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SOME CERTAIN UNIFIED INTEGRAL FORMULAE'S INVOLVING ALEPH FUNCTION AND GENERALIZED POLYNOMIALS

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Abstract. A large number of integral formulas involving a variety of special functions and polynomials have been developed by many authors. Here, in this paper, we aim at establishing some integral formulas involving the product of Aleph-function and general class polynomials. The H-functions, introduced by Fox in 1961, the well-known Mellin inversion theorem and hypergeometric function ${}_2F_1(\alpha,\beta;\gamma;x)$ have also been considered in the article. Some interesting special cases of main result are given in the form of corollaries. All the results derived here being of general character, they are seen to yield a number of results (known and new) in theory ofspecial functions.

1. Introduction and Preliminaries

A significantly large number of works on the subject of special functions give interesting account of theory and application of Hypergeometric function, G-function, H-function, I-function and Aleph function in many different areas of mathematical analysis, mathematical physics, and applicable mathematics. Extensions of a number of well known special functions were investigated by many authors (see, e.g. [4],[9] , [11], [20]-[22]).

Definition 1.1. The H-functions, introduced by Fox [3] in 1961as symmetrical Fourier kernels, can be regarded as the extreme generalization of the generalized hypergeometric functions pFq, beyond the Meijer G-functions. Like the Meijer G-functions, the Fox H-function turns out to be related to the Mellin-Barnes integrals and to the Mellin transforms, but in a more general way. After Fox, the H-functions were carefully investigated by Braaksma [2], who provided their convergent and asymptotic expansions in the complex plane, based on their Mellin-Barnes integral representation.
According to standard notation, the Fox H-function is defined as

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$$H_{p,q}^{m,n}[z] = \frac{1}{2\pi i}\int_L H_{p,q}^{m,n}(s) z^s ds \tag{1}$$

where L is suitable path in the complex plane C to be disposed later, $z^s = exp\{s(log|z| + iarg\ z)\}$, L is a suitable contour of the Mellin-Barnes type running $\gamma - i\alpha$ to $\gamma + i\alpha, \gamma \in R$, in the complex s-plane.

$$H_{p,q}^{m,n}[s] = \frac{A(s)B(s)}{C(s)D(s)} \tag{2}$$

where

$$\left.\begin{array}{c} A(s) = \prod_{j=1}^{m}\Gamma(b_j - \beta_j s), B(s) = \prod_{j=1}^{n}\Gamma(1 - a_j + \alpha_j s) \\ C(s) = \prod_{j=m+1}^{q}\Gamma(1 - b_j + \beta_j s), D(s) = \prod_{j=n+1}^{p}\Gamma(a_j - \alpha_j s) \end{array}\right\} \tag{3}$$

with $0 \leq n \leq p$, $1 \leq n \leq q, \{a_j, b_j\} \in C, \{\alpha_j, \beta_j\} \in R^+$. An empty product, when it occurs, is taken to be one so.

$$n = 0 \Leftrightarrow B(s) = 1, m = q \Leftrightarrow C(s) = 1, n = p \Leftrightarrow D(s) = 1$$

Due to the occurrence of the factor z^s in the integrand of (1) , the H-function is, in general, multi-valued, but it can be made one-valued on the Riemann surface of $log\ z$ by choosing a proper branch. We also note $\alpha's$ and $\beta's$ that when the are equal to 1, we obtain the Meijer's G-function $G_{p,q}^{m,n}(z)$.

The above integral representation on the H-functions, by involving products and ratios of Gamma functions, is known to be Mellin-Barnes integral type. A compact notation is usually adopted for (1)

$$H_{p,q}^{m,n}[z] = H_{p,q}^{m,n}\left[\ z \left| \begin{array}{c} (a_j, \alpha_j)_{j=1,\dots,p} \\ (b_j, \beta_j)_{j=1,\dots,q} \end{array} \right.\right] \tag{4}$$

Detail regarding existence conditions and various parametric restrictions of H-function we may refer ([2],[5],[7]and [13]).

Definition 1.2. The Aleph $(\aleph)-$ function, introduced by Südland [10] however the notation and complete definition is presented here in the following manner in terms of the Mellin-Barnes type integrals

$$\aleph[z] = \aleph_{x_i,y_i,\tau_i:r}^{m,n}\left[\ z \left| \begin{array}{c} (a_j, A_j)_{1,n}, [\tau_i\,(a_{ji}, A_{ji})]_{n+1,x_i}\,; r \\ (b_j, B_j)_{1,m}, [\tau_i\,(b_{ji}, B_{ji})]_{m+1,y_i}\,; r \end{array} \right.\right]$$
$$= \frac{1}{2\pi\omega}\int_L \Omega_{x_i,y_i,\tau_i:r}^{m,n}(s) z^{-s} ds. \tag{5}$$

for all $z \neq 0$ where $\omega = \sqrt{(-1)}$ and

$$\Omega_{x_i,y_i,\tau_i:r}^{m,n}(s) = \frac{\prod_{j=1}^{m}\Gamma(b_j + B_j s)\prod_{j=1}^{n}\Gamma(1 - a_j - A_j s)}{\sum_{i=1}^{r}\tau_i\prod_{j=n+1}^{x_i}\Gamma(a_{ji} + A_{ji}s)\prod_{j=m+1}^{y_i}\Gamma(1 - b_{ji} - B_{ji}s)} \tag{6}$$

The integration with $L = L_{i\gamma\infty}, \gamma \in R$ extends from $\gamma - i\infty$ to $\gamma + i\infty$ and is such that the poles, assumed to be simple, of $\Gamma(1 - a_j - A_j s), j = 1, ..., n$ do not coincide with the pole of $(b_j + B_j s), j = 1, ..., m$ the parameter p_i, q_i are non-negative integers satisfying $0 \leq n \leq x_i$, $1 \leq m \leq y_i$, $\tau_i > 0$ for $i = 1, ..., r$ and $A_j, B_j, A_{ji}, B_{ji} > 0$ and $a_j, b_j, a_{ji}, b_{ji} \in C$. The empty product is interpreted as unity. The existence conditions for the defining integral are given below

$$\phi_l > 0, |arg(z)| < \frac{\pi}{2}\phi_l, l = 1, ..., r, \tag{7}$$

$$\phi_l \geq 0, |arg(z)| < \frac{\pi}{2}\phi_l, Re\{\xi_l\} + 1 < 0, \tag{8}$$

where

$$\phi_l = \prod_{j=1}^{n} A_j + \prod_{j=1}^{m} B_j - \tau_i \left(\prod_{j=n+1}^{x_l} A_{jl} + \prod_{j=m+1}^{y_l} B_{jl} \right), \tag{9}$$

and

$$\xi_l = \prod_{j=1}^{n} b_j - \prod_{j=1}^{m} a_j + \tau_l \left(\prod_{j=n+1}^{x_l} b_{jl} - \prod_{j=m+1}^{y_l} a_{jl} \right) + \frac{1}{2}(x_l - y_l), l = 1, ..., r. \tag{10}$$

For detail account of Aleph [ℵ]-function see [4] and [6].

Definition 1.3. $S_N^M[x]$ occurring in the sequel denotes the general class of polynomials introduced by Srivastava [17]:

$$S_N^M[x] = \sum_{k=0}^{[N/M]} \frac{(-N)_{Mk}}{k!} A_{N,k} x^k, N = 0, 1, 2, ... \tag{11}$$

where M is an arbitrary positive integer and the coefficients $A_{N,k}(N, k \geq 0)$ are arbitrary constants, real or complex. On suitably specialize the coefficients $A_{N,k}, S_N^M[x]$ yields a number of known polynomials as its special cases (see Srivastava and Singh [16], pp.158-161).

Definition 1.4. The hypergeometric function $F(a, b; c; x)$ is defined as

$$F(a, b; c; x) =_2 F_1(a, b; c; x) = F(b, a; c; x) = 1 + \frac{abx}{c} + \frac{a(a+1)b(b+1)}{c(c+1)} \frac{x^2}{2!} + ...$$

$$= \sum_{n=0}^{\infty} \frac{(a)_n (b)_n}{(c)_n} \frac{x^n}{n!}, |x| < 1, C \neq 0, -1, -2, ... \tag{12}$$

where
2-refers to number of parameters in numerator and
1-refers to number of parameters in denominator.
$(a)_n$ the Pochhammer symbol defined by (for $a \in C$) (see [18], p.2 and pp.4-6):

$$(a)_n = \begin{cases} 1 & , n = 0 \\ a(a+1)...(a+(n-1)), & n \in N \end{cases} = \frac{\Gamma(a+1)}{\Gamma(a)} (a \in C/z_0^-). \tag{13}$$

Here Γ denotes the familiar gamma function.

Important Results:

(a) From Rainville [8],we have

$$\sum_{n=0}^{\infty}\sum_{K=0}^{\infty}P(K,n)=\sum_{n=0}^{\infty}\sum_{K=0}^{n}P(K,n-K) \tag{14}$$

(b)The Mellin transform of the H-function follows from the definition (1) in view of well known Mellin inversion theorem,we have

$$\int_0^{\infty}x^{s-1}H_{p,q}^{m,n}\left[\ ax\middle|\begin{array}{l}(a_j,\alpha_j)_{1,p}\\(b_j,\beta_j)_{1,q}\end{array}\right]=a^{-s}\theta(-s)$$

$$=\frac{a^{-s}\prod_{j=1}^{m}\Gamma(b_j+\beta_j s)\prod_{j=1}^{n}\Gamma(1-a_j-\alpha_j s)}{\prod_{j=n+1}^{p}\Gamma(a_j+\alpha_j s)\prod_{j=m+1}^{q}\Gamma(1-b_j-\beta_j s)} \tag{15}$$

where

$$A=\sum_{j=1}^{n}\alpha_j-\sum_{j=n+1}^{p}\alpha_j+\sum_{j=1}^{m}\beta_j-\sum_{j=m+1}^{q}\beta_j>0$$

$$|arg\ a|<\frac{1}{2}A\pi,\delta=\sum_{j=1}^{q}\beta_j-\sum_{j=1}^{n}\alpha_j>0$$

and

$$min_{1\le j\le m}\left[Re\left(\frac{b_j}{\beta_j}\right)\right]<Re(s)<min_{1\le j\le n}\left[Re\left(\frac{-a_j}{\alpha_j}\right)\right]$$

Remark 1.1 The exponential function $e^{(t-x)Z}$ can be expressed by

$$\sum_{W=0}^{\infty}\frac{(t-x)^W Z^W}{W!} \tag{16}$$

2. The Main Integral Formuale

Theorem 2.1 Let $\alpha_j,\beta_j,\alpha_{ji},\beta_{ji}\in R^+$ and $a_j,b_j,a_{ji},b_{ji}\in C;\eta>0$, the integers m,n,p,q satisfy the inequalities $0\le n\le p_i$ $0\le m\le q_i,\tau_i>0;1,2,...,r$,then

$$\int_0^{\infty}y^{\lambda-1}e^{hx}S_N^M[y]\,{}_2F_1(\alpha,\beta;\gamma;hy^{\eta})\aleph_{p_i,q_i,\tau_i:r}^{m_1,n_1}\left[\ zy^{\eta}\middle|\begin{array}{l}(a_j,\alpha_j)_{1,n_1};[\tau_i\,(a_{ji},\alpha_{ji})]_{n_1+1,p_i};r\\(b_j,\beta_j)_{1,m_1};[\tau_i\,(b_{ji},\beta_{ji})]_{m_1+1,q_i};r\end{array}\right]$$

$$\times H_{p,q}^{m,n}\left[\ \omega y\middle|\begin{array}{l}(c_j,\gamma_j)_{1,n};(c_j,\gamma_j)_{n+1,p}\\(d_j,\delta_j)_{1,m},(d_i,\delta_j)_{m+1,q}\end{array}\right]dy$$

$$=W^{-\lambda}\sum_{k=0}^{[N/M]}\frac{(-N)_{M,k}}{k!}A_{N,k}W^{-k}\sum_{W=0}^{\infty}\sum_{K=0}^{n}\frac{(\alpha)_K(\beta)_K}{(\gamma)_K}\frac{h^K z^{(W-K)}}{K!(W-K)!}W^{(1-\eta)K-W}$$

$$\times\aleph_{p_i+q,q_i+p,\tau_i:r}^{m_1+n,n_1+m}\left[\ \frac{z}{\omega^{\eta}}\middle|\begin{array}{l}(a_j,\alpha_j)_{1,n_1},(1-d_j-(\lambda+(\eta-1)K+k+W)\delta_j,\sigma\delta_j)_{1,m}:\\(b_j,\beta_j)_{1,m_1},(1-c_j-(\lambda+(\eta-1)K+k+W)\gamma_j,\sigma\gamma_j)_{1,n}:\end{array}\right.$$

$$\left.\begin{array}{l}(\tau_i(a_{ji},\alpha_{ji})_{n_1+1,p_i});r,(1-d_j-(\lambda+(\eta-1)K+k+W)\delta_j,\sigma\delta_j)_{m+1,q}\\(\tau_i(b_{ji},\beta_{ji})_{m_1+1,q_i});r,(1-c_j-(\lambda+(\eta-1)K+k+W)\gamma_j,\sigma\gamma_j)_{n+1,p}\end{array}\right],\eta>0 \tag{17}$$

Provided: For Aleph function
$(i)\phi_l > 0, |arg(z)| < \frac{\pi}{2}\phi_l, l = 1, ..., r$
$(ii)\phi_l \geq 0, |arg(z)| < \frac{\pi}{2}\phi_l, Re\{\xi_l\} + 1 < 0.$
where

$$\phi_l = \prod_{j=1}^{n_1} A_j + \prod_{j=1}^{m_1} B_j - \tau_i \left(\prod_{j=n_1+1}^{p_i} \alpha_{ji} + \prod_{j=m_1+1}^{q_i} \beta_{ji} \right),$$

$$\xi_l = \prod_{j=1}^{n_1} b_j - \prod_{j=1}^{m_1} a_j + \tau_i \left(\prod_{j=n_1+1}^{p_i} a_{ji} - \prod_{j=m_1+1}^{q_i} b_{ji} \right) + \frac{1}{2}(p_i - q_i).$$

Also for H-function
$(i)\psi_i > 0, |arg(z)| < \frac{\pi}{2}\psi_i,$
$(ii)\psi_i \geq 0, |arg(z)| < \frac{\pi}{2}\psi_i, Re\{\xi_i\} < 0.$
and

$$\psi_i = \prod_{j=1}^{m} \delta_j + \prod_{j=1}^{n} \gamma_j - \prod_{j=m+1}^{q} - \prod_{j=n+1}^{p} \gamma_j,$$

$$\zeta_i = \frac{1}{2}(p - q) + \prod_{j=1}^{q} d_j - \prod_{j=1}^{p} C_j.$$

Also the coefficient $A_{N,K}$ are arbitrary constants,real or complex.

Proof. To establish (17),express the Aleph function ,general polynomial and hypergeometric function with help of (5),(11) and (12) respectively,then left hand side of result reduces to,

$$\int_0^\infty y^{\lambda-1} \sum_{W=0}^{\infty} \frac{h^W y^W}{W!} \sum_{k=0}^{[N/M]} \frac{(-N)_{M,k}}{k!} A_{N,k} y^k \sum_{K=0}^{\infty} \frac{(\alpha)_k (\beta)_k}{(\gamma)_k} \frac{h^K}{K!} y^{\eta K}$$

$$\times \frac{1}{2\pi i} \int_L \Omega_{p_i,q_i,\tau_i;r}^{m,n}(\xi) z^{-\xi} y^{-\eta\xi} \times H_{p,q}^{m,n} \left[\omega y \middle| \begin{array}{c} (c_j, \gamma_j)_{1,n}; (c_j, \gamma_j)_{n+1,p} \\ (d_j, \delta_j)_{1,m}; [(d_j, \delta_j)]_{m+1,q} \end{array} \right] d\xi dy$$

Now using (14),then interchange the order of summation,after that we use the Mellin transform of h-function by virtue of (15),we get

$$I = \omega^{-\lambda} \sum_{k=0}^{[N/M]} \frac{(-N)_{M,k}}{k!} A_{N,k}.\omega^{-k} \sum_{W=0}^{\infty} \sum_{K=0}^{\infty} \frac{(\alpha)_k (\beta)_k}{(\gamma)_k} \frac{h^K}{K!} \frac{Z^{W-K}}{(W-K)!} \omega^{-(\eta-1)K-W}$$

$$\times \frac{1}{2\pi i} \int_L \Omega_{p_i,q_i,\tau_i;r}^{m,n}(\xi)$$

$$\times \frac{\prod_{j=1}^{m} \Gamma\left(d_j + (\lambda + (\eta-1)K + k + W)\delta_j - \sigma\delta_j\xi\right)}{\prod_{j=m+1}^{q} \Gamma\left(1 - d_j - (\lambda + (\eta-1)K + k + W)\delta_j + \sigma\delta_j\xi\right)}$$

$$\times \frac{\prod_{j=1}^{n} \Gamma\left(1 - c_j + (\lambda + (\eta-1)K + k + W)\gamma_j + \sigma\gamma_j\xi\right)}{\prod_{j=n+1}^{p} \Gamma\left(c_j + (\lambda + (\eta-1)K + k + W)\gamma_j - \sigma\gamma_j\xi\right)} z^{-\xi} \omega^{\eta\xi} d\xi$$

Finally interpreting the contour integral by virtue of (6),we arrive at the desired result (17).

Corollary 2.1 Let $\alpha_j, \beta_j, \alpha_{ji}, \beta_{ji} \in R^+$ and $a_j, b_j, a_{ji}, b_{ji} \in C; \eta > 0$, the integers m,n,p,q satisfy the inequalities $0 \le n \le p_i$ $0 \le m \le q_i, \tau_i > 0; 1, 2, ..., r$,then

$$\int_0^t x^{\lambda-1}(t-x)^{\mu-1}e^{-xz}S_N^M[x(t-x)]\ {}_2F_1(\alpha,\beta;\gamma;hx^\rho(t-x)^\eta)$$

$$\times\aleph^{m,n}_{p_i,q_i,\tau_i:r}\left[\ yx^{-\sigma}(t-x)^{-v}\middle|\begin{array}{l}(a_j,\alpha_j)_{1,n},[\tau_i(a_{ji},\alpha_{ji})]_{n+1,p_i};r\\(b_j,\beta_j)_{1,m},[\tau_i(b_{ji},\beta_{ji})]_{m+1,q_i};r\end{array}\right]dx$$

$$= e^{-zt}t^{\lambda+\mu-1}\sum_{k=0}^{[N/M]}\frac{(-N)_{M,k}}{k!}A_{N,k}.t^{2k}\sum_{W=0}^{\infty}\sum_{K=0}^{\infty}\frac{(\alpha)_k(\beta)_k}{(\gamma)_k}\frac{h^K}{K!}\frac{Z^{(W-K)}}{(W-K)!}t^{(\rho+\eta-1)K+W}$$

$$\times\aleph^{m+2,n}_{p_i+2,q_i+2,\tau_i:r}\left[\ \frac{y}{t^{\sigma+\gamma}}\middle|\begin{array}{c}(a_j,\alpha_j)_{1,n},[\tau_i(a_{ji},\alpha_{ji})]_{n+1,p_i};r,\\(\lambda+\rho K+k,\sigma),(\mu+(\eta-1)K+k+W,v),\end{array}\right.$$

$$\left.\begin{array}{c}(\lambda+\mu+(\rho+\eta-1)K+2k+W,\sigma+v)\\(b_j,\beta_j)_{1,m},[\tau_i(b_{ji},\beta_{ji})]_{m+1,q_i};r\end{array}\right] \tag{18}$$

where
(i) $\sigma \ge 0, v \ge 0 \Longrightarrow \sigma + v \ge 0$,
(ii)$\phi_i > 0, |arg(z)| < \frac{\pi}{2}\phi_i, i = 1, ..., r$,
(iii) $\phi_i \ge 0, |arg(z)| < \frac{\pi}{2}\phi_i, Re\{\xi_i\} + 1 < 0$.

provided

$$\phi_i = \prod_{j=1}^{n}\alpha_j + \prod_{j=1}^{m}\beta_j - \tau_i\left(\prod_{j=n+1}^{p_i}\alpha_{ji} + \prod_{j=m+1}^{q_i}\beta_{ji}\right),$$

and

$$\xi_i = \prod_{j=1}^{n}b_j - \prod_{j=1}^{m}a_j + \tau_i\left(\prod_{j=n+1}^{p_i}a_{ji} - \prod_{j=m+1}^{q_i}b_{ji}\right) + \frac{1}{2}(p_i - q_i).$$

(iv) ρ and η are non-negative integers such that $\rho + \eta \ge 1$.
(v) The coefficient $A_{N.k}$ are arbitrary constants,real or complex.
Proof. To establish (18),express the Aleph function ,general polynomial and hypergeometric function with help of (5),(11) and (12) respectively,then left hand side of result reduces to,

$$I = e^{-zt}\int_0^t x^{\lambda-1}(t-x)^{\mu-1}\sum_{k=0}^{[N/M]}\frac{(-N)_{M,k}}{k!}A_{N,k}.x^k(t-x)^k$$

$$\sum_{W=0}^{\infty}\sum_{K=0}^{n}\frac{(t-x)^W Z^W}{W!}\frac{(\alpha)_k(\beta)_k}{(\gamma)_k}\frac{h^K}{K!}x^{\rho K}(t-x)^{\eta K}$$

$$\times\frac{1}{2\pi i}\int_L\Omega^{m,n}_{p_i,q_i,\tau_i;r}(\xi)y^{-\xi}x^{\sigma\xi}(t-x)^{v\xi}d\xi dx$$

Now using (14),then interchanging the order of summation,we obtain

$$I = e^{-zt}\sum_{k=0}^{[N/M]}\frac{(-N)_{M,k}}{k!}A_{N,k}\sum_{W=0}^{\infty}\sum_{K=0}^{n}\frac{(\alpha)_k(\beta)_k}{(\gamma)_k}\frac{h^K}{K!}\frac{Z^{W-K}}{(W-K)!}$$

$$\times\frac{1}{2\pi i}\int_L \Omega^{m,n}_{p_i,q_i,\tau_i;r}(\xi)y^{-\xi}\ \left\{\int_0^t x^{\lambda+\rho K+\sigma K+k-1}(t-x)^{\mu+(\eta-1)K+W+v\xi+k-1}dx\right\}\ d\xi.$$

we substitute x=ts in the inner integral and inerpreting the contour integral by virtue of(6),we arrive at desired result (18).

Corollary 2.2 Let $\alpha_j,\beta_j,\alpha_{ji},\beta_{ji}\in R^+$ and $a_j,b_j,a_{ji},b_{ji}\in C;\eta>0$, the integers m,n,p,q satisfy the inequalities $0\le n\le p_i\ 0\le m\le q_i,\tau_i>0;1,2,...,r$,then

$$\int_0^t x^{\lambda-1}(t-x)^{\mu-1}e^{-xz}S_N^M[x(t-x)]\ {}_2F_1(\alpha,\beta;\gamma;hx^\rho(t-x)^\eta)$$

$$\times\aleph^{m,n}_{p_i,q_i,\tau_i:r}\left[\ \frac{y(t-x)^v}{x^\sigma}\left|\begin{array}{l}(a_j,\alpha_j)_{1,n},[\tau_i(a_{ji},\alpha_{ji})]_{n+1,p_i};r\\(b_j,\beta_j)_{1,m},[\tau_i(b_{ji},\beta_{ji})]_{m+1,q_i};r\end{array}\right.\right]dx$$

$$=e^{-zt}t^{\lambda+\mu-1}\sum_{k=0}^{[N/M]}\frac{(-N)_{M,k}}{k!}A_{N,k}.t^{2k}\sum_{W=0}^{\infty}\sum_{K=0}^{n}\frac{(\alpha)_k(\beta)_k}{(\gamma)_k}\frac{h^K}{K!}\frac{Z^{W-K}}{(W-K)!}t^{(\rho+\eta-1)K+W}$$

$$\times\aleph^{m+1,n+1}_{p_i+2,q_i+1,\tau_i;r}\left[\ yt^{v-\sigma}\left|\begin{array}{c}(1-\mu-(\eta-1)K-W-k,v),(a_j,\alpha_j)_{1,n};\\(\lambda+\rho K+k,\sigma),(b_j,\beta_j)_{1,m};\end{array}\right.\right.$$

$$\left.\begin{array}{l}[\tau_i(a_{ji},\alpha_{ji})]_{n+1,p_i};r,(\lambda+\mu+(\rho+\eta-1)K+2k+W,\sigma-v)\\ [\tau_i(b_{ji},\beta_{ji})]_{m+1,q_i};r...\end{array}\right] \tag{19}$$

where
(i) $\sigma\ge 0,v\ge 0\Longrightarrow\sigma-v\ge 0$.
(ii) The condition of validity of the above result easily follows from Aleph function.
(iii) ρ and η are non-negative integers such that $\rho+\eta\ge 1$.
(iv) The coefficient $A_{N.k}$ are arbitrary constants,real or complex.
Proof. To establish (19), express the Aleph function, general polynomial, hypergeometric function with help of (5),(11) and (12) respectively. Applying formula (14), then interchange the order of summation and finally we interpreting the contour integral by virtue of (6) after a straightforward calculation, we finally arrive at (19).

Corollary 2.3 Let $\alpha_j,\beta_j,\alpha_{ji},\beta_{ji}\in R^+$ and $a_j,b_j,a_{ji},b_{ji}\in C;\eta>0$, the integers m,n,p,q satisfy the inequalities $0\le n\le p_i\ 0\le m\le q_i,\tau_i>0;1,2,...,r$ and $\sigma\ge 0,v\ge 0$ such that $-\sigma+v\ge 0,t>0$, then

$$\int_0^t x^{\lambda-1}(t-x)^{\mu-1}e^{-xz}S_N^M[x(t-x)]\ {}_2F_1(\alpha,\beta;\gamma;hx^\rho(t-x)^\eta)$$

$$\times\aleph^{m,n}_{p_i,q_i,\tau_i:r}\left[\ \frac{y(t-x)^v}{x^\sigma}\left|\begin{array}{l}(a_j,\alpha_j)_{1,n},[\tau_i(a_{ji},\alpha_{ji})]_{n+1,p_i};r\\(b_j,\beta_j)_{1,m},[\tau_i(b_{ji},\beta_{ji})]_{m+1,q_i};r\end{array}\right.\right]dx$$

$$= e^{-zt}t^{\lambda+\mu-1}\sum_{k=0}^{[N/M]}\frac{(-N)_{M,k}}{k!}A_{N,k}.t^{2k}\sum_{W=0}^{\infty}\sum_{K=0}^{n}\frac{(\alpha)_k(\beta)_k}{(\gamma)_k}\frac{h^K}{K!}\frac{Z^{W-K}}{(W-K)!}t^{(\rho+\eta-1)K+W}$$

$$\times\aleph^{m+1,n+1}_{p_i+1,q_i+2,\tau_i;r}\left[\begin{array}{c|c} \frac{y}{t^{\sigma-v}} & \begin{array}{c}(1-\lambda-(\eta-1)K-W-k,v),(a_j,\alpha_j)_{1,n};\\ (\lambda+\rho K+k,\sigma),(b_j,\beta_j)_{1,m};[\tau_i(b_{ji},\beta_{ji})]_{m+1,q_i};r,\end{array}\end{array}\right.$$

$$\left.\begin{array}{c}[\tau_i(a_{ji},\alpha_{ji})]_{n+1,p_i};r.............................\\ (1-\lambda-\mu+(\rho+\eta-1)K-2k-W,v-\sigma)\end{array}\right] \tag{20}$$

where
(i) $\sigma \geq 0, v \geq 0 \Longrightarrow v-\sigma \geq 0$.
(ii) The condition of validity of the above result easily follows from Aleph function.
(iii) ρ,$\eta \in Z^+$ such that $\rho+\eta \geq 1$.
(iv) The coefficient $A_{N.k}$ are arbitrary constants,real or complex.
Proof. The proof of the (20) would run parallel to the integral formula (18) asserted by Corollary 2.2. Therefore, we omit the details.

Corollary 2.4 Let $\alpha_j,\beta_j,\alpha_{ji},\beta_{ji} \in R^+$ and $a_j,b_j,a_{ji},b_{ji} \in C;\eta > 0$, the integers m,n,p,q satisfy the inequalities $0 \leq n \leq p_i$ $0 \leq m \leq q_i,\tau_i > 0;1,2,...,r, for\ t > 0$, then

$$\int_0^t x^{\lambda-1}(t-x)^{\mu-1}e^{-xz}S_N^M[x(t-x)]\ {}_2F_1(\alpha,\beta:\gamma;hx^{\rho}(t-x)^{\eta})$$

$$\times\aleph^{m,n}_{p_i,q_i,\tau_i:r}\left[\begin{array}{c|c}\frac{yx^{\sigma}}{(t-x)^{v}} & \begin{array}{c}(a_j,\alpha_j)_{1,n},[\tau_i(a_{ji},\alpha_{ji})]_{n+1,p_i};r\\ (b_j,\beta_j)_{1,m},[\tau_i(b_{ji},\beta_{ji})]_{m+1,q_i};r\end{array}\end{array}\right]dx$$

$$= e^{-zt}t^{\lambda+\mu-1}\sum_{k=0}^{[N/M]}\frac{(-N)_{M,k}}{k!}A_{N,k}.t^{2k}\sum_{W=0}^{\infty}\sum_{K=0}^{\infty}\frac{(\alpha)_k(\beta)_k}{(\gamma)_k}\frac{h^K}{K!}\frac{Z^{W-K}}{(W-K)!}t^{(\rho+\eta-1)K+W}$$

$$\times\aleph^{m+1,n+1}_{p_i+1,q_i+2,\tau_i;r}\left[\begin{array}{c|c} yt^{\sigma-v} & \begin{array}{c}(1-\lambda-\rho K-k,\sigma),(a_j,\alpha_j)_{1,n};\\ (\lambda+(\eta-1)K+W+k,v),(b_j,\beta_j)_{1,m};\end{array}\end{array}\right.$$

$$\left.\begin{array}{c}[\tau_i(a_{ji},\alpha_{ji})]_{n+1,p_i};r...\\ [\tau_i(b_{ji},\beta_{ji})]_{m+1,q_i};r,(1-\lambda-\mu+(\rho+\eta-1)K-2k-W,\sigma-v)\end{array}\right] \tag{21}$$

where
(i) $\sigma \geq 0, v \geq 0 \Longrightarrow \sigma-v \geq 0$.
(ii) The condition of validity of the above result easily follows from Aleph function.
(iii) ρ,$\eta \in Z^+$ such that $\rho+\eta \geq 1$.
(iv) The coefficient $A_{N.k}$ are arbitrary constants,real or complex.
Proof. On applying the same procedure as result (18) as given in section 2,the result (21) is established.

Corollary 2.5 Let $\alpha_j, \beta_j, \alpha_{ji}, \beta_{ji} \in R^+$ and $a_j, b_j, a_{ji}, b_{ji} \in C; \eta > 0$, the integers m,n,p,q satisfy the inequalities $0 \leq n \leq p_i$ $0 \leq m \leq q_i, \tau_i > 0; 1, 2, ..., r$, then

$$\int_0^t x^{\lambda-1}(t-x)^{\mu-1}e^{-xz}S_N^M[x(t-x)]\ {}_2F_1(\alpha,\beta;\gamma;hx^\rho(t-x)^\eta)$$

$$\times\aleph^{m,n}_{p_i,q_i,\tau_i:r}\left[\frac{yx^\sigma}{(t-x)^v}\middle|\begin{array}{l}(a_j,\alpha_j)_{1,n},[\tau_i(a_{ji},\alpha_{ji})]_{n+1,p_i};r\\(b_j,\beta_j)_{1,m},[\tau_i(b_{ji},\beta_{ji})]_{m+1,q_i};r\end{array}\right]dx$$

$$= e^{-zt}t^{\lambda+\mu-1}\sum_{k=0}^{[N/M]}\frac{(-N)_{M,k}}{k!}A_{N,k}.t^{2k}\sum_{W=0}^{\infty}\sum_{K=0}^{\infty}\frac{(\alpha)_k(\beta)_k}{(\gamma)_k}\frac{h^K}{K!}\frac{Z^{(W-K)}}{(W-K)!}t^{(\rho+\eta-1)K+W}$$

$$\times\aleph^{m+1,n+1}_{p_i+2,q_i+1,\tau_i;r}\left[yt^{\sigma-v}\middle|\begin{array}{c}(1-\lambda-\rho K-k,\sigma),(a_j,\alpha_j)_{1,n},\\(\lambda+(\eta-1)K+W+k,v),(b_j,\beta_j)_{1,m},\end{array}\right.$$

$$\left.\begin{array}{l}[\tau_i(a_{ji},\alpha_{ji})]_{n+1,p_i};r,(\lambda+\mu+(\rho+\eta-1)K+2k+W,v-\sigma)\\ [\tau_i(b_{ji},\beta_{ji})]_{m+1,q_i};r...\end{array}\right] \tag{22}$$

where
(i) $\sigma \geq 0, v \geq 0$ such that $\sigma - v \geq 0$.
(ii) The condition of validity of the above result easily follows from Aleph function.
(iii) $\rho,\eta \in Z^+$ such that $\rho + \eta \geq 1$.
(iv) The coefficient $A_{N.k}$ are arbitrary constants,real or complex.
Proof. On applying the same procedure as result (18) as given in section 2,the result (22) is established.

Corollary 2.6 Let $\alpha_j, \beta_j, \alpha_{ji}, \beta_{ji} \in R^+$ and $a_j, b_j, a_{ji}, b_{ji} \in C; \eta > 0$, the integers m,n,p,q satisfy the inequalities $0 \leq n \leq p_i$ $0 \leq m \leq q_i, \tau_i > 0; 1, 2, ..., r$, then

$$\int_0^t x^{\lambda-1}(t-x)^{\mu-1}e^{-xz}S_N^M[x(t-x)]\ {}_2F_1(\alpha,\beta;\gamma;hx^\rho(t-x)^\eta)$$

$$\times\aleph^{m,n}_{p_i,q_i,\tau_i:r}\left[yx^\sigma(t-x)^v\middle|\begin{array}{l}(a_j,\alpha_j)_{1,n},[\tau_i(a_{ji},\alpha_{ji})]_{n+1,p_i};r\\(b_j,\beta_j)_{1,m},[\tau_i(b_{ji},\beta_{ji})]_{m+1,q_i};r\end{array}\right]dx$$

$$= e^{-zt}t^{\lambda+\mu-1}\sum_{k=0}^{[N/M]}\frac{(-N)_{M,k}}{k!}A_{N,k}.t^{2k}\sum_{W=0}^{\infty}\sum_{K=0}^{\infty}\frac{(\alpha)_k(\beta)_k}{(\gamma)_k}\frac{h^K}{K!}\frac{Z^{W-K}}{(W-K)!}t^{(\rho+\eta-1)K+W}$$

$$\times\aleph^{m,n+2}_{p_i+2,q_i+2,\tau_i;r}\left[yt^{\sigma+v}\middle|\begin{array}{c}(1-\lambda-\rho K-k,\sigma),(1-\mu-(\eta-1)K-k-W,v),\\(b_j,\beta_j)_{1,m};[\tau_i(b_{ji},\beta_{ji})]_{m+1,q_i};r,\end{array}\right.$$

$$\left.\begin{array}{c}(a_j,\alpha_j)_{1,n};[\tau_i(a_{ji},\alpha_{ji})]_{n+1,p_i};r\\(1-\lambda-\mu-(\rho+\eta-1)K-2k-W,\sigma+v)\end{array}\right] \tag{23}$$

where
(i) $\sigma \geq 0, v \geq 0$ such that $\sigma + v \geq 0$.
(ii) The condition of validity of the above result easily follows from Aleph function.
(iii) $\rho,\eta \in Z^+$ such that $\rho + \eta \geq 1$.
(iv) The coefficient $A_{N.k}$ are arbitrary constants,real or complex.

Proof. On applying the same procedure as result (18) as given in section 2,the result (23) is established.

3. Special Cases and Application

Since the polynomials $A_{N,k}$ in (11), H-function in (1) and Aleph-function in (5) are very general, the main result (17) can be specialized to yield a large number of integral formulas involving familiar polynomials and special functions; we consider only the following two examples.

Choosing $M = 2$ and $A_{N,k} = (-1)^k$ in (11), the polynomials $S_N^2(x)$ become the Harmite polynomials $H_N(x)$ (see [16];see also [[8], p.187])

$$S_N^2(x) = x^{\frac{N}{2}} H_N\left[\frac{1}{2\sqrt{x}}\right] \tag{24}$$

Example 1. Let $\alpha_j, \beta_j, \alpha_{ji}, \beta_{ji} \in R^+$ and $a_j, b_j, a_{ji}, b_{ji} \in C; \eta > 0$, the integers m,n,p,q satisfy the inequalities $0 \le n \le p_i$ $0 \le m \le q_i, \tau_i > 0; 1, 2, ..., r$,then

$$\int_0^\infty y^{\lambda-1} e^{hx} y^{\frac{N}{2}} H_N\left[\frac{1}{2\sqrt{x}}\right] {}_2F_1(\alpha,\beta;\gamma;hy^\eta)$$
$$\times \aleph^{m_1,n_1}_{p_i,q_i,\tau_i:r}\left[zy^\eta \middle| \begin{matrix} (a_j,\alpha_j)_{1,n_1}; [\tau_i(a_{ji},\alpha_{ji})]_{n_1+1,p_i}; r \\ (b_j,\beta_j)_{1,m_1}; [\tau_i(b_{ji},\beta_{ji})]_{m_1+1,q_i}; r \end{matrix} \right]$$
$$\times H^{m,n}_{p,q}\left[\omega y \middle| \begin{matrix} (c_j,\gamma_j)_{1,n}; (c_j,\gamma_j)_{n+1,p} \\ (d_j,\delta_j)_{1,m}, (d_i,\delta_j)_{m+1,q} \end{matrix} \right] dy$$

$$= W^{-\lambda} \sum_{k=0}^{[N/2]} \frac{(-1)^k(-N)_{2k}}{k!} W^{-k} \sum_{W=0}^{\infty}\sum_{K=0}^{n} \frac{(\alpha)_K(\beta)_K}{(\gamma)_K} \frac{h^K z^{(W-K)}}{K!(W-K)!} W^{(1-\eta)K-W}$$

$$\times \aleph^{m_1+n,n_1+m}_{p_i+q,q_i+p,\tau_i:r}\left[\frac{z}{\omega^\eta} \middle| \begin{matrix} (a_j,\alpha_j)_{1,n_1}, (1-d_j-(\lambda+(\eta-1)K+k+W)\delta_j, \sigma\delta_j)_{1,m} : \\ (b_j,\beta_j)_{1,m_1}, (1-c_j-(\lambda+(\eta-1)K+k+W)\gamma_j, \sigma\gamma_j)_{1,n} : \end{matrix} \right.$$

$$\left. \begin{matrix} (\tau_i(a_{ji},\alpha_{ji})_{n_1+1,p_i}); r, (1-d_j-(\lambda+(\eta-1)K+k+W)\delta_j, \sigma\delta_j)_{m+1,q} \\ (\tau_i(b_{ji},\beta_{ji})_{m_1+1,q_i}); r, (1-c_j-(\lambda+(\eta-1)K+k+W)\gamma_j, \sigma\gamma_j)_{n+1,p} \end{matrix} \right], \eta > 0 \tag{25}$$

Choosing $M = 1$ and $A_{N,k} = \binom{N+\mu}{N}\frac{1}{(\mu+1)_k}$ in (11) , we have

$$S_N^1[x] = L_N^{(\mu)}[x] \tag{26}$$

where $L_N^{(\mu)}[x]$ are Laguerre polynomials (see [16]; see also [[8] p.200]).

Example 1. Let $\alpha_j, \beta_j, \alpha_{ji}, \beta_{ji} \in R^+$ and $a_j, b_j, a_{ji}, b_{ji} \in C; \eta > 0$, the integers m,n,p,q satisfy the inequalities $0 \le n \le p_i$ $0 \le m \le q_i, \tau_i > 0; 1, 2, ..., r$,then

$$\int_0^\infty y^{\lambda-1} e^{hx} y^{\frac{N}{2}} L_N^{(\mu)}[y]\, {}_2F_1(\alpha,\beta;\gamma;hy^\eta)$$
$$\times \aleph^{m_1,n_1}_{p_i,q_i,\tau_i:r}\left[zy^\eta \middle| \begin{matrix} (a_j,\alpha_j)_{1,n_1}; [\tau_i(a_{ji},\alpha_{ji})]_{n_1+1,p_i}; r \\ (b_j,\beta_j)_{1,m_1}; [\tau_i(b_{ji},\beta_{ji})]_{m_1+1,q_i}; r \end{matrix} \right]$$

$$\times H_{p,q}^{m,n}\left[\omega y\left|\begin{array}{c}(c_j,\gamma_j)_{1,n};(c_j,\gamma_j)_{n+1,p}\\(d_j,\delta_j)_{1,m},(d_i,\delta_j)_{m+1,q}\end{array}\right.\right]dy$$

$$=W^{-\lambda}\sum_{k=0}^{[N]}\frac{1}{k!(\mu+1)_k}\binom{N+\mu}{N}W^{-k}\sum_{W=0}^{\infty}\sum_{K=0}^{n}\frac{(\alpha)_K(\beta)_K}{(\gamma)_K}\frac{h^K z^{(W-K)}}{K!(W-K)!}W^{(1-\eta)K-W}$$

$$\times\aleph_{p_i+q,q_i+p,\tau_i:r}^{m_1+n,n_1+m}\left[\frac{z}{\omega^\eta}\left|\begin{array}{c}(a_j,\alpha_j)_{1,n_1},(1-d_j-(\lambda+(\eta-1)K+k+W)\delta_j,\sigma\delta_j)_{1,m}:\\(b_j,\beta_j)_{1,m_1},(1-c_j-(\lambda+(\eta-1)K+k+W)\gamma_j,\sigma\gamma_j)_{1,n}:\end{array}\right.\right.$$

$$\left.\begin{array}{c}(\tau_i(a_{ji},\alpha_{ji})_{n_1+1,p_i});r,(1-d_j-(\lambda+(\eta-1)K+k+W)\delta_j,\sigma\delta_j)_{m+1,q}\\(\tau_i(b_{ji},\beta_{ji})_{m_1+1,q_i});r,(1-c_j-(\lambda+(\eta-1)K+k+W)\gamma_j,\sigma\gamma_j)_{n+1,p}\end{array}\right],\eta>0 \tag{27}$$

Remark 3.1. If we put product of polynomials S_N^M to unity, $\tau_i=1, i=1,...,r$ in (17)-(23),then we can easily obtain the known results given by Saha and Arora [1].
Remark 3.2. Putting $\tau_i, i=1,...,r, h=0$ in (17) and set product of polynomials S_N^M and hypergeometric function to unity,then we can easily obatin the known result given by Sexena [[15],p.66,eq.(4.5.1)]:

$$\int_0^\infty y^{\lambda-1}I_{p_i,q_i,\tau_i:r}^{m_1,n_1}\left[zy^\eta\left|\begin{array}{c}(a_j,\alpha_j)_{1,n_1},(a_{ji},\alpha_{ji})_{n_1+1,p_i}\\(b_j,\beta_j)_{1,m_1},(b_{ji},\beta_{ji})_{m_1+1,q_i}\end{array}\right.\right]$$

$$\times H_{p,q}^{m,n}\left[\omega y\left|\begin{array}{c}(c_j,\gamma_j)_{1,n},(c_j,\gamma_j)_{n+1,p}\\(d_j,\delta_j)_{1,m},(d_j,\delta_j)_{m+1,q}\end{array}\right.\right]dy$$

$$=w^{-\lambda}I_{p_i+q,q_i+p:r}^{m_1+n,n_1+m}\left[\frac{z}{\omega^\eta}\left|\begin{array}{c}(a_j,\alpha_j)_{1,n_1},(1-a_j-\eta\delta_j,\sigma\delta_j)_{1,m};\\(b_j,\beta_j)_{1,m_1},(1-c_j-\eta\gamma_j,\sigma v_j)_{1,n};\end{array}\right.\right.$$

$$\left.\begin{array}{c}(a_{ji},\alpha_{ji})_{n_1+1,p_i},(1-d_j-\eta\delta_j,\sigma\delta_j)_{m+1,q}\\(b_{ji},\beta_{ji})_{m_1+1,q_i},(1-c_j-\eta\gamma_j,\sigma\gamma_j)_{n+1,q}\end{array}\right] \tag{28}$$

Remark 3.3. When $\tau_i=1, i=1,...,r, r=1, t=1, \eta=0$ and we set product of polynomials S_N^M to unity,then results (2),(3) and (7) leads to the known results given by Arora and Saha [1].
Remark 3.4. Putting $r=1, t=1, \eta=0, \tau_i=1, i=1,...,r$ and we set product of polynomials $S_N^M=1$,then Corollaries 2.3, 2.4, 2.5 and 2.6 ,we may obtain new results.

4. Concluding Remark

We conclude this paper with the remark that the results obtained in this paper are useful in deriving certain formulas involving Aleph-function, H-function and general class of polynomials. The main integral formula, whose integrand being the products of Aleph-function and generals class of polynomials as shown in section 2, can be specialized to yield a large number of simpler results. The results are general in nature and can be having applications in variety of area.

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