

## GENERAL SOLUTIONS OF SPACE FRACTIONAL FISHER'S NONLINEAR DIFFUSION EQUATION

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ABSTRACT. In this paper, we develop a framework to obtain exact solutions to Fisher's nonlinear diffusion equation by using the modified Adomian's decomposition method. Some examples are provided to verify the effectiveness of the method. The new modification introduces a promising tool for many linear and nonlinear models.

### 1. INTRODUCTION

In recent past, the Fractional calculus has been extensively investigated due to their broad applications in mathematics, physics and engineering[1, 2, 3], such as anomalous transport in disordered systems, some percolations in porous media, and the diffusion of biological populations. But these nonlinear fractional differential equations are difficult to get their exact solutions[4, 5, 6]. An effective method for solving such equations is needed. The Adomian decomposition method[7]for solving differential and integral equations, linear or nonlinear, has been the subject of extensive analytical and numerical studies. The method, well addressed in[8, 9, 10], provide the solutions in the form of a power series with easily computed terms. The method has many advantages over the classical technique mainly, it provides an efficient numerical solution with high accuracy and minimal calculations. And in this paper, we make a simple modification of the initial conditions, so that the decomposition method is easy to calculate.

Biological population problem are widely investigated in many papers[11, 12]. [13] considered the spatial diffusion of biological populations and obtain its corresponding numerical solution using variational iteration method. [14] considered the explicit solutions of travelling waves with a front of Fisher equation. [15] obtained an exact solution describing self-similar growth of the initially inhabited domain for a special case. [16] develop a framework to obtain exact solutions to Fisher's equation and to a nonlinear diffusion equation of the Fisher type by employing Adomian's decomposition method.

We extend the modified Adomian's decomposition method to time-fractional Fisher's equation, a representative Fisher's equation is  $u_t = u_{xx} + u(1 - u)$  was first

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introduced by Fisher as a model for the propagation of a mutant gene[17], where  $u(x, t)$  denotes the population density and  $u(1-u)$  represents the population supply due to births and deaths. In this paper, we propose a generalized space-fractional Fisher's biological population diffusion equation as follows

$$\frac{\partial^\alpha u}{\partial t^\alpha} = \frac{\partial^2 u}{\partial x^2} + F(u), \quad u(x, 0) = \phi(x) \quad (1)$$

where  $t > 0$ ,  $x \in R$ ,  $F(u)$  is a continuous nonlinear function which satisfies the conditions  $F(0) = F(1) = 0, F'(0) > 0 > F'(1)$ . The derivatives in Eq.(1) is the Caputo derivative.

This paper is devoted to study the Fisher's equation, the general Fisher's equation, and the nonlinear diffusion equation of the Fisher type. Our work here stems mainly from Adomian's decomposition method, that has been widely used in applied sciences, which is capable of handing a wider class of diffusion problems. Numerical solutions of Fisher's biological population shall be presented to demonstrate the effectiveness of the algorithm.

## 2. FRACTIONAL CALCULUS

There are several approaches to define the fractional calculus, e.g. Riemann-Liouville, Gruünwald-Letnikov, Caputo, and Generalized Functions approach. Riemann-Liouville fractional derivative is mostly used by mathematicians but this approach is not suitable for real world physical problems since it requires the definition of fractional order initial conditions, which have no physically meaningful explanation yet, Caputo introduced an alternative definition, which has the advantage of defining integer order initial conditions for fractional order differential equations.

**Definition 1.** The Riemann-Liouville fractional integral operator  $J^\alpha (\alpha \geq 0)$  of a function  $f(t)$ , is defined as

$$J^\alpha f(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-\tau)^{\alpha-1} f(\tau) d\tau, \quad (\alpha \geq 0) \quad (2)$$

where  $\Gamma(\cdot)$  is the well-known gamma function, and some properties of the operator  $J^\alpha$  are as follows

$$J^\alpha J^\beta f(t) = J^{\alpha+\beta} f(t), \quad (\alpha \geq 0, \beta \geq 0) \quad (3)$$

$$J^\alpha t^\gamma = \frac{\Gamma(1+\gamma)}{\Gamma(1+\gamma+\alpha)} t^{\alpha+\gamma}, \quad (\gamma \geq -1) \quad (4)$$

**Definition 2.** The Caputo fractional derivative  $D^\alpha$  of a function  $f(t)$  is defined as

$${}_0D_t^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)} \int_0^t \frac{f^{(n)}(\tau) d\tau}{(t-\tau)^{\alpha+1-n}}, \quad (n-1 < Re(\alpha) \leq n, n \in N) \quad (5)$$

the following are two basic properties of the Caputo fractional derivative

$${}_0D_t^\alpha t^\beta = \frac{\Gamma(1+\beta)}{\Gamma(1+\beta-\alpha)} t^{\beta-\alpha}, \quad (6)$$

$$(J^\alpha D^\alpha) f(t) = f(t) - \sum_{k=0}^{n-1} f^{(k)}(0^+) \frac{t^k}{k!}, \quad (7)$$

we have chosen to the Caputo fractional derivative because it allows traditional initial and boundary conditions to be included in the formulation of the problem. And some other properties of fractional derivative can be found in [1, 2].

### 3. DESCRIPTION OF THE METHOD

The Adomian's decomposition method which provides an analytical approximate solution is applied to various nonlinear problems[9, 10, 11]. In an operator form, Eq.(1) can be written as

$$Lu = \frac{\partial^\beta u}{\partial x^\beta} + F(u), \quad (8)$$

where the differential operator  $L$  is given by  $L = \frac{\partial}{\partial t}$ , so that the inverse operator  $L^{-1}$  exists and defined by  $L^{-1}(\cdot) = \int_0^t (\cdot) dt$ , operating with  $L^{-1}$  on (8) and using the initial condition gives

$$u(x, t) = \phi(x) + L^{-1}\left(\frac{\partial^\beta u}{\partial x^\beta} + F(u)\right), \quad (9)$$

The Adomian's decomposition method decomposes the solution  $u(x, t)$  by an infinite series of components

$$u(x, t) = \sum_{n=0}^{\infty} u_n(x, t), \quad (10)$$

and the nonlinear term  $F(u)$  by an infinite series of polynomials

$$F(u) = \sum_{n=0}^{\infty} A_n, \quad (11)$$

where the component  $u_n$  of the solution  $u(x, t)$  will be determined recurrently.  $A_n$  is the so-called Adomian's polynomials. substituting (10) and (11) into (9) yields

$$\sum_{n=0}^{\infty} u_n(x, t) = \phi(x) + L^{-1}\left(\frac{\partial^\beta}{\partial x^\beta} \sum_{n=0}^{\infty} u_n(x, t) + \sum_{n=0}^{\infty} A_n\right), \quad (12)$$

*Adomian's decomposition method.* The standard Adomian decomposition method begins by setting the zeroth component  $u_0(x, t) = \phi(x)$ , in addition, Adomian's methodology allows us to introduce the recursive relation

$$u_0(x, t) = \phi(x), \quad (13)$$

$$u_{k+1}(x, t) = L^{-1}\left(\frac{\partial^\beta}{\partial x^\beta} u_k(x, t) + A_k\right), k \geq 0 \quad (14)$$

*Modified Adomian's decomposition method.* In the new modification, we suggest that  $\phi(x)$  be expressed in a series of infinite components.

$$u(x, 0) = \phi(x) = \sum_{n=0}^{\infty} u_n(x, 0), \quad (15)$$

moreover, we suggest a new recursive relationship expressed in the form

$$u_0 = u_0(x, 0), \quad (16)$$

$$u_{k+1} = u_{k+1}(x, 0) + L^{-1}\left(\frac{\partial^\beta}{\partial x^\beta} u_k(x, t) + A_k\right), k \geq 0 \quad (17)$$

To determine the components  $u_n(x, t)$ , it is useful to list the first few Adomian decomposition polynomials. Followin[8], Adomian polynomials can be derived by  $A_0 = F(u_0)$ ,  $A_1 = u_1 F'(u_0)$ ,  $A_2 = u_2 F'(u_0) + \frac{u_1^2}{2!} F''(u_0)$ ,  $A_3 = u_3 F'(u_0) +$

$u_1 u_2 F''(u_0) + \frac{u_1^3}{3!} F'''(u_0)$ . With the Adomian's polynomials, and using the recursive relation (17) the components  $u_n(x, t)$  of the series solution of  $u(x, t)$  follow immediately.

#### 4. SPACE FRACTIONAL EQUATION

In order to assess the advantages and the accuracy of the modified Adomian's decomposition method presented in this paper for nonlinear fractional fisher's equation, we have applied it to the following several problems.

*Case 1:* In this case, we will examine the Fisher's equation

$$\frac{\partial u}{\partial t} = \frac{\partial^\beta u}{\partial x^\beta} + 6u(1 - u), \quad (18)$$

subject to the initial condition

$$u(x, 0) = \frac{1}{(1 + e^x)^2}, \quad (19)$$

According to the modified Adomian's decomposition method, we suggest  $u(x, 0)$  be expressed in the Taylor series

$$\frac{1}{(1 + e^x)^2} = \frac{1}{4} - \frac{x}{4} + \frac{x^2}{16} + \frac{x^3}{48} - \frac{x^4}{96} + \cdots, \quad (20)$$

and we know that  $\frac{1}{4} - \frac{x}{4} + \frac{x^2}{16} + \frac{x^3}{48} - \frac{x^4}{96}$  is the best polynomial approximation of  $\frac{1}{(1+e^x)^2}$  in the interval  $[0, 1]$ . Operating with  $L^{-1}$  on both sides of (19) gives

$$u(x, t) = \frac{1}{(1 + e^x)^2} + L^{-1}\left(\frac{\partial^\beta u}{\partial x^\beta} + 6u - 6u^2\right) \quad (21)$$

substituting (10),(11) and (20) into (21),we find

$$\sum_{n=0}^{\infty} u_n(x, t) = \sum_{n=0}^{\infty} u_n(x, 0) + L^{-1}\left(\frac{\partial^\beta}{\partial x^\beta} \sum_{n=0}^{\infty} u_n(x, t) + 6\left(\sum_{n=0}^{\infty} u_n(x, t)\right) - 6\sum_{n=0}^{\infty} A_n\right), \quad (22)$$

Following the analysis discussed above, we set the recursive relation

$$u_0 = 1/4, u_{k+1}(x, t) = u_{k+1}(x, 0) + L^{-1}\left(\frac{\partial^\beta}{\partial x^\beta} u_k(x, t) + 6u_k(x, t) - 6A_k\right), \quad (23)$$

where  $A_k$  are the Adomian's polynomials for the nonlinear term  $u^2$ . The first few components of  $u_n(x, t)$

$$u_0 = \frac{1}{4}, \quad (24)$$

$$u_1 = -\frac{x}{4} + \frac{9t}{8}, \quad (25)$$

$$u_2 = \frac{x^2}{16} - \frac{x^{1-\beta}}{4\Gamma(2-\beta)}t - \frac{3x}{4}t + \frac{27}{16}t^2, \quad (26)$$

$$u_3 = \frac{x^3}{48} + c_1(x)t + c_2(x)t^2 - \frac{27}{32}t^3, \quad (27)$$

$$u_4 = -\frac{x^4}{96} + c_3(x)t + c_4(x)t^2 + c_5(x)t^3 - \frac{405}{64}t^4, \quad (28)$$

where

$$\begin{aligned} c_1(x) &= -\frac{3x^2}{16} + \frac{x^{2-\beta}}{8\Gamma(3-\beta)}, c_2(x) = \frac{9x}{16} - \frac{3x^{1-\beta}}{4\Gamma(2-\beta)} + \frac{x^{1-2\beta}}{8\Gamma(2-2\beta)}, \\ c_3(x) &= \frac{x^3}{4} + \frac{x^{3-\beta}}{8\Gamma(4-\beta)}, c_4(x) = -\frac{117x^2}{64} + \frac{x^{2-2\beta}}{16\Gamma(3-2\beta)} - \frac{3x^{2-\beta}}{8\Gamma(2-\beta)}, \\ c_4(x) &= \frac{45x}{8} - \frac{x^{1-3\beta}}{24\Gamma(2-3\beta)} - \frac{3x^{1-2\beta}}{8\Gamma(2-2\beta)} + \frac{9x^{1-\beta}}{16\Gamma(2-\beta)}, \end{aligned}$$

the solution in a series form is given by

$$u(x, t) = \frac{1}{4} - \frac{x}{4} + \frac{9t}{8} + \frac{x^2}{16} - \frac{x^{1-\beta}}{4\Gamma(2-\beta)}t - \frac{3x}{4}t + \frac{27}{16}t^2 + \dots \quad (29)$$

which is in full agreement with the result in [16].

*Case 2:* we will consider the generalized Fisher's equation

$$\frac{\partial u}{\partial t} = \frac{\partial^\beta u}{\partial x^\beta} + u(1 - u^6), \quad (30)$$

subject to the initial condition

$$u(x, 0) = \frac{1}{(1 + e^{(3/2)x})^{1/3}}, \quad (31)$$

$u(x, 0)$  be expressed in the Taylor series

$$\frac{1}{(1 + e^{(3/2)x})^{1/3}} = \frac{1}{\sqrt[3]{2}} - \frac{x}{4\sqrt[3]{2}} - \frac{x^2}{16\sqrt[3]{2}} + \frac{x^3}{48\sqrt[3]{2}} + \dots, \quad (32)$$

and we know that  $\frac{1}{\sqrt[3]{2}} - \frac{x}{4\sqrt[3]{2}} - \frac{x^2}{16\sqrt[3]{2}} + \frac{x^3}{48\sqrt[3]{2}}$  is the best polynomial approximation of  $\frac{1}{(1+e^{(3/2)x})^{1/3}}$  in the interval  $[0, 1]$ . Operating with  $L^{-1}$  on both sides of (29) gives

$$u(x, t) = \frac{1}{(1 + e^{(3/2)x})^{1/3}} + L^{-1}\left(\frac{\partial^\beta u}{\partial x^\beta} + u - u^7\right) \quad (33)$$

substituting (10),(11) and (31) into (32), we find

$$\sum_{n=0}^{\infty} u_n(x, t) = \sum_{n=0}^{\infty} u_n(x, 0) + L^{-1}\left(\frac{\partial^\beta}{\partial x^\beta} \sum_{n=0}^{\infty} u_n(x, t) + \sum_{n=0}^{\infty} u_n(x, t) - \sum_{n=0}^{\infty} A_n\right), \quad (34)$$

Following the analysis discussed above, we set the recursive relation

$$u_0 = 1/\sqrt[3]{2}, u_{k+1}(x, t) = u_{k+1}(x, 0) + L^{-1}\left(\frac{\partial^\beta}{\partial x^\beta} u_k(x, t) + u_k(x, t) - A_k\right), \quad (35)$$

where  $A_k$  are the Adomian's polynomials for the nonlinear term  $u^7$ . The first few components of  $u_n(x, t)$

$$u_0 = \frac{1}{\sqrt[3]{2}}, \quad (36)$$

$$u_1 = -\frac{x}{4\sqrt[3]{2}} + \frac{3}{4\sqrt[3]{2}}t, \quad (37)$$

$$u_2 = -\frac{x^2}{16\sqrt[3]{2}} + \left(-\frac{x^{1-\beta}}{4\sqrt[3]{2}\Gamma(2-\beta)} + \frac{3x}{16\sqrt[3]{2}}\right)t - \frac{9}{32\sqrt[3]{2}}t^2, \quad (38)$$

$$u_3 = \frac{x^3}{48\sqrt[3]{2}} - \left(\frac{9x^2}{32\sqrt[3]{2}} + \frac{x^{2-\beta}}{8\sqrt[3]{2}\Gamma(3-\beta)}\right)t + \left(\frac{117x}{128\sqrt[3]{2}} - \frac{x^{1-2\beta}}{8\sqrt[3]{2}\Gamma(2-2\beta)} + \frac{3x^{1-\beta}}{16\sqrt[3]{2}\Gamma(2-\beta)}\right)t^2 - \frac{117}{128\sqrt[3]{2}}t^3, \quad (39)$$

Then the approximate solution in a series form is

$$u(x, t) = 1/\sqrt[3]{2} + \frac{x}{4\sqrt[3]{2}} + \frac{3}{4\sqrt[3]{2}}t - \frac{x^2}{16\sqrt[3]{2}} + \left(-\frac{x^{1-\beta}}{4\sqrt[3]{2}\Gamma(2-\beta)} + \frac{3x}{16\sqrt[3]{2}}\right)t - \frac{9}{32\sqrt[3]{2}}t^2 + \dots \quad (40)$$

*Case 3:* we will consider the nonlinear diffusion equation of the Fisher type

$$\frac{\partial^\alpha u}{\partial t^\alpha} = \frac{\partial^2 u}{\partial x^2} + u(1-u)(u-a), \quad 0 < a < 1 \quad (41)$$

subject to the initial condition

$$u(x, 0) = \frac{1}{1 + e^{-(1/\sqrt{2})x}}, \quad (42)$$

$u(x, 0)$  be expressed in the Taylor series

$$u(x, 0) = \frac{1}{2} + \frac{x}{4\sqrt{2}} - \frac{x^2}{96\sqrt{\sqrt{2}}} + \dots, \quad (43)$$

$\frac{1}{2} + \frac{x}{4\sqrt{2}} - \frac{x^2}{96\sqrt{\sqrt{2}}}$  is the best polynomial approximation of  $\frac{1}{1+e^{-(1/\sqrt{2})x}}$  in the interval  $[0, 1]$ . Operating with  $L^{-1}$  on both sides of (41) gives

$$u(x, t) = \frac{1}{1 + e^{-(1/\sqrt{2})x}} + L^{-1}\left(\frac{\partial^\beta u}{\partial x^\beta} - au + u^2 + au^2 - u^3\right) \quad (44)$$

substituting (10),(11) and (43) into (44), we find

$$\sum_{n=0}^{\infty} u_n(x, t) = \sum_{n=0}^{\infty} u_n(x, 0) + L^{-1}\left(\frac{\partial^\beta}{\partial x^\beta} \sum_{n=0}^{\infty} u_n(x, t) + a \sum_{n=0}^{\infty} u_n(x, t) - (1+a) \sum_{n=0}^{\infty} A_n - \sum_{n=0}^{\infty} B_n\right), \quad (45)$$

Following the analysis discussed above, we set the recursive relation

$$u_0 = 1/2, u_{k+1}(x, t) = u_{k+1}(x, 0) + L^{-1}\left(\frac{\partial^\beta}{\partial x^\beta} u_k(x, t) - au_k(x, t) + (1+a)A_k - B_k\right), \quad (46)$$

where  $A_k, B_k$  are the Adomian's polynomials for the nonlinear term  $u^2, u^3$  respectively. The first few components of  $u_n(x, t)$

$$u_0 = \frac{1}{2}, \quad (47)$$

$$u_1 = \frac{x}{4\sqrt{2}} + \left(\frac{1}{8} - \frac{a}{4}\right)t, \quad (48)$$

$$u_2 = -\frac{x^2}{96\sqrt{2}} + \left(\frac{x}{16\sqrt{2}} + \frac{x^{1-\beta}}{4\sqrt{2}\Gamma(2-\beta)}\right)t + \left(\frac{1}{64} - \frac{a}{32}\right)t^2, \quad (49)$$

$$u_3 = d_1(x)t + d_2(x)t^2 + d_3(x)t^3, \quad (50)$$

$$d_1(x) = -\frac{x^2}{64} - \frac{x^2}{384\sqrt{2}} + \frac{ax^2}{32} - \frac{x^{2-\beta}}{48\sqrt{2}\Gamma(3-\beta)}t,$$

$$d_2(x) = \left(-\frac{x}{128\sqrt{2}} + \frac{ax}{16\sqrt{2}} - \frac{a^2x}{16\sqrt{2}} + \frac{x^{1-2\beta}}{8\sqrt{2}\Gamma(2-2\beta)} + \frac{x^{1-\beta}}{16\sqrt{2}\Gamma(2-\beta)}\right)t^2,$$

$$d_3(x) = \left(-\frac{1}{768} + \frac{5a}{384} - \frac{a^2}{32} + \frac{a^3}{48}\right)t^3$$

TABLE 1. numerical values and exact solutions when  $\beta = 2, a = 0.1$  for Eq.(51)

t	x	numerical value by ADM	exact solution	absolute error
0.02	0.25	0.5456	0.5460	0.0004
	0.50	0.5886	0.5894	0.0008
	0.75	0.6306	0.6314	0.0008
	1.0	0.6717	0.6715	0.0002
0.06	0.25	0.5495	0.5500	0.0005
	0.50	0.5928	0.5932	0.0004
	0.75	0.6351	0.6351	0.0000
	1.0	0.6763	0.6750	0.0013
0.2	0.25	0.5633	0.5638	0.0005
	0.50	0.6077	0.6067	0.0010
	0.75	0.6509	0.6480	0.0029
	1.0	0.6928	0.6872	0.0056
0.4	0.25	0.5838	0.5834	0.0004
	0.50	0.6299	0.6256	0.0043
	0.75	0.6703	0.6660	0.0043
	1.0	0.7071	0.7041	0.0030

TABLE 2. numerical values and exact solutions when  $\beta = 2, t = 0.06$  for Eq.(51)

a	x	numerical value by ADM	exact solution	absolute error
0.3	0.25	0.5465	0.5470	0.0005
	0.50	0.5898	0.5903	0.0005
	0.75	0.6323	0.6323	0.0000
	1.0	0.6737	0.6724	0.0013
0.5	0.25	0.5435	0.5440	0.0005
	0.50	0.5869	0.5874	0.0005
	0.75	0.6294	0.6295	0.0001
	1.0	0.6710	0.6697	0.0013
0.8	0.25	0.5390	0.5396	0.0006
	0.50	0.5825	0.5831	0.0006
	0.75	0.6252	0.6253	0.0001
	1.0	0.6671	0.6657	0.0014

Then the approximate solution in a series form is

$$u(x, t) = \frac{1}{2} + \frac{x}{4\sqrt{2}} + \left(\frac{1}{8} - \frac{a}{4}\right)t - \frac{x^2}{96\sqrt{2}} + \left(\frac{x}{16\sqrt{2}} + \frac{x^{1-\beta\beta}}{4\sqrt{2}\Gamma(2-\beta\beta)}\right)t + \left(\frac{1}{64} - \frac{a}{32}\right)t^2 + \dots \quad (51)$$

Table 1 shows the approximate solutions for Eq.(51) by using the modified Adomian's decomposition method and the exact solution[16] when  $\beta = 2, a = 0.1$ , and the values of  $\beta = 2$  is the only case for which we know the exact solution  $u(x, t) = 1/(1 + e^{-\zeta/\sqrt{2}})$ , where  $\zeta = x + ct$  and  $c = \sqrt{2}(1/2 - a)$ . It is to be noted that only the third-order of the modified Adomian's decomposition solution were

used in evaluating the approximate solutions for Table 1, and it is evident that the method used in this paper has high accuracy. Table 2 shows the approximate solutions for Eq.(51) using the modified Adomian decomposition method and the exact solutions[16] when  $\beta = 2, t = 0.4$ .

## 5. CONCLUSION

In this paper, analytical solutions for the space fractional Fisher's nonlinear diffusion equation have been obtained, and the modified Adomian's decomposition method was successfully used to these solutions. The reliability of this method and reduction in computations give this method a wider applicability. The corresponding solutions are obtained according to the recurrence relation using Mathematica.

## REFERENCES

- [1] I. Podlubny, Fractional differential equations: an introduction to fractional derivatives, fractional differential equations, to methods of their solution and some of their applications. New York: Academic Press; 1999.
- [2] R. Hilfer, Applications of fractional calculus in physics, Singapore: World Scientific; 2000.
- [3] R. Metzler, J. Klafter, The random walks guide to anomalous diffusion: a fractional dynamics approach, Phys. Rep. Vol. 339(2000) No.1, pp.1-77.
- [4] S.W. Wang, M.Y. Xu, Axial coquette flow of two kinds of fractional viscoelastic fluids in an annulus, Nonlinear Anal: Real World Appl. Vol.10(2009) No.2, pp.1087-1096.
- [5] X.Y. Jiang, M.Y. Xu, Analysis of fractional anomalous diffusion caused by an instantaneous point source in disordered fractal media, Int. J. Nonlinear Mech. Vol.41(2006)No.1, pp.156-165.
- [6] Y.Q. Liu, J.H. Ma, On the Solutions of Time-fractional Bacterial Chemotaxis in a Diffusion Gradient Chamber, Int. J. of Nonlinear Science, Vol.7(2009) No.4, pp.485-492.
- [7] G. Adomian, J. Math. A review of the decomposition method in applied mathematics, J. Math. Anal. Appl. Vol.135(1988)No.2, pp.501-544.
- [8] A.M. Wazwaz, S.M. El-Sayed, A new modification of the Adomian decomposition method for linear and nonlinear operators, Appl. Math. Comput. Vol.122(2001)No.3, pp.393-405.
- [9] S.S. Ray, Analytical solutions for the space fractional diffusion equation by two-step Adomian decomposition method, Commun. Nonlinear. Sci. Numer. Simul. Vol.14(2009)No.4, pp.1295-1306.
- [10] S. Momani, Z. Odibat, Analytical solutions of a time-fractional Navier-Stokes equation by Adomian decomposition method, Appl. Math. Comput. Vol.177(2006)No.2, pp.488-494.
- [11] A.M. Wazwaz, A reliable modification of Adomian decomposition method, Appl. Math. Comput. Vol.102(1999)No.1, pp.77-86.
- [12] A.M.A. El-Sayed, S.Z. Rida, A.A.M. Arafa, Exact Solutions of Fractional-Order Biological Population Model, Commun. Theor. Phys. Vol.52(2009)No.6, pp.992-996.
- [13] F. Shakeri, M. Dehghan, Numerical solution of a biological population model using He's variational iteration method, Comput. And Maths. With Appl. Vol.54(2007)No.7-8, pp.1197-1209.
- [14] Y. Tan, H. Xu, S.J. Liao, Explicit Series Solution of Travelling waves with a front of Fisher equation, Chaos, Solitons and Fractals, Vol.31(2007)No.2, 2007, pp.462-472.
- [15] S. Petrovskii, N. Shigesada, Some exact solutions of a generalized Fisher equation related to the problem of biological invasion, Mathematical Biosciences, Vol.172(2001)No.2, pp. 73-94.
- [16] A.M. Wazwaz, A. Gorguis, Exact solutions for heat-like and wave-like equations with variable coefficients, Appl. Math. Comput. Vol.149(2004)No.1, pp.15-29.
- [17] R.A. Fisher, The wave of advance of advantageous genes, Ann Eugen, Vol.7(1937)No.4, pp.355-369.

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