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# ON FRACTIONAL $q$ - DERIVATIVE INTEGRAL FORMULAE OF PRASAD'S I-FUNCTION I

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Abstract. In the present research work, we have derived two theorems which involves integral operators of Erdélyi-Kober type and a q- analogue of modified multivariable I-function. The related averment for the Riemann-Liouville and Weyl fractional basic integral transforms are also deduced. A number of corollaries concerning the basic analogue of modified multivariable H-function, q - analogue of multivariable H-function and remarks are given at the end of this paper.

## 1. Introduction

The idea of fractional calculus is considered to have emerged from a question asked by L'Hospital to Leibniz in 1695 [19]. This has obtained more attention during last century because of its vatious specific applications in numerous diverse fields ( [15], [16], [17], [31]). The q-calculus was also came in to existence in twentieth century. A detailed theory are given in the books by Slater [35], Exton [8], Gasper [11] and a thesis [7]. The q-extension of the ordinary fractional calculus is known as the fractional q-calculus . In recent times the theory of q-calculus operators have been uesd in several areas. The idea of fractional q-calculus was introduced by Al-Salam. From the basic analogue of Cauchy's formula ([4], [5], [6]), Agarwal [3] studied some fractional basic integral operators and q-derivatives. Later that, Isogawa et al. [13] studied some basic properties of fractional q-derivatives. The notion of the left fractional q-integral operators and fractional q-derivatives was generalized by Rajkovic et al. [21] by introducing variable lower limit and proved the semigroup properties. Garg et al. [10] introduced basic analogues of hyper-Bessel type Kober fractional derivatives. Saxena et al. [33], Yadav et al. ([38]-[43]) have found values of different basic special functions by using fractional q- operators. Inspired by this apporach of applicability, some researchers have applied these integral operators to find value of q - fractional calculus formulae for various special functions. One can see the recent publications [9]-[11] and [33], [22]-[25], [26]-[28], [38]-[43] in this subject.

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In the present research work, we have derived two theorems including the fractional basic integral operator of Erdelyi-Kober type. These theorems generalizes the Riemann-Liouville and Weyl fractional basic integral operators.
For real or complex $a$ and $|q| < 1$, the $q$-shifted factorial is defined as :

$$(a;q)_n = \prod_{i=1}^{n-1}(1-aq^i) = \frac{(a;q)_\infty}{(aq^n;q)_\infty}, \qquad (n \in \mathbb{N}). \tag{1.1}$$

so that $(a;q)_0 = 1$, or equivalently

$$(a,q)_n = \frac{\Gamma_q(a+n)(1-q)^n}{\Gamma_q(a)} \qquad (a \neq 0, -1, -2, \cdots). \tag{1.2}$$

The basic analogue of Riemann-Liouville operator of a function $f(x)$ by Agarwal [3], is given by

$$I_q^\alpha\{f(x)\} = \frac{1}{\Gamma_q(\alpha)}\int_0^x (x-tq)_{\alpha-1}f(t)d_qt \qquad (Re(\alpha) > 0, |q| < 1). \tag{1.3}$$

The basic analogue of the Kober operator, see Al-Salam [7, 34] is defined by

$$I_q^{\eta,\alpha}\{f(x)\} = \frac{x^{-\eta-\alpha}}{\Gamma_q(\alpha)}\int_0^x (x-tq)_{\alpha-1}t^\eta f(t)d_qt \qquad (Re(\alpha) > 0, , \eta \in \mathbb{R}, |q| < 1). \tag{1.4}$$

A q-analogue of the Weyl integral operator due to Al-Salam [7] is given by

$$K_q^\alpha\{f(x)\} = \frac{q^{(\alpha-1)/2}}{\Gamma_q(\alpha)}\int_x^\infty (t-x)_{\alpha-1}f(tq^{1-\alpha})d_qt \qquad (Re(\alpha) > 0, |q| < 1). \tag{1.5}$$

Al-Salam [4] defined the following basic analogue:

$$K_q^{\eta,\alpha}\{f(x)\} = \frac{q^{-\eta}x^\eta}{\Gamma_q(\alpha)}\int_x^\infty (t-x)_{\alpha-1}t^{-\eta-\alpha}f(t)d_qt \; (Re(\alpha) > 0, \eta \in \mathbb{R}, |q| < 1). \tag{1.6}$$

The q - integral, see Gasper and Rahman [11] are given by

$$\int_0^x f(t)d_qt = x(1-q)\sum_{k=0}^\infty q^k f(xq^k). \tag{1.7}$$

$$\int_x^\infty f(t)d_qt = x(1-q)\sum_{k=1}^\infty q^{-k} f(zq^{-k}). \tag{1.8}$$

$$\int_0^\infty f(t)d_qt = x(1-q)\sum_{k=-\infty}^\infty q^k f(zq^k). \tag{1.9}$$

## 2. Base Formula

In this section, we will derive two fractional q-integral formulae for the q - analogue of multivariable I-function defined by Prasad [18]. We note

$$G(q^a) = \left[\prod_{n=0}^\infty (1-q^{a+n})\right]^{-1} = \frac{1}{(q^a;q)_\infty}. \tag{2.1}$$

We have

$$I(z_1,\cdots,z_r;q)=I^{0,n_2;0,n_3;\cdots;0,n_r:m^{'}n^{'};\cdots;m^{(r)},n^{(r)}}_{p_2,q_2,p_3,q_3;\cdots;p_r,q_r:p^{'},q^{'};\cdots;p^{(r)},q^{(r)}}\left(\begin{array}{c|c} z_1 & (a_{2j};\alpha^{'}_{2j},\alpha^{''}_{2j})_{1,p_2};\cdots; \\ \vdots \;\; ;q & \\ z_r & (b_{2j};\beta^{'}_{2j},\beta^{''}_{2j})_{1,q_2};\cdots; \end{array}\right.$$

$$\left.\begin{array}{c} (a_{rj};\alpha^{'}_{rj},\cdots,\alpha^{(r)}_{rj})_{1,p_r}:(a^{'}_j,\alpha^{'}_j)_{1,p^{'}};\cdots;(a^{(r)}_j,\alpha^{(r)}_j)_{1,p^{(r)}} \\ (b_{rj};\beta^{'}_{rj},\cdots,\beta^{(r)}_{rj})_{1,q_r}:(b^{'}_j,\beta^{'}_j)_{1,q^{'}};\cdots;(b^{(r)}_j,\beta^{(r)}_j)_{1,q^{(r)}} \end{array}\right)$$

$$=\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\pi^r\xi(s_1,\cdots,s_r;q)\prod_{i=1}^{r}\phi_i(s_i;q)t_i^{s_i}ds_1\cdots ds_r, \tag{2.2}$$

where

$$\xi(s_1,\cdots,s_r;q)=\frac{\prod_{j=1}^{n_2}G(q^{1-a_{2j}+\sum_{i=1}^{2}\alpha_{2j}^{(i)}\xi})\prod_{j=1}^{n_3}G(q^{1-a_{3j}+\sum_{i=1}^{3}\alpha_{3j}^{(i)}\xi})\cdots}{\prod_{j=n_2+1}^{p_2}G(q^{a_{2j}-\sum_{i=1}^{2}\alpha_{2j}^{(i)}\xi})\prod_{j=n_3+1}^{p_3}G(q^{a_{3j}-\sum_{i=1}^{3}\alpha_{3j}^{(i)}\xi})\cdots}$$

$$\frac{\cdots\prod_{j=1}^{n_r}G(q^{1-a_{rj}+\sum_{i=1}^{r}\alpha_{rj}^{(i)}s_i})}{\cdots\prod_{n_r+1}^{p_r}G(q^{a_{rj}-\sum_{i=1}^{r}\alpha_{rj}^{(i)}s_i})\prod_{j=1}^{q_2}G(q^{1-b_{2j}-\sum_{i=1}^{2}\beta_{2j}^{(i)}s_i})\cdots\prod_{j=1}^{q_r}G(q^{1-b_{rj}-\sum_{i=1}^{r}\beta_{rj}^{(i)}s_i})}. \tag{2.3}$$

$$\phi(s_i,q)=\frac{\prod_{j=1}^{m^{(i)}}G(q^{b_j^{(i)}-\beta_j^{(i)}s})\prod_{j=1}^{n^{(i)}}G(q^{1-a_j^{(i)}+\alpha_j^{(i)}s_i})}{\prod_{j=1+m^{(i)}}^{q^{(i)}}G(q^{1-b_j^{(i)}+\beta_j^{(i)}s_i})\prod_{j=n^{(i)}+1}^{p^{(i)}}G(q^{a_j^{(i)}-\alpha_j^{(i)}s_i})G(1-q^{s_i})\sin\pi s_i}, \tag{2.4}$$

$\alpha_j^{(i)},\beta_j^{(i)},\alpha_{kj}^{(i)},\beta_{kj}^{(i)}(i=1,\cdots,r),(k=1,\cdots,r)$ are positive numbers.
$a_j^{(i)},b_j^{(i)}(i=1,\cdots,r),\alpha_{kj},\beta_{kj}(k=2,\cdots,r)$ are complex numbers and here $m^{(i)}$, $n^{(i)}$, $p^{(i)}$, $q^{(i)}$ $(i=1,\cdots,r)$, $n_k$, $p_k$, $q_k(k=2,\cdots,r)$ are non-negative integers where $0\leq m^{(i)}\leq q^{(i)};0\leq n^{(i)}\leq p^{(i)}(i=1,\cdots,r),0\leq q^{(k)}$ and $0\leq n_k\leq p_k$.
Here $(i)$ denotes the numbers of dashes. The contour $L_k$ is in the $s_k(k=1,\cdots,r)$-plane and varies from $\sigma-i\infty$ to $\sigma+i\infty$ where $\sigma$ if is a real number, if necessary to ensure that the poles of $G(q^{1-a_{2j}+\sum_{k=1}^{2}\alpha_{2j}^{(k)}s_k}),(j=1,\cdots,n_2),G(q^{1-a_{3j}+\sum_{k=1}^{3}\alpha_{3j}^{(k)}s_k})$, $(j=1,\cdots,n_3),G(q^{1-a_{rj}+\sum_{k=1}^{r}\alpha_{rj}^{(k)}s_k})$, $(j=1,\cdots,n_r)$, $G(q^{1-c_j^{(k)}+\gamma_j^{(k)}s_k}),(j=1,\cdots,n^{(k)}),(k=1,\cdots,r)$ to the left of the contour $L_k$ and the poles of $G(q^{d_j^{(k)}-\delta_j^{(k)}s_k})$, $(j=1,\cdots,m^{(k)}),(k=1,\cdots,r)$ lie to the right of the contour$L_k$. For further details and asymptotic expansion of the I-function one can refer by Prasad [18]. For large values of $|s_i|$ $Re(s_i log(z_i)-log\sin\pi s_i)<0$, $i=1,\cdots,r$. It is assumed that integrand function has simple poles.
We note

$$A=(a_{2j};\alpha'_{2j},\alpha''_{2j})_{1,p_2};\cdots;(a_{(r-1)j};\alpha'_{(r-1)j},\cdots,\alpha^{r-1}_{(r-1)j})_{1,p_{r-1}}. \tag{2.5}$$

$$B=(b_{2j};\beta'_{2j},\beta''_{2j})_{1,q_2};\cdots;(b_{(r-1)j};\beta'_{(r-1)j},\cdots,\beta^{r-1}_{(r-1)j})_{1,q_{r-1}}. \tag{2.6}$$

$$\mathbb{A}=(a_{rj};\alpha'_{rj},\cdots,\alpha^{(r)}_{rj})_{1,p_r}:\mathfrak{A}=(a'_j,\alpha'_j)_{1,p'};\cdots;(a_j^{(r)},\alpha_j^{(r)})_{1,p^{(r)}}. \tag{2.7}$$

$$\mathbb{B}=(b_{rj};\beta'_{rj},\cdots,\beta^{(r)}_{rj})_{1,q_r}:\mathfrak{B}=(b'_j,\beta'_j)_{1,q'};\cdots;(b_j^{(r)},\beta_j^{(r)})_{1,q^{(r)}}. \tag{2.8}$$

$$U=p_2,q_2;p_3,q_3;\cdots;p_{r-1},q_{r-1};\;V=0,n_2;0,n_3;\cdots;0,n_{s-1}. \tag{2.9}$$

$$W=(p',q');\cdots;(p^{(r)},q^{(r)});X=(m',n');\cdots;(m^{(r)},n^{(r)}). \tag{2.10}$$

## 3. Results

We shall derive two fractional basic integral formulae for the q- analogue of multivariable I-function.

**Theorem 3.1.** *Let* $Re(\mu) > 0, |q| < 1, \eta \in \mathbb{R}, \rho_i > 0 (i = 1, \cdots, r)$ *and* $I_q^{\eta,\alpha}\{.\}$ *be the Kober fractional q-integral operator (1.4), then the following result holds :*

$$I_q^{\eta,\mu}\left\{ x^{\lambda-1} I \left( \begin{array}{c} z_1 x^{\rho_1} \\ \vdots \\ z_r x^{\rho_r} \end{array} ; q \middle| \begin{array}{c} A; \mathbb{A} : \mathfrak{A}; \\ \\ B; \mathbb{B} : \mathfrak{B} \end{array} \right) \right\} = (1-q)^{\mu} x^{\lambda-1}$$

$$I_{U;p_r+1,q_r+1:W}^{V;0,n_r+1:X} \left( \begin{array}{c} z_1 x^{\rho_1} \\ \vdots \\ z_r x^{\rho_r} \end{array} ; q \middle| \begin{array}{c} A; (1-\lambda-\eta; \rho_1, \cdots, \rho_r), \mathbb{A} : \mathfrak{A} \\ \\ B; \mathbb{B}(1-\lambda-\mu-\eta; \rho_1, \cdots, \rho_r) : \mathfrak{B} \end{array} \right), \quad (3.1)$$

*where* $Re(s_i \log(z_i) - \log \sin \pi s_i) < 0, (i = 1, \cdots, r)$.

*Proof.* To prove this theorem, we use equation (3.1) (say I) and using the definitions (1.4) and (2.2), we get

$$I = \frac{x^{-\eta-\alpha}}{\Gamma_q(\alpha)} \int_0^x (x-yq)_{\alpha-1} y^{\eta} \left\{ y^{\lambda-1} \frac{1}{(2\pi\omega)^r} \int_{L_1} \cdots \int_{L_r} \pi^r \xi(s_1, \cdots, s_r; q) \right.$$

$$\left. \prod_{i=1}^r \phi_k(s_{i_r}; q)(z_i y^{\rho_i})^{s_i} d_q s_1 \cdots d_q s_r \right\} = \frac{x^{-\eta-\alpha}}{(2\pi\omega)^r \Gamma_q(\alpha)} \int_{L_1} \cdots \int_{L_r} \pi^r \xi(s_1, \cdots, s_r; q)$$

$$\prod_{i=1}^r \phi_k(s_i; q) z_i^{s_i} \left\{ \int_0^x (x-yq)_{\alpha-1} y^{\eta} y^{\lambda + \sum_{i=1}^r \rho_i s_i - 1} d_q y \right\} d_q s_1 \cdots d_q s_r$$

$$= \frac{1}{(2\pi\omega)^r} \int_{L_1} \cdots \int_{L_r} \pi^r \xi(s_1, \cdots, s_r; q) \prod_{i=1}^r \theta_k(s_i; q) z_i^{s_i} I_q^{\eta,\mu}\{x^{\lambda + \sum_{i=1}^r \rho_i s_i - 1}\} d_q s_1 \cdots d_q s_r$$

Applying the formula by Yadav and Purohit ([39], p. 440, eq. (19))

$$I_q^{\eta,\mu}\{x^{\lambda-1}\} = \frac{\Gamma_q(\lambda+\eta)}{\Gamma_q(\lambda+\eta+\mu)} x^{\eta+\lambda-1}, \ (Re(\lambda+\mu) > 0). \quad (3.2)$$

Substituting (3.2) in the above equation, we obtain

$$\frac{1}{(2\pi\omega)^r} \int_{L_1} \cdots \int_{L_r} \pi^r \xi(s_1, \cdots, s_r; q) \prod_{i=1}^r \theta_k(s_i; q) z_i^{s_i}$$

$$\frac{\Gamma_q(\lambda+\eta+\sum_{i=1}^r \rho_i s_i)}{\Gamma_q(\lambda+\eta+\mu+\sum_{i=1}^r \rho_i s_i)} x^{\lambda+\eta+\sum_{i=1}^r \rho_i s_i - 1} d_q s_1 \cdots d_q s_r. \quad (3.3)$$

Now, deducing the q- Mellin- Barnes double contour integrals in terms of the q - analogue of Aleph- function of several variables, the required result (3.1) can be obtained. □

For $Re(\mu) > 0, |q| < 1, \ \rho_i (i = 1, \cdots, r)$ positive integers, the Riemann Liouville fractional q-integral of the product of two basic functions exists.

**Theorem 3.2.** *Let $Re(\mu) > 0, |q| < 1, \eta \in R$, $\rho_i > 0 (i = 1, \cdots, r)$ then the generalized Weyl q-integral operator for the basic analogue multivariable I-function is given by*

$$K_q^{\eta,\mu}\left\{ x^{\lambda-1} I \left( \begin{array}{c} z_1 x^{-\rho_1} \\ \vdots \\ z_r x^{-\rho_r} \end{array} ; q \left| \begin{array}{l} A; \mathbb{A} : \mathfrak{A}; \\ \\ B; \mathbb{B} : \mathfrak{B} \end{array} \right. \right) \right\} = x^{\lambda}(1-q)^{\mu} q^{-\mu\lambda}$$

$$I^{V;0,n_r+1:X}_{U;p_r+1,q_r+1:W} \left( \begin{array}{c} z_1 (xq^{-\mu})^{-\rho_1} \\ \vdots \\ z_r (xq^{-\mu})^{-\rho_r} \end{array} ; q \left| \begin{array}{l} A; (1+\lambda-\eta; \rho_1, \cdots, \rho_r), \mathbb{A} : \mathfrak{A}; \\ \\ B; \mathbb{B}(1+\lambda-\mu-\eta; \rho_1, \cdots, \rho_r) : \mathfrak{B} \end{array} \right. \right), \tag{3.4}$$

*where $Re(\xi_i log(z_i) - log \sin \pi\xi_i) < 0, (i = 1, \cdots, r)$.*

*Proof.* To prove this result we apply equation (1.6) in equation (3.4). Then we write in integral form using equation (2.2). After that using equation (3.2) we can obtain the required theorem. □

## 4. Special Cases:

**Corollary 4.1.** *Let $Re(\mu) > 0, |q| < 1, \eta \in R$ and $I_q^{\mu}\{.\}$ be the Riemann-Liouville fractional q-integral operator (1.3), then the following result holds :*

$$I_q^{\mu}\left\{ x^{\lambda-1} I \left( \begin{array}{c} z_1 x^{\rho_1} \\ \vdots \\ z_r x^{\rho_r} \end{array} ; q \left| \begin{array}{l} A; \mathbb{A} : \mathfrak{A} \\ \\ B; \mathbb{B} : \mathfrak{B} \end{array} \right. \right) \right\} = (1-q)^{\mu} x^{\lambda+\mu-1}$$

$$I^{V;0,n_r+1:X}_{U;p_r+1,q_r+1:W} \left( \begin{array}{c} z_1 x^{\rho_1} \\ \vdots \\ z_r x^{\rho_r} \end{array} ; q \left| \begin{array}{l} A; (1-\lambda; \rho_1, \cdots, \rho_r), \mathbb{A} : \mathfrak{A}; \\ \\ B; \mathbb{B}(1-\lambda-\mu; \rho_1, \cdots, \rho_r) : \mathfrak{B} \end{array} \right. \right), \tag{4.1}$$

*where $Re(s_i log(z_i) - log \sin \pi s_i) < 0, (i = 1, \cdots, r)$.*

**Corollary 4.2.** *Let $Re(\mu) > 0, |q| < 1, \eta \in R$ and $K_q^{\mu}\{.\}$ be the Weyl fractional q - integral operator (1.5) then the following result holds:*

$$K_q^{\mu}\left\{ x^{\lambda-1} I \left( \begin{array}{c} z_1 x^{-\rho_1} \\ \vdots \\ z_r x^{-\rho_r} \end{array} ; q \left| \begin{array}{l} A; \mathbb{A} : \mathfrak{A} \\ \\ B; \mathbb{B} : \mathfrak{B} \end{array} \right. \right) \right\} = x^{\mu+\lambda}(1-q)^{\mu} q^{-\mu\lambda-\mu(\mu+1)/2}$$

$$I^{V;0,n_r+1:X}_{U;p_r+1,q_r+1:W} \left( \begin{array}{c} z_1 (xq^{-\mu})^{-\rho_1} \\ \vdots \\ z_r (xq^{-\mu})^{-\rho_r} \end{array} ; q \left| \begin{array}{l} A; (1+\lambda+\mu; \rho_1, \cdots, \rho_r), \mathbb{A} : \mathfrak{A} \\ \\ B; \mathbb{B}(1+\lambda; \rho_1, \cdots, \rho_r) : \mathfrak{B} \end{array} \right. \right), \tag{4.2}$$

*where $Re(s_i log(z_i) - log \sin \pi s_i) < 0, (i = 1, \cdots, r)$.*

## 5. Basic of Multivariable H- Function

Setting $A = B = 0 = U = V$, q - analogue I of several variables converts in q-analogue of H-function of several variables defined by Prasad and Singh [19]. We obtain

**Corollary 5.1.**

$$I_q^{\eta,\mu}\left\{ x^{\lambda-1} H\left( \begin{array}{c} z_1 x^{\rho_1} \\ \vdots \\ z_r x^{\rho_r} \end{array} ; q \left| \begin{array}{c} \mathbb{A} : \mathfrak{A} \\ \\ \mathbb{B} : \mathfrak{B} \end{array} \right. \right) \right\} = (1-q)^{\mu} x^{\lambda-1}$$

$$H^{0,n_r+1:X}_{p_r+1,q_r+1:W}\left( \begin{array}{c} z_1 x^{\rho_1} \\ \vdots \\ z_r x^{\rho_r} \end{array} ; q \left| \begin{array}{c} (1-\lambda-\eta;\rho_1,\cdots,\rho_r), \mathbb{A} : \mathfrak{A} \\ \\ \mathbb{B}; (1-\lambda-\mu-\eta;\rho_1,\cdots,\rho_r) : \mathfrak{B} \end{array} \right. \right), \tag{5.1}$$

*where* $Re(s_i log(z_i) - log \sin \pi s_i) < 0, (i = 1, \cdots, r)$.

**Corollary 5.2.**

$$K_q^{\eta,\mu}\left\{ x^{\lambda-1} H\left( \begin{array}{c} z_1 x^{-\rho_1} \\ \vdots \\ z_r x^{-\rho_r} \end{array} ; q \left| \begin{array}{c} \mathbb{A} : \mathfrak{A} \\ \\ \mathbb{B} : \mathfrak{B} \end{array} \right. \right) \right\} = x^{\lambda}(1-q)^{\mu} q^{-\mu\lambda}$$

$$H^{0,n_r+1:X}_{p_r+1,q_r:W}\left( \begin{array}{c} z_1 (xq^{-\mu})^{-\rho_1} \\ \vdots \\ z_r (xq^{-\mu})^{-\rho_r} \end{array} ; q \left| \begin{array}{c} (1+\lambda-\eta;\rho_1,\cdots,\rho_r), \mathbb{A} : \mathfrak{A} \\ \\ \mathbb{B}; (1+\lambda-\mu-\eta;\rho_1,\cdots,\rho_r) : \mathfrak{B} \end{array} \right. \right), \tag{5.2}$$

*where* $Re(\xi_i log(z_i) - log \sin \pi \xi_i) < 0, (i = 1, \cdots, r)$.

**Corollary 5.3.**

$$I_q^{\mu}\left\{ x^{\lambda-1} H\left( \begin{array}{c} z_1 x^{\rho_1} \\ \vdots \\ z_r x^{\rho_r} \end{array} ; q \left| \begin{array}{c} \mathbb{A} : \mathfrak{A} \\ \\ \mathbb{B} : \mathfrak{B} \end{array} \right. \right) \right\} = (1-q)^{\mu} x^{\lambda+\mu-1}$$

$$H^{0,n_r+1:X}_{p_r+1,q_r+1:W}\left( \begin{array}{c} z_1 x^{\rho_1} \\ \vdots \\ z_r x^{\rho_r} \end{array} ; q \left| \begin{array}{c} (1-\lambda;\rho_1,\cdots,\rho_r), \mathbb{A} : \mathfrak{A} \\ \\ \mathbb{B}; (1-\lambda-\mu;\rho_1,\cdots,\rho_r) : \mathfrak{B} \end{array} \right. \right), \tag{5.3}$$

*where* $Re(s_i log(z_i) - log \sin \pi s_i) < 0, (i = 1, \cdots, r)$.

**Corollary 5.4.**

$$K_q^{\mu}\left\{ x^{\lambda-1} H\left( \begin{array}{c} z_1 x^{-\rho_1} \\ \vdots \\ z_r x^{-\rho_r} \end{array} ; q \left| \begin{array}{c} \mathbb{A} : \mathfrak{A} \\ \\ \mathbb{B} : \mathfrak{B} \end{array} \right. \right) \right\} = x^{\mu+\lambda}(1-q)^{\mu} q^{-\mu\lambda-\mu(\mu+1)/2}$$

$$H^{0,n_r+1:X}_{p_r+1,q_r+1:W}\left( \begin{array}{c} z_1 (xq^{-\mu})^{-\rho_1} \\ \vdots \\ z_r (xq^{-\mu})^{-\rho_r} \end{array} ; q \left| \begin{array}{c} (1+\lambda+\mu;\rho_1,\cdots,\rho_r), \mathbb{A} : \mathfrak{A} \\ \\ \mathbb{B}; (1+\lambda;\rho_1,\cdots,\rho_r) : \mathfrak{B} \end{array} \right. \right), \tag{5.4}$$

*where* $Re(s_i log(z_i) - log \sin \pi s_i) < 0, (i = 1, \cdots, r)$.

Taking $r = 2$, then the q - analogue of H - function of several variables reduces to q - analogue of H - function of two variables [12] given by Saxena et al.[33], Let

$$C_2 = \{(a_i;\alpha_i, A_i)\}_{1,p_1} D_2 = \{(e_i;E_i)\}_{1,p_2}, \{(g_i;G_i)\}_{1,p_3}. \tag{5.5}$$

$$E_2 = \{(b_i;\beta_i, B_i)\}_{1,q_1} F_2 = \{(f_i;F_i)\}_{1,q_2}, \{(h_i;H_i)\}_{1,q_3}. \tag{5.6}$$

We get:

**Corollary 5.5.**

$$I_q^{\eta,\mu}\left\{ x^{\lambda-1} H^{m_1,n_1;m_2,n_2,m_3,n_3}_{p_1,q_1;p_2,q_2;p_3,q_3}\left(\begin{array}{c} z_1x^{\rho} \\ \vdots \\ z_2x^{\sigma}\end{array}; q \left| \begin{array}{c} C_2 : D_2 \\ \\ E_2 : F_2 \end{array}\right.\right)\right\} = (1-q)^{\mu}x^{\lambda-1}$$

$$H^{m_1,n_1+1;m_2,n_2,m_3,n_3}_{p_1+1,q_1+1;p_2,q_2;p_3,q_3}\left(\begin{array}{c} z_1x^{\rho} \\ \vdots \\ z_2x^{\sigma}\end{array}; q \left| \begin{array}{c} (1-\lambda-\eta;\rho,\sigma), C_2 : D_2 \\ \\ E_2, (1-\lambda-\mu-\eta;\rho,\sigma) : F_2 \end{array}\right.\right), \tag{5.7}$$

*under the conditions verified by the corollary 5.1 and* $r = 2$*, see Yadav et al.* [42].

**Corollary 5.6.**

$$K_q^{\eta,\mu}\left\{ x^{\lambda-1} H^{m_1,n_1;m_2,n_2,m_3,n_3}_{p_1,q_1;p_2,q_2;p_3,q_3}\left(\begin{array}{c} z_1x^{-\rho} \\ \vdots \\ z_2x^{-\sigma}\end{array}; q \left| \begin{array}{c} C_2 : D_2 \\ \\ E_2 : F_2 \end{array}\right.\right)\right\} = x^{\lambda}(1-q)^{\mu}q^{-\mu\lambda}$$

$$H^{m_1+1,n_1;m_2,n_2,m_3,n_3}_{p_1+1,q_1+1;p_2,q_2;p_3,q_3}\left(\begin{array}{c} z_1(xq^{-\mu})^{-\rho} \\ \vdots \\ z_2(xq^{-\mu})^{-\sigma}\end{array}; q \left| \begin{array}{c} (1+\lambda-\eta;\rho,\sigma), C_2 : D_2 \\ \\ E_2, (1+\lambda-\mu-\eta;\rho,\sigma) : F_2 \end{array}\right.\right), \tag{5.8}$$

*under the conditions verified by the corollary 5.2 and* $r = 2$*, see Yadav et al.* [42].

We suppose

$$(\alpha_j)_{1,p_1} = (A_j)_{1,p_1} = (E_j)_{1,p_2} = (G_j)_{1,p_3} = (\beta_j)_{1,q_1} = (B_j)_{1,q_1} = (F_j)_{1,q_2} = (H_j)_{1,q_3} = 1. \tag{5.9}$$

The q - analogue of Meijer's G - function of two variables given by Agarwal [2], can also be obtained by setting the following parameters:

$$A_2^{'} = (a_j)_{1,p_1} : B_2^{'} = (e_j)_{1,p_2}, (g_j)_{1,p_3}. \tag{5.10}$$

$$C_2^{'} = (b_j)_{1,q_1} : D_2^{'} = (f_j)_{1,q_2}, (h_j)_{1,q_3}. \tag{5.11}$$

and we obtain the formulas as follows:

**Corollary 5.7.**

$$I_q^{\eta,\mu}\left\{ x^{\lambda-1} \left(\begin{array}{c} z_1x^{\rho} \\ \vdots \\ z_2x^{\sigma}\end{array}; q \left| \begin{array}{c} A_2^{'} : C_2^{'} \\ \\ B_2^{'} : D_2^{'} \end{array}\right.\right)\right\} = (1-q)^{\mu}x^{\lambda-1}$$

$$G^{m_1,n_1+1;m_2,n_2,m_3,n_3}_{p_1+1,q_1+1;p_2,q_2;p_3,q_3}\left(\begin{array}{c} z_1x^{\rho} \\ \vdots \\ z_2x^{\sigma}\end{array}; q \left| \begin{array}{c} (1-\lambda-\eta;\rho,\sigma), C_2^{'} : D_2^{'} \\ \\ E_2^{'}, (1-\lambda-\mu-\eta;\rho,\sigma) : F_2^{'} \end{array}\right.\right), \tag{5.12}$$

*with the conditions verified by the corollary 5.7 and* $(\alpha_j)_{1,p_1} = (A_j)_{1,p_1} = (E_j)_{1,p_2} = (G_j)_{1,p_3} = (\beta_j)_{1,q_1} = (B_j)_{1,q_1} = (F_j)_{1,q_2} = (H_j)_{1,q_3} = 1.$

**Corollary 5.8.**

$$K_q^{\eta,\mu}\left\{ x^{\lambda-1} G^{0,n_1;m_2,n_2,m_3,n_3}_{p_1,q_1;p_2,q_2;p_3,q_3}\left(\begin{array}{c} z_1 x^{-\rho} \\ \vdots \\ z_2 x^{-\sigma}\end{array} ;(p,q)\left| \begin{array}{c} A_2^{'} : B_2^{'} \\ \\ C_2^{'} : D_2^{'}\end{array}\right.\right)\right\} = x^{\lambda}(1-q)^{\mu} q^{-\mu\lambda}$$

$$G^{0,n_1+1;m_2,n_2,m_3,n_3}_{p_1+1,q_1+1;p_2,q_2;p_3,q_3}\left(\begin{array}{c} z_1 (xq^{-\mu})^{-\rho} \\ \vdots \\ z_2 (xq^{-\mu})^{-\sigma}\end{array} ;q\left| \begin{array}{c} (1+\lambda-\eta;\rho,\sigma), A_2^{'} : B_2^{'} \\ \\ C_2^{'}, (1+\lambda-\mu-\eta;\rho,\sigma) : D_2^{'}\end{array}\right.\right), \tag{5.13}$$

*with the conditions verified by the corollary 5.6 and* $(\alpha_j)_{1,p_1} = (A_j)_{1,p_1} = (E_j)_{1,p_2} = (G_j)_{1,p_3} = (\beta_j)_{1,q_1} = (B_j)_{1,q_1} = (F_j)_{1,q_2} = (H_j)_{1,q_3} = 1.$

If $r = 1$, the q - analogue of multivariable I - function converts in q - analogue of I - function defined by Rathie [29].
Let $A_1 = (a_j, \alpha_j : A_j)_{1,p}; B_1 = (b_j, \beta_j : B_j)_{1,q}$, we obtain the following result

**Corollary 5.9.**

$$I_q^{\eta,\mu}\left\{ x^{\lambda-1} I^{m,n}_{p,q'}\left( zx^{\rho} \; ;q\left| \begin{array}{c} A_1 \\ . \\ B_1\end{array}\right.\right)\right\} = (1-q)^{\mu} x^{\lambda-1}$$

$$I^{m,n+1}_{p+1,q'+1}\left( zx^{\rho} \; ;q\left| \begin{array}{c} (1-\lambda-\eta;\rho;1), A_1 \\ \\ B_1, (1-\lambda-\mu-\eta;\rho;1)\end{array}\right.\right), \tag{5.14}$$

*where* $Re(slog(z_1) - log \sin \pi s) < 0.$

**Corollary 5.10.**

$$K_q^{\eta,\mu}\left\{ x^{\lambda-1} I^{m,n}_{p,q'}\left( zx^{-\rho} \; ;q\left| \begin{array}{c} A_1 \\ . \\ B_1\end{array}\right.\right)\right\} = x^{\lambda}(1-q)^{\mu} q^{-\mu\lambda}$$

$$I^{m,n+1}_{p+1,q'+1}\left( z(xq^{-\mu})^{-\rho} \; ;q\left| \begin{array}{c} (1+\lambda-\eta;\rho;1), A_1 \\ \\ B_1, (1+\lambda-\mu-\eta;\rho;1)\end{array}\right.\right), \tag{5.15}$$

*where* $Re(slog(z_1) - log \sin \pi s) < 0.$

Let $A_1^{'} = (a_j, \alpha_j)_{1,p}$ and $B_1^{'} = (b_j, \beta_j)_{1,q}$, the q - analogue of I - function changes to q - analogue of H - function of one variable defined by Saxena et. al.[32]. We have the following results:

**Corollary 5.11.**

$$I_q^{\eta,\mu}\left\{ x^{\lambda-1} H^{m,n}_{p,q}\left( zx^{\rho} \; ;q\left| \begin{array}{c} A_1^{'} \\ B_1^{'}\end{array}\right.\right)\right\}$$

$$= (1-q)^{\mu} x^{\lambda-1} H^{m,n+1}_{p+1,q+1}\left( zx^{\rho} \; ;q\left| \begin{array}{c} (1-\lambda-\eta;\rho), A_1^{'} \\ B_1^{'}, (1-\lambda-\mu-\eta;\rho)\end{array}\right.\right), \tag{5.16}$$

*under the condition verified by the corollary 5.9 and* $(A_j)_{1,p} = (B_j)_{1,q} = 1.$

**Corollary 5.12.**

$$K_q^{\eta,\mu}\left\{\; x^{\lambda-1}H_{p,q}^{m,n}\left(\; zx^{-\rho} \;\; ;q\Big|\; \begin{array}{c} A_1^{'} \\ B_1^{'} \end{array}\right)\;\right\} = x^{\lambda}(1-q)^{\mu}q^{-\mu\lambda}$$

$$H_{p+1,q+1}^{m,n+1}\left(\; z(xq^{-\mu})^{-\rho} \;\; ;q\Big|\; \begin{array}{l} (1+\lambda-\eta;\rho), A_1^{'} \\ B_1^{'}, (1+\lambda-\mu-\eta;\rho) \end{array}\right), \tag{5.17}$$

*under the condition verified by the corollary 5.10 and* $(A_j)_{1,p} = (B_j)_{1,q} = 1$.

Let $A_1^{''} = (a_j)_{1,p}$ and $B_1^{''} = (b_j)_{1,q}$, the basic of H- function of one variable reduce to basic of Meijer's G-functions, this gives:

**Corollary 5.13.**

$$I_q^{\eta,\mu}\left\{\; x^{\lambda-1}G_{p,q}^{m,n}\left(\; zx^{\rho} \;\; ;q\Big|\; \begin{array}{c} A_1^{''} \\ B_1^{''} \end{array}\right)\;\right\}$$

$$= (1-q)^{\mu}x^{\lambda-1}G_{p+1,q+1}^{m,n+1}\left(\; zx^{\rho} \;\; ;q\Big|\; \begin{array}{l} (1-\lambda-\eta;\rho), A_1^{''} \\ B_1^{''}, (1-\lambda-\mu-\eta;\rho) \end{array}\right), \tag{5.18}$$

*under the condition verified by the corollary 5.11 and* $(\alpha_j)_{1,p} = (\beta_j)_{1,q} = 1$.

**Corollary 5.14.**

$$K_q^{\eta,\mu}\left\{\; x^{\lambda-1}G_{p,q}^{m,n}\left(\; zx^{-\rho} \;\; ;q\Big|\; \begin{array}{c} A_1^{''} \\ B_1^{''} \end{array}\right)\;\right\} = x^{\lambda}(1-q)^{\mu}q^{-\mu\lambda}$$

$$G_{p+1,q+1}^{m,n+1}\left(\; z(xq^{-\mu})^{-\rho} \;\; ;q\Big|\; \begin{array}{l} (1+\lambda-\eta;\rho), A_1^{''} \\ B_1^{''}, (1+\lambda-\mu-\eta;\rho) \end{array}\right), \tag{5.19}$$

*under the condition verified by the corollary 5.12 and* $(\alpha_j)_{1,p} = (\beta_j)_{1,q} = 1$.

## 6. Conclusion

The results obtained in this research paper have various applications due to its general nature. By putting some particular values of the parameters and variables in the q- analogue of multivariable I-function, we can deduce various results. These results include various type of basic functions and can be expressed in terms of q-analogue of H-function [32], q- analogue of Meijer's G-function, q- analogue of E-function, q- analogue of hypergeometric function of one variable and multivariable [43], special basic functions of one and several variables. The results proved in this research paper are generalised results of multivariable I - functions. Therefore a number of results in literature can be expressed in terms of these results.

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