasing function, i.e., the per capita growth rate of the population decreased with increase in population levels. The delay in this instance can indicates a gestation or maturation period, so the number of individuals entering the population depends on the levels of the population at a previous instance of time. The function $d(x)$ is a nondecreasing and positive function. This represents the per capita death rate. Here we concentrate on fuzzification, steady states and stability.

3.1. Linear Fuzzification.

Let fuzzify the linear part of the system (6) by triangular fuzzy number, which is symmetric and let $x(t)$is a non negative fuzzy functions. Let

$$\widetilde{1} = (1 - (1-\alpha)\sigma_1, 1 + (1-\alpha)\sigma_1)$$

$$\widetilde{d} = (d - (1-\alpha)\sigma_1, d + (1-\alpha)\sigma_1)$$

By using Definition 5 system (6) can be written as follows:

$$\begin{bmatrix} {}^{C}_{t_0}D^p_t \underline{x}_\alpha(t) \\ {}^{C}_{t_0}D^p_t \overline{x}_\alpha(t) \end{bmatrix} = \begin{bmatrix} a_1 - a_2 & 0 \\ 0 & a_1 - a_2 \end{bmatrix} \begin{bmatrix} \underline{x}_\alpha \\ \overline{x}_\alpha \end{bmatrix} + \begin{bmatrix} -\underline{x}^2_\alpha(t) + be^{-c\tau}\underline{x}_\alpha(t-\tau) \\ -\overline{x}^2_\alpha(t) + be^{-c\tau}\overline{x}_\alpha(t-\tau) \end{bmatrix} \tag{7}$$

where

$$a_1 = (1-\mu)(1 - (1-\alpha)\sigma_1) + \mu(1 + (1-\alpha)\sigma_1),$$

$$a_2 = (1-\mu)(d - (1-\alpha)\sigma_1) + \mu(d + (1-\alpha)\sigma_1),$$

$0 \leq \mu \leq 1$ and $0 \leq p \leq 1$
Then (7) is known as fuzzy fractional delay predator-prey (FDPP) system.

3.2. **Steady States.** To find the steady states of the system (7), assume that the constant $(\underline{x}^*, \overline{x}^*)_\alpha$, is a solution and our aim is to determine the values of these constant. The equations for determining steady states are

$$\underline{x}_\alpha((a_1 - a_2) - x_\alpha + be^{-c\tau}) = 0 \tag{8}$$

$$\overline{x}_\alpha((a_1 - a_2) - x_\alpha + be^{-c\tau}) = 0 \tag{9}$$

If $\underline{x}_\alpha = 0$ and $\overline{x}_\alpha = 0$ then the first and the second equations of (8-9) are satisfied. Here (0,0) $_\alpha$ is trivial steady state.
If $\underline{x}_\alpha$ and $\overline{x}_\alpha$ are not equal zero then the steady state equations are:

$$(a_1 - a_2) - \underline{x}_\alpha + be^{-c\tau} = 0 \tag{10}$$

$$(a_1 - a_2) - \overline{x}_\alpha + be^{-c\tau} = 0 \tag{11}$$

where $\underline{a_1} = (1-(1-\alpha)\sigma_1)$ and $\overline{a_1} = 1 + (1-\alpha)\sigma_1)$ So, if the equation (10-11) are satisfied, then the system (7) has a nontrivial steady state$(\underline{x}^*, \overline{x}^*)_\alpha$. Thus, the system (7) has the steady state solutions such that ;$(0,0)_\alpha$ and the nontrivial steady state $(\overline{x}^*, \overline{x}^*)_\alpha$

3.3. **Linear Stability.** The linearization of the fuzzy system (7) about the trivial steady state $(0,0)_\alpha$ is

$$\begin{bmatrix} {}^C_{t_0}D^p_t \underline{x}_\alpha(t) \\ {}^C_{t_0}D^p_t \overline{x}_\alpha(t) \end{bmatrix} = \begin{bmatrix} a_1 - a_2 & 0 \\ 0 & a_1 - a_2 \end{bmatrix} \begin{bmatrix} \underline{x}_\alpha \\ \overline{x}_\alpha \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \underline{x}_\alpha(t) \\ \overline{x}_\alpha(t) \end{bmatrix}$$

where $x_t\alpha = x_\alpha(t-\tau)$. The characteristic equation is obtained as

$$\lambda = a_1 - a_2 \tag{12}$$

If $a_1 < a_2 \to$ stable, $a_1 > a_2 \to$ un stable.

Proposition 1

A trivial steady state $(0,0)_\alpha$ with characteristic equation (12) is stable or unstable using some condition for all value delay.
Similarly, for the non trivial steady state $(\underline{a_1} - \underline{a_2}, \overline{a_1} - \overline{a_2})$
consider $(\underline{a_1} - \underline{a_2}, \overline{a_1} - \overline{a_2})$=$(\underline{a}, \overline{a})$

$$\begin{bmatrix} {}^C_{t_0}D^p_t \underline{x}_\alpha(t) \\ {}^C_{t_0}D^p_t \overline{x}_\alpha(t) \end{bmatrix} = \begin{bmatrix} a - 2\underline{a} & 0 \\ 0 & a - 2\underline{a} \end{bmatrix} \begin{bmatrix} \underline{x}_\alpha \\ \overline{x}_\alpha \end{bmatrix} + \begin{bmatrix} be^{-c\tau} & 0 \\ 0 & be^{-c\tau} \end{bmatrix} \begin{bmatrix} \underline{x}_\alpha(t) \\ \overline{x}_\alpha(t) \end{bmatrix} \tag{13}$$

The characteristic equation for (13) is

$$(\lambda^2 + A\lambda + B) + e^{-(c+\lambda)\tau}(C\lambda + D) + e^{-2(c+\lambda)t}(E) = 0 \tag{14}$$

where
$A = -2(a_1 - a_2) + 2(\underline{a} + \overline{a})$
$B = (a_1 - a_2)^2 + 4\underline{a}\overline{a} - 2(a_1 - a_2)(\underline{a} + \overline{a})$
$C = -2b$
$D = 2b(a_1 - a_2 - (\underline{a} + \overline{a}))$
$E = b^2$
The steady state is stable in the absence of delay if the roots of $\lambda^2 + (A+C)\lambda + (B+D+E) = 0$ have negative real parts. This occurs if and only if

$$(A+C) > 0, (B+D+E) > 0 \tag{15}$$

The steady state $(\underline{a}, \overline{a})$ is stable in the absence of delay if and only if (15) are satisfied. Now for increasing $\tau, \tau \neq 0$, we first assume that the root of the characteristic equation (14) is $\lambda = i\mu$ and $\mu > 0$. Substitute $\lambda = i\mu$ in (14), we obtain,

$(-\mu^2+Ai\mu+B)+e^{-C\tau}(cos(\mu\tau)-isin(\mu\tau))(Ci\mu+D)+e^{-2C\tau}(cos(2\mu\tau)-isin(2\mu\tau)) = 0$

Separating the real and imaginary parts, we get

$$\begin{aligned} \mu^2 - B &= e^{-C\tau}(Dcos(\mu\tau) + C\mu sin(\mu\tau)) + e^{-2c\tau}(cos(2\mu\tau)), \\ A\mu &= e^{-C\tau}(Dsin(\mu\tau) - C\mu cos(\mu\tau)) + e^{-2c\tau}(sin(2\mu\tau)). \end{aligned}$$

Squaring and adding both sides gives the polynomial of degree four as follows:

$$\begin{aligned} (\mu^2 - B)^2 + (A\mu)^2 =& (e^{-C\tau}(Dcos(\mu\tau) + C\mu sin(\mu\tau)) + e^{-2c\tau}(cos(2\mu\tau)))^2 \\ &+ (e^{-C\tau}(Dsin(\mu\tau) - C\mu cos(\mu\tau)) + e^{-2c\tau}(sin(2\mu\tau)))^2 \quad (16) \end{aligned}$$

As $\tau \to \infty$, the right hand side of (16)$\to$0 and let $\gamma = \mu^2$ the equation (16) can be written in terms of γ as follows:

$$S(\gamma) = \gamma^2 + (A^2 - 2B)\gamma + B^2 = 0 \quad (17)$$

This can be simplified by substituting the known values of A,and B. For the γ coefficient, we have

$$\begin{aligned} A^2 - 2B &= (-2(a_1 - a_2) + 2(\underline{a} + \overline{a}))^2 - 2(a_1 - a_2)^2 + 4\underline{a}\overline{a} - 2(a_1 - a_2)(\underline{a} + \overline{a}) \\ &= (2(a_1 - a_2))^2 - 2(a_1 - a_2)^2 = 2(a_1 - a_2)^2 \geq 0 \quad (18) \end{aligned}$$

which is always positive. Equation (18) are positive coefficient if the right hand side of (18) are greater than zero for certain value of α. Finally, the constant term B^2 is always positive.

Therefore all the coefficients of the polynomial (17) are positive and it has no positive real roots. In other words iμ is not a root of the characteristic equation (14) for increasing delay. Hence, the system (7) cannot lead to a bifurcation. It means that the non trivial steady state is locally asymptotically stable for all values of delay . We conclude the following proposition:

Proposition 2

A non-trivial steady state $(\underline{a}, \overline{a})$ with characteristic equation (14) is locally asymptotically stable for all values of delay if and only if

- $(A + C) > 0, (B + D + E) > 0$, where A,B,C,D,E are previously given.
- $(2(a_1 - a_2) - 4d + 4)^2 + 2((a_1 - a_2)^2 - 4a + 4ad + 4[(1 - (1 - \alpha)\sigma_1) - (d - (1 - \alpha)\sigma_2)][(1 + (1 - \alpha)\sigma_1) - (d + (1 - \alpha)\sigma_2)]) > 0$ for certain value of α

4. Numerical Examples

Example 4.1. *Consider the problem without (7) fuzzy and an ordinary system. Let $\tau = 0$. That is the given system will turn into an ordinary system with constant co-efficients.*

$$^{C}_{t_0}D^p_t x(t) = x(t)(1 - x(t)) - dx(t) + b * x(t) \quad (19)$$

Case 1: If we fix the parameter as b=0.1, d=1.4 with the initial conditions $x(0) = 1$. Then it is possible to analyze the give system by using the concept of characteristic polynomial and eigen values. For this problem eigen value is -0.3, which has the negative real part,also,the non negative term satisfies the condition $f(t, 0) = 0$ For details see [20]. Hence the given system is asymptotically stable. See Fig 1

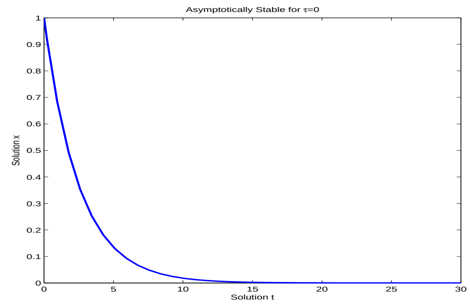

Fig.1

Case 2: If we fix the parameter as b=0.1, d=0.4 with the initial conditions $x(0) = 1$. Same way one may find an eigen value as 0.7, which does not the negative real part, hence it is not asymptotically stable. See Fig 2

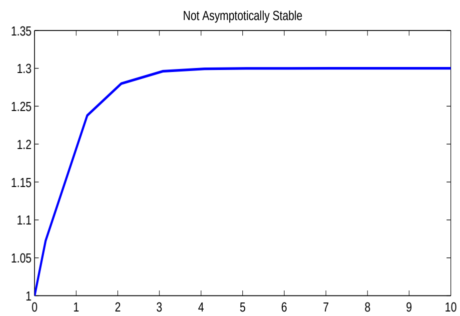

Fig.2

Example 4.2. *Consider the model (7) with $\alpha = 1$ and $\tau = 1$.*

Since $\alpha = 1$, the given system will turn into a ordinary system with delay. Lets fix the parameter as b=0.1, c=0.5, d=0.4, with the initial condition$(\underline{x}_\alpha, \overline{x}_\alpha) = (m_1 - (1-\alpha)\sigma_4, m_1 + (1-\alpha)\sigma_5)$ with $m_1 = 2, \sigma_4 = 0.2, \sigma_5 = 0.2,$. For $\alpha = 1$ and $\tau = 1$,the non-trivial steady state of the model is given by (0.6607,0.6607)
Straight forward computation gives $A = 1.4428, B = 0.5204, C = -0.2000, D = -0.1443, E = 0.0100, A + C = 1.2428 \geq 0, B + D + E = 0.3861 \geq 0$. Eigen values are given by -0.6276 and -0.6152. It may be noted that eigenvalues satisfies the necessary condition, that is negativity of a real part. Hence the given system is asymptotically stable, one can see in the Fig 3.

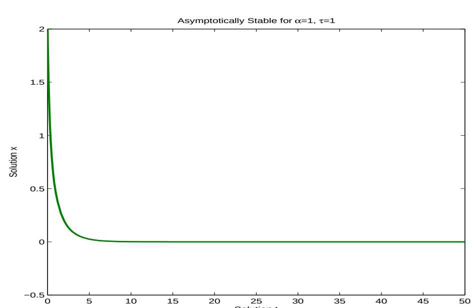

Fig.3

Example 4.3. *Consider the model (7) with $\alpha = 1/2$ and $\tau = 1$.*

Since $\alpha \neq 1$, we may fuzzify the given system. Lets fix the parameter as b=0.1, c=0.5, d=0.4, $\sigma_1 = 1.4$, $\sigma_2 = 0.1$, $\sigma_3 = 0.4$, $\sigma_4 = -1.0$, $\sigma_5 = 2.0$, μ=1 with the initial conditions $(\underline{x}_\alpha, \overline{x}_\alpha) = (m_1 - (1-\alpha)\sigma_4, m_1 + (1-\alpha)\sigma_5)$ where $m_1 = 1$. For $\alpha = 1/2$ and $\tau = 1$,the non-trivial steady state of the model is (0.6226,2.1652). Straight forward computation gives $A = 1.4756, B = 1.8353, C = -0.2000, D = 0.1476, E = 0.0100, A + C = 1.2756 \geq 0, B + D + E = 1.9930 \geq 0$. Eigenvalues are given by $-0.6378 \pm 1.2594i$. It may be noted that eigenvalues has the negative real part. Hence the given system is asymptotically stable, one can see in the Fig 4.

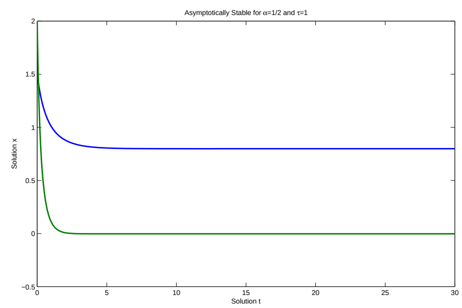

Fig.4

Example 4.4. *Consider the model (7) with $\alpha = 0.8$ and $\tau = 5$.*

Since $\alpha \neq 1$, we may fuzzify the given system. Lets fix the parameter as b=-0.01, c=0.5, d=0.4, $\sigma_1 = 1.4$, $\sigma_2 = 0.1$, $\sigma_3 = 0.4$, $\sigma_4 = 5.0$, $\sigma_5 = -5.0$, μ=1 with the initial conditions $(\underline{x}_\alpha, \overline{x}_\alpha) = (m_1 - (1-\alpha)\sigma_4, m_1 + (1-\alpha)\sigma_5)$ where $m_1 = 3$. For $\alpha = 0.8$ and $\tau = 5$,the non-trivial steady state of the model is (1.0854,1.7025). Straight forward computation gives $A = 2.2558, B = 0.8913, C = 0.0200, D = 0.0226, E = 1.0000e{-}004, A{+}C = 2.2758 \geq 0, B{+}D{+}E = 0.9140 \geq 0$. Furthrmore, eignvalues are -1.7552 and -0.5206. Hence the given system is asymptotically stable, one can see in the Fig 5.

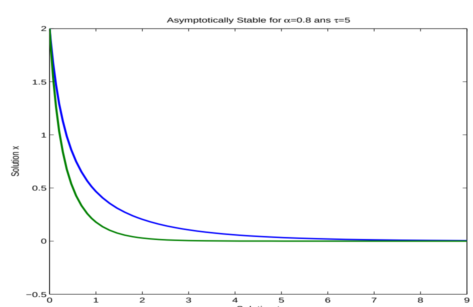

Fig.5

5. Conclusion

The main part of this work deals with stability criteria for time-delay systems. A system of fuzzy delay predator-prey (FDPP) equations analyzed by using fuzzy number. The FDPP system has trivial, and nontrivial steady states. In this case the characteristic equation is of degree 2. The fuzzy system proposed leads to the difficulty of locating the roots of the characteristic equation since the system becomes larger compare with the crisp system. Generally, the situation is more complex to arrive at general conditions on the coefficients of characteristic equation such that it describes a locally asymptotically stable for non trivial steady state for all values of delay, and the trivial steady state is stable or unstable using certain condition. Examples are given to illustrate the efficiency of the proposed method. In that the given system without delay is compared with different values of delay, also, different values of α are considered, especially when $\alpha = 1$,the system will turn into a crisp system, when $\alpha \neq 1$, the given system may be fuzzified. It may be noted that from example , particularly when $b = 0.4$ the given system is not asymptotically stable with out delay, but in presence of delay, the same system is asymptotically stable. This shows the efficiency of the proposed system. Finally, the results are concluded in Propositions 1 and 2.

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