UNITED ARAB REPUBLIC

THE INSTITUTE OF NATIONAL PLANNING

التخطيط القومي

Memo. No. 682

THE CLASSICAL TRANSPORTATION PROBLEM AND ITS APPLICATION IN IRON-ORE INDUSTRY (U.A.R.)

By

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14th August 1966.

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CLASSICAL TRANSPORTATION

Problem, and its application in

IRON-ORE INDUSTRY.

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1965

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PREFACE

This thesis is concerned with the theory, and solution of classical transportation problem. This field should be just interesting to mathematicians, managers, and engineers.

To provide motivation, the first chapter has been devoted to the mathematical formulation of the transportation problem, method of solution, and finally some concepts or remarks to enable us to proceed for the optimal solution.

The view point of the second chapter is, in brief, a satisfcory approach to the computer and one of its languages "FORTRAN , FORTRAN program which translates our problem expressed as a series of algebraic statements, this in turn is translated by the computer into a complete machine language program, generating the step-by-step instructions necessary to solve the problem, and then left the computer to solve general example on transportation.

Before leaving this point, I like to note something about programmers, in order to show how they face repeated failiure with great patience, this is because the "first-time correct" program is extremely rare, and the word "first" can be changed to "second"., and "fourth" with the statement still true. However, programmers are those who are deep, silent thinkers, and have an astonishing ability to endure repeated failure.

The reader may ask me, why did you write this paragraph about programmers??

The answer will be; in order to show that preparation of programs is not an easy task, but it is more difficult than what any one could immagine. So, the reader now can expect how much the second chapter needs good effort and more time.

To provide insight into application in a "real" inviroment. the third and fourth chapters on application conclude this thesis.

The third chapter deals with the "Iron-Ore" problem, which is a very important mining problem, specially, in building our industrial country, because, as we all know our needs to iron ore.

This problem was found to be a combination of two problems; the first is a decision problem, and the second is transportation problem. So before treating this problem, a general knowledge about types of decision and Engineering Economic concepts have been shown in, and then a mathematical model of solution have been built.

Finally,

The fourth chapter deals with the final solution of the problem numerically - I hope it will do its purpose.

To the readers who are not interested in the paper from all its different aspects, I shall say that:

The first chapter needs mathematical background, so it is for mathematicans, the second one is for those who are interested in computer techinques, specially for programmers, and the last two ones are written specially for managers and engineers of Iron-Ore industry.

My love thanks to Eng. Fikri El Moursi, for his assistance in providing me with all the information requried for iron-ore problem.

Also;

I am indebted to Dr. Salah Hamid, the general manager of Operations Research Center" for his permission to me to have the honour to be one of the members of the fourth long training period of The National Planning Institute.

I am grateful to Dr. Roshdi Amer, the great experience in linear programming, for his many constructive suggestions, concepts, origins and for his undertaking to revise this thesis, also I am most grateful for his acceptance to be the supervisor of my thesis, and I have the honour to be one of his students in this field of science.

Finally, Thanks to Miss. Elen Zaki, and Mrs. Nadia Amer, the typists of the Operations Research Center for their good efforts in typing this thesis in beautiful form.

To all of these people , I say simply

"Thank you very much".

OKTOBER 15, 1965

A. L. Boctor.

CHAPTER -1-

THE CLASSICAL TRANSPORTATION PROBLEM.

1.1 - Introduction.

In this chapter we will take up a special problem called "Transportation", and show how it may be solved. This problem is an important class of operations research problems which can be formulated in terms of a net-work composed of points (shipping stations or sources, and receiving terminals or destinations), connected by routes, over which various kinds of transport take place.

The classical transportation problem exhibits a structure which can be represented by a mathematical equivalent, called a mathematical model, and then some computational method will be evolved for determining an optimal shedule of shipments under the following conditions:

a) fixed stockpiles of commodity must be available at sources.

b) Various fixed amounts are required at the final destinations.

c) total demand then must equals total supply.

The objective of the system happens to be the minimization of total costs measured in monetary units, which must satisfy a linear objective function.

Finally;

as a result of this summary

Our problem is to determine an optimal shipping shedule--one that has least costs.

1.2 - The Network Representation.

If we consider that we have "i" shipping stations, where (i = 1,2,3, ... , n) , "J" receiving terminals where (J = 1,2,3, ... , m) , and the costs to ship one unit from i to J are C(i,J)

Therefore the net work will be represented as shown in the following figure :-

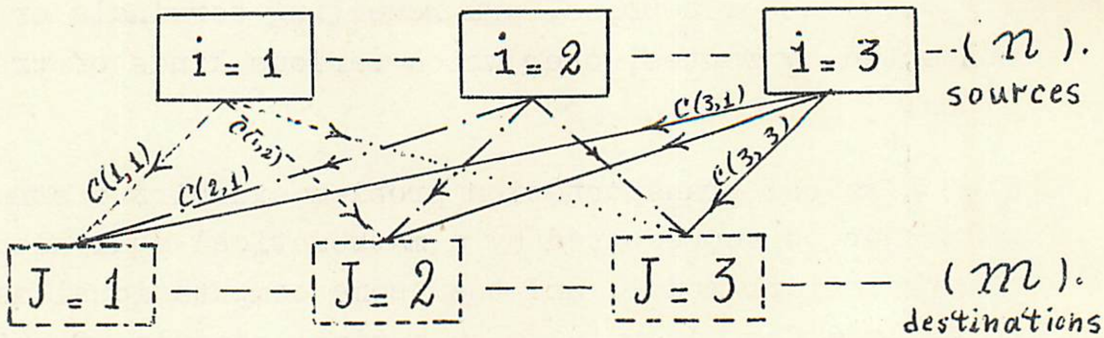


Fig 1-2-I Net work Representation of transportation problem.

The above fig gives us a general picutre about the number of possible routes which link every source "i" with all destinations "J". The arrows show the direction of shipment, and the costs of shipping are shown on the relevant link, thus for example C(1,2) is the cost from source 1 to destination 2.

1.3 - Elementary Transportation theory.

The classical transportation problem is defined as follows: Suppose that n (sources) contain various amounts of a commodity, which must be transported to m (destinations). let the quantity available at the ith (origin) is a_i (i.e: the ith source must dispose of exactly the quantity a_i), and the demand at the Jth destination is d_J (i.e: the Jth destination must receive exactly the quantity d_J.

It is assumed that:

(a) $$\sum_{i=1}^{n} a_i = \sum_{J=1}^{m} d_J \qquad (1)$$

(b) the numbers a_i, d_J are nonnegative.

Now;

Our problem is to determine X_{iJ} : (the number of units to be shipped from i to J) at an over-all minimum cost.

Therefore the above problem can be written in the standard form as follows:

2)

$$\begin{array}{l|r}
x_{11} + x_{12} + \cdots + x_{1m} & = a_1 \\
\quad + x_{21} + x_{22} + \cdots + x_{2m} & = a_2 \\
\text{-----------------------------------} & = \cdots \\
\quad\quad + x_{n1} + x_{n2} + \cdots + x_{nm} & = a_n \\
\hline
x_{11} \quad + x_{21} \quad + x_{n1} & = d_1 \\
+ x_{12} \quad + x_{22} \quad + x_{n2} & = d_2 \\
\text{-----------------------------------} & = \cdots \\
+ x_{1m} \quad + x_{2m} \quad + x_{nm} & = d_m \\
\hline
C_{11} x_1 + \cdots + C_{1m} x_{1m} + C_{21} x_{21} + \cdots + C_{2m} x_{2m} + C_{n1} x_{n1} + \cdots + C_{nm} x_{nm} & = Z
\end{array}$$

Each part of the matrix represents a group of equations:

3) The First Group Equations: $\sum_{J=1}^{m} x_{iJ} = a_i$, $i=1,2,\ldots,n$

4) The Second Group Equations: $\sum_{i=1}^{n} x_{iJ} = d_J$, $J=1,2,\ldots,m$

5) The Third Group Equation (Objective Function):-

$$\min. \quad Z = \sum_{i=1}^{n} \sum_{J=1}^{m} C_{iJ} \quad x_{iJ}$$

<u>Remark</u>:- The rank of the system is exactly $(m + n-1)$

The above model can be summedup into the form of rectangular array (6) called "<u>THE STANDARD TRANSPORTATION ARRAY</u>".

6)

					Row Total
	X_{11} C_{11}	X_{12} C_{12}	- - -	X_{1m} C_{1m}	a_1
	X_{21} C_{21}	X_{22} C_{22}	- - -	X_{2m} C_{2m}	a_2
	- - -	- - -	X_{iJ} C_{ij}	- - -	...
	- - -	- - -	- - -	- - -	...
	X_{n1}	X_{n2} C_{n2}	- - -	X_{nm} C_{nm}	a_n
Column Total	d_1	d_2	- - -	d_m	

1.4 - Method Of Solution, or the Method Of Finding a Basic Feasible Solution :-

Step 1: We know now that, the sum of the number of units required should be equal to the sum of the number of units available.

This is an important point in begining our method, but we can not expect this condition is satisfied in all the transportation problems. (i.e. the problem now is not balanced), so what will be the case if the total sum is not equall??

In this case a (Dummy) sources or destinations are added such that:

(a) If $\sum_{i=1}^{n} a_i > \sum_{J=1}^{m} dJ$

therefore a dummy destination $d_m = \sum_{i=1}^{n} a_i - \sum_{J=1}^{m} d_J$

is utilized, and the costs of this dummy column must be zero's since this dummy unit do not exist, and hence it will not be distributed.

$$\therefore \sum_{i=1}^{n} C_{im} = 0.0$$

(b) If $\sum_{i=1}^{n} a_i < \sum_{J=1}^{m} d_J$

therfore a dummy source $a_n = \sum_{J=1}^{m} d_J - \sum_{i=1}^{n} a_i$

is utilized, and the costs of this dummy row must be zero's

$$\therefore \sum_{J=1}^{m} C_{Jn} = 0.0$$

Now, we have a feasible solution <u>without</u> regard to the associated costs.

<u>Numerical Example</u>:-

Since it is easier to represent the method of transforming a general transportation problem into a classical one by means of an example, so consider the following numerical example:-

7)

i \ J	1	2	3	Quantity Available at Sources.
1	50	100	120	2
2	20	80	150	7
3	100	50	40	3
Quantity Required at destinations	4	5	3	12

Here in this example the quantity required is equal to the quantity available i.e. $\sum_{i=1}^{n} a_i = \sum_{J=1}^{m} d_J$

Now if: $\sum_{i=1}^{n} a_i < \sum_{J=1}^{m} d_J$

8)

(Before)

50	100	120	2
20	80	150	7
100	50	40	2
4	5	3	12 \ 11

→ Dummy Row

(After)

50	100	120	2
20	80	150	7
100	50	40	2
0	0	0	1
4	5	3	12

But if : $\sum_{i=1}^{n} a_i > \sum_{J=1}^{m} d_J$

9)

(Before)

50	100	120	2
20	80	150	7
100	50	40	4
4	5	3	12 \\ 13

→

(After)

			Dummy Column	
50	100	120	0	2
20	80	150	0	7
100	50	40	0	4
4	5	3	1	13

Step 2: To begin a starting feasible solution, there are many Methods:

First Method :-

It is called "North-West corner method" or the "Stepping-Stone Method."

(1) Start with the upper left-hand corner i.e (square(1,1), and allocate as much as possible: if $a_1 > d_1$, then allocate $x_{11} = d_1$, and if $a_1 < d_1$, then allocate $x_{11} = a_1$

(2) Therfore row or column 1 must be satisfied, and the other leaved unsatisfied. We will follow then the same steps as in (1) for the neighbouring square of this unsatified row or column ..... etc.

This rule will select many variables for the basic set.

Since on the last step, when one row and one column remain, both must be dropped after the last variable is evaluated. i.e. the last remaining variable can aquire a value consistent with the totals for the single row and column still remaining in the final reduced array. This number of Basic Variables provides us with the first basic solution, and by multiplying them by their associated costs, it will give us the "Total cost".

This cost is not the minimum cost, so how to obtain the optimum cost; this will be discussed later.

Second Method:-

It is called "Least-Cost Method"

It deals as follows:-

(1) Choose the smallest cost in the Unit Cost Array:

$$\begin{bmatrix} C_{11} & C_{22} & \cdots & C_{2n} \\ C_{21} & C_{22} & \cdots & C_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ C_{m1} & C_{m2} & & C_{mn} \end{bmatrix}$$

let this cost be $C_{pq} = \underset{(i,J)}{\text{Min}}\ C_{iJ}$

(2) Determine the value of x_{pq} as the minimum of its row or column total i.e:

Case 1: If $a_p < d_q$, then all the other variables in the pth row are to be given the value zero, and designated as non-basic. Next delete the pth row, reduce the value of d_q to $(d_q - a_p)$, and then find the smallest cost factor among those squaresin column q, and proceed in the same manner to evaluate the corresponding variable.

Case 2: If $a_p > d_q$, then, similarly, the qth column is to be deleted and a_p is replaced by $(a_p - d_q)$, ... etc.

Case 3: If $a_p = d_q$, then delete either the row or column, but not both. However, if several columns, but only one row, remain in the reduced array, then drop the qth column, and conversly, if several rows and one column, drop the pth row.

Remarks:-

1. In this method, (m+n-1) entries will be made.
2. This method gives us a good starting solution than the first, since the number of iterations required to acheive optimality can be greatly reduce if the basic set is selected with some reference to the values of the coefficients in the objective form.

Application of First Method (North-West Corner Method) :-

10)

2 (50)	(100)	(120)	2
2 (20)	5 (80)	(150)	7
(100)	(50)	3 (40)	3
4	5	3	

Table (10) shows the solution of the numerical example given by table (7) by "the North-West corner method". The basic solution in this case is:

$$x_{11} = 2 \ , \ x_{21} = 2 \ , \ x_{22} = 5 \ , \ x_{33} = 3$$

Therefore, the total cost is,

$$= 2 \times 50 + 2 \times 20 + 5 \times 80 + 3 \times 40 = 640$$

Important Remark:-

Here the basic variables is $<(m + n - 1)$

Application of Second Method (Least Cost Method):-

11)

50	100	2 120	2
4 20	3 80	150	7
100	2 50	1 40	3
4	5	3	

Table (11) shows the solution of the same example by the "Least-Cost Method".

The basic solution in this case is

$x_{13} = 2$, $x_{21} = 4$, $x_{22} = 3$, $x_{32} = 2$, $x_{33} = 1$

Therefore, the total cost is,

$$= 2 \times 120 + 4 \times 20 + 3 \times 80 + 2 \times 50 + 1 \times 40 = 700$$

Important Remark:-

It is clear here that total costs given by the first method is less than that given by the second Method, but if we look for a moment, we will see that the basic variables in second method are $= m + n - 1$. from which it is easy to continue the solution.

Step 3 :

We have now $(m + n - 1)$ variables. Assume that the multiplier of the ith row equation is U_i, and that of the Jth column equation is v_J. These $(m+n)$ numbers U_i , v_J are called "shadow costs"

Since any equation may be considered redundant, we can assign an arbitrary value to one of the multipliers, and then evaluate the set of multipliers, ther by rendered unique, which will cause the vanishing of all the relative cost factors correspon-ding to the basic variables, i.e., (we have $(m + n - 1)$ equat-ions in $(m + n)$ unknowns), So we suppose any one of these unknowns equals zero, let this be;

(12) $v_n = o$ (*)

The values for u_i and v_J are chosen to make the coefficients of the basic variables vanish, i.e.

(13) $C_{iJ} = U_i + v_J$ for x_{ij}, a basic variable.

Note that (13) defines a system of equations in which the multipliers play the part of variables.

Now the $(m + n)$ shadow numbers are determined.

For the non basic variables x_{iJ}, we will determine their shadow costs.,

(14) $\bar{C}_{iJ} = U_i + v_J$ for x_{ij}, non basic variable.

Noting that c_{iJ} may be larger or smaller than $\bar{c}_{iJ}$ but not equal.

Therefore the shadow costs table will contain C_{iJ} for basic variables, and $\bar{C}_{iJ}$ for non basic variables.

(*) It is very important to show that u_i,v_J may be replaced by (u_i+k) and (v_J-k) without affecting, hence, any one of$(m+n)$ multipliers may be given an arbitrary value in determining the remainder.

Step 4 :

In this step we will determine the routes which will yield less costs. This will be done by subtracting the elements of the shadow-costs table from those of the real-costs table.

Now, for the basic variables, the result will be zero, but for those non basic, it will be either negative or positive; this means the following:-

(a) If it were positive, this means that the costs of shipping one unit through this route will increased by this amount.

(b) If it were negative, this means that the costs of shipping one unit through this route will decreased by this amount.

The new table is called "Real-Shadow Costs table", in it: if all the entries are more or equal to zero, therefore Optimal solution is obtained !! (because there are no routes minimize the costs, but all increases in this case), but if they are not so (i.e. not all of them are 0), therefore we seek for the most negative value in the "R.S. costs table". This means that a new basic solution other than the old one will be deduced. In this new solution the total costs will be decreased.

Remark:-

The principle of this new solution depends upon:

If a certain route used rather than another, the costs will be decreased.

This will lead us to step 5.

If we apply steps 3,4 to our numerical example (7), we get

(15)

		120	$U_1 = 70$
20	80		$U_2 = 20$
	50	40	$U_3 = -10$
$V_1 = o$	$V_2 = 60$	$V_3 = 50$	

1. In table (15), u_i, v_i were determined only for the relative cost factors (C_{iJ}) Corresponding to the basic variables.
2. We will determine now $\bar{C}_{iJ}$ for the non-basic variables:-

(16)

70	130	
		70
-10		

3. Tables (15),(16) constitute the Shadow-Cost table (17):-

70	130	120
20	80	70
-10	50	40

4. Subtracting table (17) from table (7), weget the Real-Shadow cost table (18):-

-20	-30	0
0	0	80
100	0	0

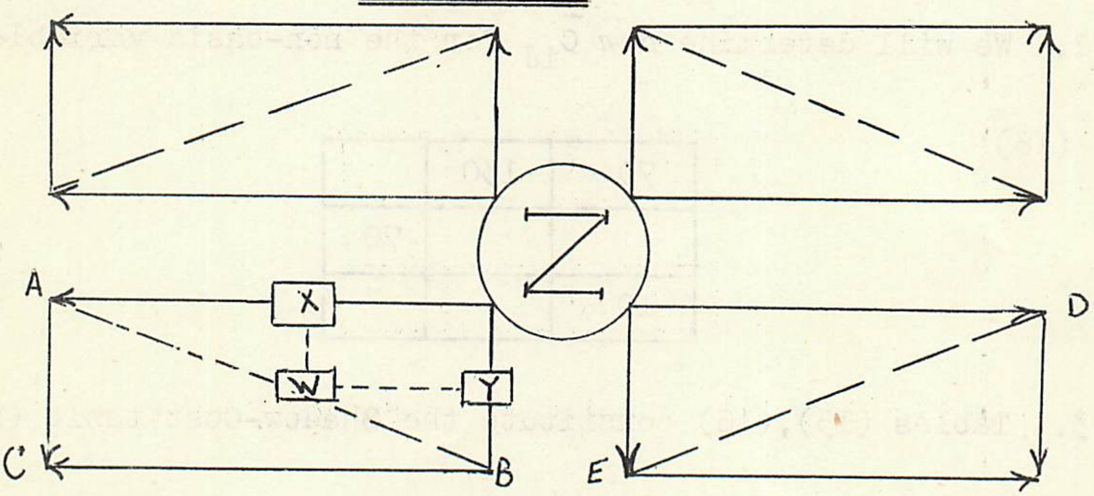
Step 5 :

We denote the most negative value by Z.

Therefore, we must allocate many units as possible as we can to this route taking care that the demand and supply constraints are always satisfied. The procedure to find how many units could we allocate to this route is as follows:-

(1) Imagine Z as a head of a right angle.

Fig 1-4-1



(2) Consider the angle (AZB) in fig 1-4-I.
on the side AZ, there are many squares, and this is also the case on the other side.
Look; if there are squares containing basic variables in both sides of the angle, if they are for ex:(x,Y), then be sure if (w) is a basic variable also, but if there are not, we seek for the another right angles (DZE) ... etc, until we find our case: (square at its ends, there are basic variables).
Let for example X, Y, W are the basic variables we observe.

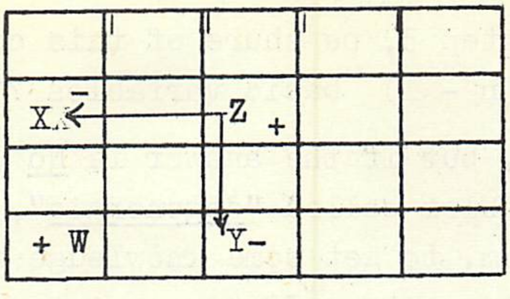


Fig 1-4-II.

(3) we will start with the positive sign in the squares defining the new route Z, W, and negative signs for both squares X, Y.

(4) We will pick the smallest value among X, Y. This value will be assigned to the new route, then subtracted from X,Y, and added to the value in W, because of not affecting the feasibility of the solution.

In this case, we have new basic solution, with costs less than the first one.

(5) But !! <u>Is this the Optimal one</u> ..... ??
We can not expect, but we will continue all the last steps (3,4) and if we see that we have been reached to a point where the costs can no longer be decreased therefore, at this point, it is said that the optimal solution is reached and the problem is finished, but if it is not so, we will continue step (5), and then 3, 4, ... , etc.

In the last lines, I asked: is this the optimal one, and answered to repeat steps 3,4, ... !!

Before begining step 3, be shure of this condition:
Do you have (m + n - 1) basic variables ??

Yes .. Continue, but if the answer is No , this will leads us to another problem called "degenerate", So we will discuss first this problem, to get some knowledge about the trick done, in order to continue steps 3, 4, ... etc.,

If we apply Step (5) to our numerical example, this show's the following:-

From table (18), the most negative value is (-30), this means that if a unit is shipped from source (1) to destination (2), costs will be decreased by 30/per unit shipped.

So Z = the square (1,2)

(19)

	Z	X=2
4	3	
	Y=2	W=1

Therefore, the new basic solution will be as follows:-

(20)

	2	
4	3	
		3

$x_{12} = 2$, $x_{21} = 4$, $x_{22} = 1$, $x_{23} = 2$, $x_{32} = 2$, $x_{33} = 1$

The Total costs = 2x100 + 4x20 + 3x80 + 3x40
= 640

Now the sum of basic variables is = 4, but (m + n - 1) = 5, So, the problem now starts to degenerate.

1-5 Degeneracy:-

As we said before, we must have (m+n-1) basic variables. So if we have less than (m+n-1), we will assign an infinitizmal amount to the empty squares required to complete (m + n - 1).

This symbol, $+\epsilon$, is entered in the square (e.g(s,t) to indicate that a value, called ϵ, will be given to the non-basic variable, x_{st}. In this case we will treat

$X_{st} = \epsilon$ as basic variable.

When we obtain the final solution, all the ϵ's are dropped from the solution.

1-6 Some Helpful Remarks, regarding programming Difficultires:-

1. If the basic variables of any solution but not these of the optimal solution have one of the following arrangements:-

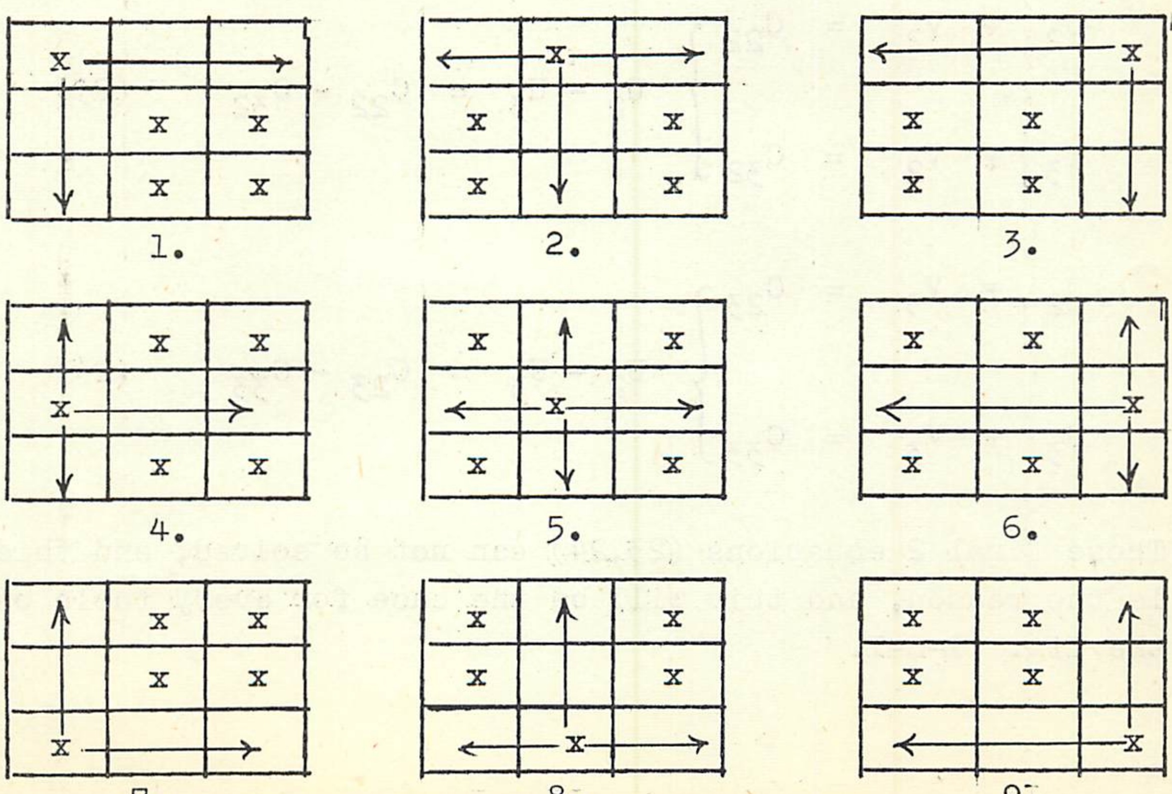


Fig. 1-6 (I)

In this case, this arrangment of basic variables will not help us in finding the Shadow-costs (U_i, v_J) and the problem in this case couldnot be solved. because for ex:-

a) Let the shadow squares are those contain basic variables.

	V(1)	V(2)	V(3)
U(1)	C_{11}	C_{12}	C_{13}
U(2)	C_{21}	C_{22}	C_{23}
U(3)	C_{31}	C_{32}	C_{33}

Fig 1-6-II.

b) Suppose $V_1 = 0$ (21)

$U_1 + V_1 = C_{11}$ (V_1 , C_{11} are known) (22)

from which u_1 can be deduced.

the other 4 equations will be:

$$\left.\begin{aligned} U_2 + V_2 &= C_{22} \\ U_3 + V_2 &= C_{32} \end{aligned}\right\} U_2 - U_3 = C_{22} - C_{32} \qquad (23)$$

$$\left.\begin{aligned} U_2 + V_3 &= C_{23} \\ U_3 + V_3 &= C_{33} \end{aligned}\right\} U_2 - U_3 = C_{23} - C_{33} \qquad (24)$$

These final 2 equations (23,24) can not be solved, and this is the reason, and this will be the case for every table of the fig. 1-6-I.

2. If the arrangment is as follows:-

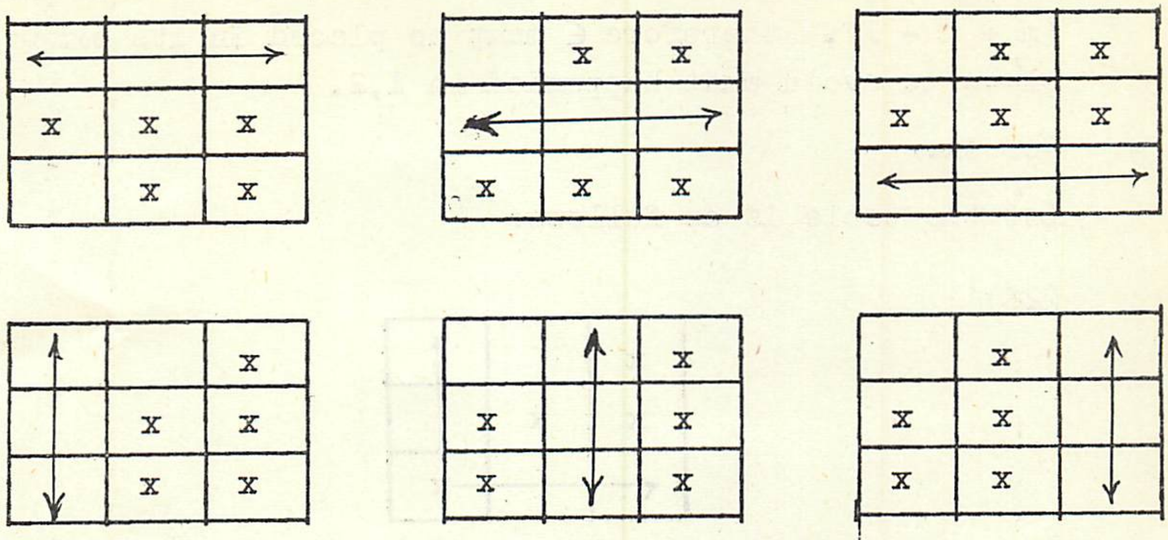


Fig 1-6 -III-

In this case also the problem could not be solved. because for ex:-

a) Also in this case, the shadow squares deal with basic variables.

(25)

	V_1	V_2	V_3
U_1			
U_2	x	x	x
U_3		x	x

b) Suppose $V_1 = o$

Can we get U_1 ?? (Of course no)

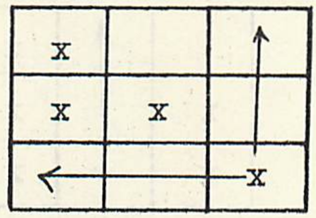
because there are no basic variables in the first row so C_{11}, C_{12}, C_{13} are not known, so if v_1 is known, then $U_1 + V_1 = C_{11}$ includes 2 unknown's, so it could not be solved. and this is also the case for the other tables in fig. 1-6 - III.

3. Now in the case where the basic variables are less than $(m + n - 1)$, therefore $\in$ must be placed in its exact place to avoid what happended in 1,2.

For ex:-

Let the table is as follows:

(26)



Therefore the exact position of $\in$ is either one of the following:-

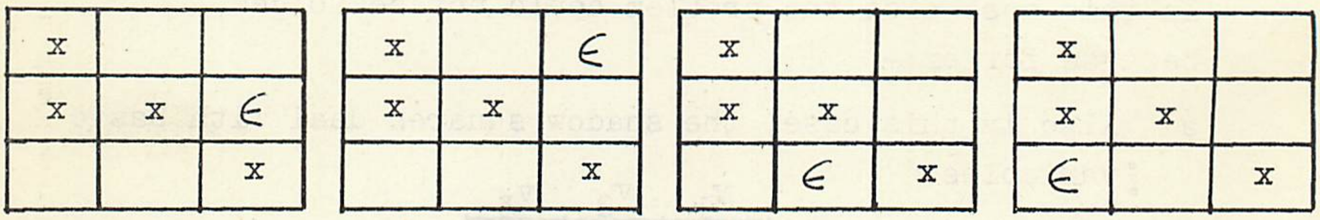
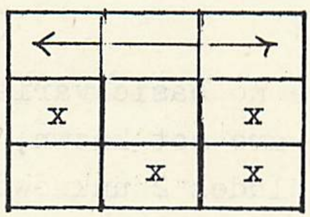


Fig 1-6 - IV.

From above we see that the suitable position of $\in$ is one of the squares covered by arrow's. but if the table is as follows:-

(27)



Therefore the exact position of ϵ is either one of the following:

ϵ		
x		x
	x	x

	ϵ	
x		x
	x	x

		ϵ
x		x
	x	x

Fig 1-6. -V-

This means that its suitable position is one of the empty squares covered by the arrow.

The Same procedure must be performed if more than one ϵ must be placed in order to complete $(m + n - 1)$ variables.

4. Suppose that no units could be shipped through one of the routes we have in our transportation problem. This will be treated by assigning a very large cost say M to this route. It must be noted that M must be greater than all the remainder costs of the other routes.

   This is only a trick help's us to reach to the final solution, and it is found to have many useful applications.

5. If the basic variables of any feasible solution are more than $(m+n-1)$; what will be the case .... ?? for example :-

   In Step (5): (Fig 1-4-I), the condition was that X,Y,W must be basic variables. If we can not observe basic variables except X,Y, what will be the case .... ?? we will treat this problem by the following example:-

(28)

x = 2 ←	Z ↓	
1		3
W = Zero (non basic)	Y=5	1

	+2	
1		3
+2	3	1

Therefore, we have (m+n) basic variables.

Now, we come to the next step to get the shadow costs table.

∴ we have 6 equations:

$$U_1 + V_2 = 2 \quad (1)$$
$$U_2 + V_1 = 1 \quad (2)$$
$$U_2 + V_3 = 3 \quad (3)$$
$$U_3 + V_1 = 2 \quad (4)$$
$$U_3 + V_2 = 3 \quad (5)$$
$$U_3 + V_3 = 1 \quad (6)$$

(29)

These 6 equations could not be solved, and also if we assume $V_1 = 0$

∴ from 2 $U_2 = 1$
from 3 $V_3 = 2$
from 4 $U_3 = 2$

therefore in (6) : Is $U_3 + V_3 = 1$?? .... of course not.

Therefore when we have (m+n-1) basic variables, the problem could not be continued, and so no solution will be observed in this case.

Remark:-

All these helpful Remarks are taken into consideration in the FØRTRAN Program

An example on treatment of Degeneracy:-

In the last example, the problem starts to degenerate. So in order to continue, and infinitiwmal number ϵ will be placed not at random, but with accuracy in order to avoid what happend at (1-6).

Therefore the, best allocation of ϵ in our example is as follows:

(30)

	2	
4	3	ϵ
		3

Now, we have (m+n-1) basic variables, which make us able to continue.

We will continue, until the optimal solution is reached:-

$V_1=0$ $V_2=60$ $V_3=130$

$U_1=40$		100	
$U_2=20$	20	80	150
$U_3=90$			40

→ The Shadow-C.Table

40	100	170
20	80	150
-90	-30	40

→ The Real-S.C.Table

10	0	-50
0	0	0
190	80	0

↓

	X=2	Z=??
4	W=3	Y=ϵ
		3

← The New-basic solution

	$2-\epsilon$	ϵ
4	$3+\epsilon$	
		3

← $V_1=0$ $V_2=60$ $V_3=80$

$U_1=40$		100	120
$U_2=20$	20	80	
$U_3=40$			40

↓

The Shadow-C.Table

40	100	120
20	80	100
-40	20	40

→ The Real-S.C.Table

10	0	0
0	0	50
140	30	0

All Positive ∴ this is the optimal

Fig. 1-6 -VI

∴ $X_{12} = 2$, $X_{21} = 4$, $X_{22} = 3$, $X_{33} = 3$

Total Costs = 640 [note that ϵ must be dropped,],

CHAPTER 2

THE COMPUTER PROGRAM.

2.1. Introduction.

In the last chapter, we discussed the classical transportation problem, and observed how many operations we are need in order to reach to the optimal solution. These operations we do, needs many computational methods, and so a good effort is required.

But;

If we look backwards, Just two or three years ago, a good device reaches our "Operations Research Center". it was the first automatic computer in the U.A.R. Just over these years ago ... Yet to day, it is at work in industrial, government, business military, and University researche's aiding men's minds with automatic calculating power & relativity high order operations per second.

The computer through its high speed alone enable man:

1. to increase his output per hour.
2. to make use of many mathematical methods that were previously impractical due to the lengthy and time-consuming calculations involved.

Therefore, we see that computers, ingeneral, increase accuracy, and increase productivity by encouraging intelligent planning.

So, why we do not use this device to solve our classical transportation problem. ?!

This is what we are going to do in this chapter Therefore, to the following pages ....

2.2. Definition of Fortran Program.

"Fortran" is a programming language closely related to the language of ordinary algebra. The name "Fortran" comes from "Formula TRANslation" and was chosen because many of the statements which this language accepts look like algebraic formulas.

These statements are used by the programmer to describe his problem.

The programmer is the person who spend much time may be month's, preparing the steps of solution mathematically, then translate it into "Fortran Ianguage", and have the computer execute the program and produce the results in few minutes.

2.3. The Flow-Chart Diagram

The general sequence of the different operations in the classical transportion problem is given by the following block diagram. This block diagram is called (Flow-Chart). The details of each part is given in the Flow-Chart.

The following symbols are used in the Flow-Chart, and in the Fortran program, they have the following meaning:-

M,N : the number of origins or sources, and the number of terminals or destinations.

EPS : an infinitizmal amount required to complete (M+N-1)

A(I) : the quantity available at sources I=1,2,....,N.

D(J) : the quantity required at destinations J=1,2, ...,M.

C(I,J) : the real costs to ship one unit from source I to destination J.

X(I,J) : the number of units to be shipped from source I to destination J.

SUMI : $\text{SUMI} = \sum_{I} A(I)$ I = i,2,...,N.

SUM J : $\text{SUM J} = \sum_{J} D(J)$ J = 1,2,...,M.

TOCO : the Total costs and =

(X(I,J) multiplied by C(I,J))

for I=1,2,...,N , J=1,2,...,M.

U(I) : the shadow costs for I = 1,2,....,N

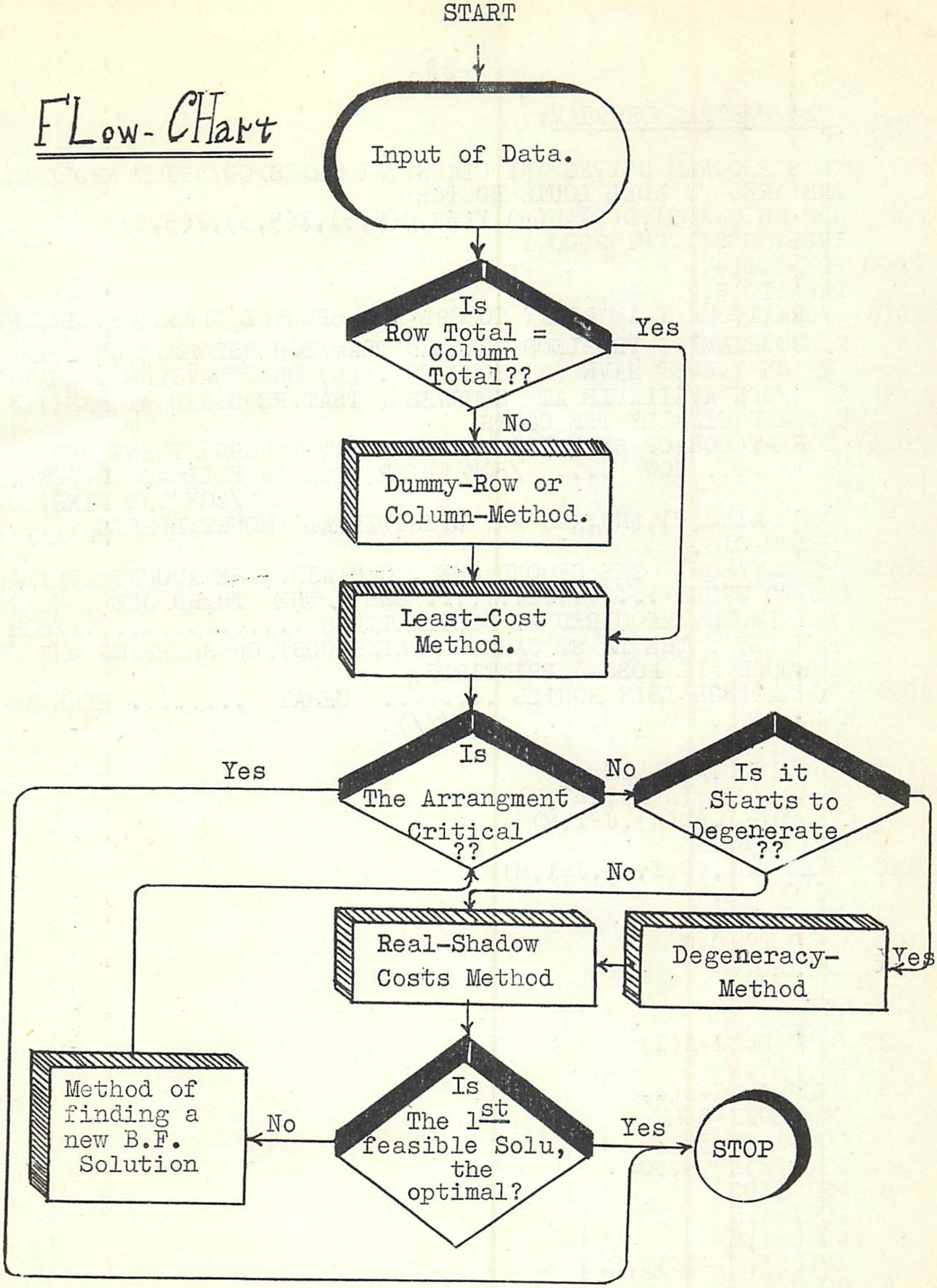
V(J) : the shadow costs for J=1,2,....,M

F(I,J) : the Real-Shadow Cost Table for I=1,2,...,N J = 1,2,...,M, and is = (Real-Costs table C(I,J))- Shadow-Costs table (U(I) + V (J)))

SUM : The number of basic variables X(I,J).

AB : the number of shadow costs U(I), V(J).

FLow-CHart


START
Input of Data.
Is Row Total = Column Total??
Yes
No
Dummy-Row or Column-Method.
Least-Cost Method.
Is The Arrangment Critical ??
Yes
No
Is it Starts to Degenerate ??
No
Yes
Real-Shadow Costs Method
Degeneracy-Method
Method of finding a new B.F. Solution
No
Is The 1st feasible Solu, the optimal?
Yes
STOP

2.4. The FØRTRAN PRØGRAM:

```
C       THIS PROGRAM SOLVES ANY CLASSICAL TRANSPORTATION PROBLEM.
        PREPARED BY ADEL LOUIS BOCTOR
        DIMENSIONA(5),D(5),U(5),V(5),C(5,5),X(5,5),F(5,5)
        IF(SENSESWITCH1)2000,1
 2000   PRINT1014,
        PRINT1015,
 1014    FORMAT(80H I AM READY TO SOLVE A SPECIAL CLASS OF LINEAR
        X PROGRAMMING PROBLEMS, CALLED TRA-/80H NSPORTATION , IN
         WHICH I MUST HAVE (N) SOURCES , (M) DESTINATIONS , QUANTITY
        X   /80H AVAILABLE AT  SOURCES , THAT REQUIRED AT DESTINATIONS
        X , AND FINALLY THE COSTS  )
 1015    FORMAT(80H OF SHIPPING PER ONE UNIT THROUGH THESE ROUTES ....
        X      .... NOW   ....  /80H ENTER DATA IN FULL AS FOLLOW .....
        X                                           /80H THE FIRST CARD ,
        XCONT AINS(M),(N),AND AN INFINITIZMAL AMOUNT EPSELON........)
         PRINT1016,
 1016    FORMAT(80H   THE SECOND ONE , CONTAINS THE AMOUNT AVAILABLE
        XAT SO URCES ................/80H  THE  THIRD ONE , CONTAINS
         THE AMOUNT REQUIRED AT DESTINATIONS ................/80H
         FINALLY , THE LA ST CARD CONTAINS COST OF SHIPPING ONE UNIT
         THROUGH ALL POSS-) PRINT1017,
 1017    FORMAT(80H LBLE ROUTES ........  OKKAY  ........ HERE WE GO
        X ........                 ////)
         PAUSE
    1   READ1000,M,N,EPS
        READ 1001,(A(I),I=1,N)
        READ1001,(D(J),J=1,M)
        DO110I=1,N
  110   READ1001,(C(I,J),J=1,M)
        DO100I=1,N
        DO100J=1,M
  100    X(I,J)=0.0
        SUMI=0.0
        SUMJ=0.0
        DO2I=1,N
    2   SUMI=SUMI+A(I)
        DO3J=1,M
    3   SUMJ=SUMJ+D(J)
        DIF=SUMI-SUMJ
        CC=+1.0E+38
        IF(DIF)28,4,30
    4   PRINT1005
        DO6I=1,N
        DO6J=1,M
        IF(C(I,J)-CC)5,6,6
    5   CC=C(I,J)
        II=I
        JJ=J
```

```
   6  CONTINUE
1050  PRINT1002,II,JJ,C(II,JJ)
   7  IF(A(II)-D(JJ))18,8,23
   8  IF(A(II))114,114,119
 119  X(II)=0.0
      D(JJ)=0.0
      PRINT1003,II,JJ,X(II,JJ)
      DO15I=1,N
       IF(A(I))9,15,9
   9  IF(II-1)10,10,14
  10  IP=II+1
  11  CC=+1.0E+38
      DO13J=1,M
      IF(J-JJ)12,13,12
  12  IF(X(IP,J))47,47,13
  47  IF(C(IP,J)-CC)39,13,13
  39  CC=C(IP,J)
      II=IP
      J1=J
  13  CONTINUE
      JJ=J1
      GOTO1050
  14   IP=II-1
      GOTO11
  15  CONTINUE
  16  TOCO=0.0
      DO17I=1,N
      DO17J=1,M
  17  TOCO=TOCO+X(I,J)*C(I,J)
      PRINT1004,TOCO
      GOTO400
  18  IF(A(II))113,114,113
 114  I 11=II
      I 11=I 11+1
      IF(I 11-N)120,120,115
 120  II=I 11
      GOTO18
 115  II=II-2
 124  IF(A(II))1050,122,1050
 122  II=II-1
      GOTO124
 113  X(II,JJ)=A(II)
```

```
      D(JJ)=D(JJ)-A(II)
      A(II)=0.0
       PRINT1003,II,JJ,X(II,JJ)
      DO19I=1,N
 19   CONTINUE
      GOTO16
 20   CC=+1.0E+38
      DO37I=1,N
      IF(I-II)21,37,21
 21   IF(X(I,JJ))46,46,37
 46   IF(C(I,JJ)
 36   CC=C(I,JJ)
      I1=I
 37   CONTINUE
      II=I1
      GOTO1050
 23   IF(D(JJ))112,111,112
111   J11=JJ
      J11=J11+1
       IF(J11-M)121,121,116
121   JJ=J11
      GOTO23
116   JJ=JJ-2
125   IF(D(JJ))1050,123,1050
123   JJ=JJ-1
      GOTO125
112   X(II,JJ)=D(JJ)
      A(II)=A(II)-D(JJ)
      D(JJ)=0.0
      PRINT1003,II,JJ,X(II,JJ)
      DO24I=1,N
      IF(A(I))25,24,25
 24   CONTINUE
      GOTO16
 25   CC=+1.0E+38
      DO34J=1,M
       IF(J-JJ)33,34,33
 33   IF(X(II,J))45,45,34
 45   IF(C(II,J)-CC)41,34,34
 41   CC=C(II,J)
      J1=J
 34   CONTINUE
       JJ=J1
      GOTO1050
 28   N1=N+1
      A(N1)=ABSF(DIF)
      DO29J=1,M
      X(N1,J)=0.0
 29   C(N1,J)=0.0
      N=N1
      GOTO4
```

```
   3Ø M1=M+I
      D(M1)=ABSF(DIF)
      DO31I=1,N
      X(1,MI)=Ø.Ø
   31 C(I,M1)=Ø.Ø
      M=M1
      GOTO4
  4ØØ PRINT 1ØØ8
      PRINT1ØØI,((X(I,J),J=1,M),I=1,N)
  199 SUM=Ø.Ø
      DO2Ø1I=1,N
      DO2Ø1J=1,M
      IF(X(I,J))2Ø1,2Ø1,2ØØ
  2ØØ SUM=SUM+1.
  2Ø1 CONTINUE
      AK=M+N-1
      PRINT1ØØ6,SUM,AK
  2Ø8 AA=SUM-AK
      I=1
 25ØØ DO2Ø11J=1,M
      IF(X(I,J))2ØØ6,2Ø11,2ØØ6
 2Ø11 CONTINUE
      IF(AA)334,2ØØ2,2ØØ2
  334 I=1
  335 DO381J=1,M
      IF(X(I,J))38Ø,381,38Ø
  38Ø I=I+1
      IF(I-N)335,335,386
  381 CONTINUE
      X(I,1)=EPS
      SUM=SUM+1.
      GOTO2Ø8
  386 DO388I=1,N
      DO388J=1,M
      IF(X(I,J))388,387,388
  387 X(I,J))388,387,388
      SUM=SUM+1.
      GOTO2Ø8
  388 CONTINUE
 2ØØ6 II=I
      JJ=J
 275Ø DO2ØØ8I=1,N
      IF(I-II)2ØØ7,2ØØ8,2ØØ7
 2ØØ7 IF(X(I,JJ))2Ø3Ø,2ØØ8,2Ø3Ø
 2Ø3Ø JP=JJ+1
      IF(JP-M)26ØØ,26ØØ,2Ø31
 26ØØ JJ=JP
      IF(X(II,JJ)275Ø,2Ø
 2Ø31 I=II+1
      IF(I-N)25ØØ,25ØØ,336
```

```
2ØØ8 CONTINUE
     DO2Ø1ØJ=1,M
     IF(J-JJ)2ØØ9,2Ø1Ø,2Ø3Ø
2ØØ9 IF(X(II,J))2Ø3Ø,2Ø1Ø,2Ø3Ø
2Ø1Ø CONTINUE
     IF(AA)2Ø3,2ØØ2,2ØØ2
2ØØ2 PRINT1ØØ9
     GOTO1
 2Ø3 I=1
 2Ø9 AI=I
     IF(ABSF(AA)-A1)2Ø7,2Ø4,2Ø7
 2Ø4 IF(II-1)2Ø5,2Ø5,2Ø12
 2Ø5 IP=II+1
 2Ø6 X(IP,JJ)=EPS
     SUM=SUM+1.
     GOTO2Ø8
2Ø12 IF(II-N)2Ø5,2Ø13,2Ø13
2Ø13 IP=II-I
     GOTO2Ø6
 2Ø7 I=I+1
     GOTO2Ø9
 336 PRINT1ØØ8,
     PRINT1ØØ1,((X(I,J),J=1,M),I=1,N)
     AB=Ø.Ø
     DO5ØØØ I=1,N
5ØØØ U(I)=Ø.Ø
     DO6ØØØJ=1,M
6ØØØ V(J)=Ø.Ø
 3ØØ J=1
     DO333I=1,N
     IF(X(I;J))3Ø1,333,3Ø1
 333 CONTINUE
     GOTO2ØØ2
 3Ø1 V(1)=Ø.Ø
     DO332I=1,N
     IF(X(I,1)3Ø2,332,3Ø2
 3Ø2 IF(U(I)332,3Ø3,332
 3Ø3 F(I,J)=C(I,J)
     U(I)=F(I,J)-V(J)
     AB=AB+1.
     IF(AB-SUM)332,31Ø,31Ø
 332 CONTINUE
 3Ø4 DO331J=2,M
     DO331I=1,N
     IF(X(I,J))3Ø5,331,3Ø6
 3Ø5 IF(V(J)3Ø6;3Ø9,3Ø6
 3Ø6 IF(U(I))331,3Ø7,331
 3Ø7 F(I,J)=C(I,J)
     U(I)=F(I,J)-V(J)
 3Ø8 AB=AB+1.
     IF(AB-SUM)39Ø,31Ø,31Ø
```

```
390 IF(1-N)331,391,331
391 IF(J-M)331,304,331
309 IF(U(I)330,331,330
330 F(I,J)=C(I,J)
    V(J)=F(I,J)-U(I)
    GOTO308
331 CONTINUE
    IF(AB-SUM)304,310,310
310 DO312I=1,N
    DO312J=1,M
    IF(X(I,J))312,311,312
311 F(I,J)=U(I)+V(J)
312 CONTINUE
    PRINT1011,(U(I),I=1,N)
    PRINT1012,(V(J),J=1,M)
    PRINT1013,(F(I,J),J=1,M),I=1,N)
    DO316I=1,N
    DO316J=1,M
316 F(I,J)=C(I,J)-F(I,J)
    PRINT1010,((F(I,J),J=1,M),I=1,N)
    DO320I=1,N
    DO320J=1,M
    IF(F(I,J))318,320,320
320 CONTINUE
    TOCO=0.0
    DO202I=1,N
    DO202J=1,M
202 TOCO=TOCO+X(I,J)*C(I,J)
    PRINT1004,TOCO
    PRINT1007,
    GOTO1
318 CC=+1.0E+38
    DO348I=1,N
    DO348J=1,M
    IF(F(I,J)-CC)319,348,348
319 CC=F(I,J)
    II=I
    JJ=J
348 CONTINUE
321 IP=II+I
450 IF(IP-N)422,422,432
422 IF(X(IP,JJ))650,437,650
650 JP=JJ+1
423 IF(JP-M)424,424,438
424 IF(X(IP,JP))425,600,425
600 JP=JP+1
    GOTO423
```

```
425 IF(X(II,JP))426,6ØØ,426
426 IF(X(II,JP)-X(IP,JJ))427,427,431
427 X(II,JJ)=X(II,JJ)+X(II,JP)
    X(IP,JJ)=X(IP,JJ)-X(II,JP)
    X(IP,JP)=X(IP,JP)+X(II,JP)
    X(II,JP)=Ø.Ø
    GOTO16
431 X(II,JJ)=X(II,JJ)+X(IP,JJ)
    X(II,JP)=X(II,JP)-X(IP,JJ)
    X(IP,JP)=X(IP,JP)+X(IP,JJ)
    X(IP,JJ)=Ø.Ø
    GOTO16
432 IM=II-1
433 IF(IM-1)341,434,434
434 IF(X(IM,JJ)435,436,435
435 IP=IM
    GOTO65Ø
436 IM=IM-1
    GOTO433
437 IP=IP+1
    GOTO45Ø
438 JM=JJ-1
439 IF(JM-1)437,44Ø,44Ø
44Ø IF(X(IP,JM)525,7ØØ,525
7ØØ JM=JM-1
    GOTO439
525 IF(X(II,JM)626,7ØØ,626
626 JP=JM
    GOTO426
341 CC=+1.ØE+38
    DO344I=1,N
    DO344J=1,M
    IF(F(I,J)-F(II,JJ))342,344,342
342 IF(F(I,J)-CC)343,344,344
343 CC=F(I,J)
    II=I
    JJ=J
344 CONTINUE
    IF(F(II,JJ)321,345,345
345 DO349I=1,N
    DO349J=1,M
    IF(X(I,J)-EPS)349,346,349
346 II=I
    JJ=J
    X(II,JJ)=Ø.Ø
    SUM=SUM-1.
    GOTO2Ø8
```

```
 349 CONTINUE
     GOTO2ØØ2
1ØØØ FORMAT(2I2,F10.3)
1ØØ1 FORMAT(5(F10.3,2x))
1ØØ2 FORMAT(2HC( I2,1H, I2,3H) = F10.3/)
1ØØ3 FORMAT(2HX( I2,1H, I2,3H) = F10.3/)
1ØØ4 FORMAT(//13HTOTAL COSTS = F10.3//)
1ØØ5 FORMAT(2ØX,22HTHE LEAST COST METHOD./21X,2ØH*************
     *******/)
1ØØ6 FORMAT(//12HSUM OF X(I)=F10.3,4X,8H(M+N-1)=F10.3//)
1ØØ7 FORMAT(/32HFINI SHED WITH SUCCESS..GOOD LUCK)
1ØØ8 FORMAT(3ØX,6HX(I,J)/29X,8H********)
1ØØ9 FORMAT(/35HF INISHED WITHOUT SUCCESS. HARD LUCH)
1Ø1Ø FORMAT(//2ØX,28HTHE REAL SHADOW COST TABLE./19X,3ØH*********
    X********************/5(F10.3,2X)/)
1Ø11 FORMAT(3ØX,4HU(I)/34X,6H******/5(F10.3,2X)/)
1Ø12 FORMAT(//3ØX,4HV(J)/34X,6H******/5(F10.3,2X)/)
1Ø13 FORMAT(//2ØX,23HTHE SHADOW COSTS TABLE./19X,25H*************
    X***********/5(F10.3,2X)/)
     END
```

2.5. <u>Example</u>:

If we apply the same example given in chapter 1, on the computer, the results will be given as follows:-

1. Explains the least-cost method.
2. Total costs according to the first feasible solution.
3. The arrangment of x_{iJ} according to the first feasible solution.
4. Is the number of basic variables according to the first feasible solution $\gtrless$ M+N-1.??
   here in our example = M+N-1
5. This stage observes any critical arrangment of basic variables, if there are, the problem will be stopped, but if there are not, the basic variables will be written as an assurance that there are no critical arrangments.
6. (a,b) explains the way to get the shadow-costs U_i, V_J.
7. Explains the way to get the shadow Costs table.
   here, there are negative costs, therefore, we will continue.

9,10. Deals with the second feasible solution, and the total costs.

11. Indicates that the basic variables of the second feasible solution is not equal M+N-1.
12. Explains the "<u>Degeneracy problem</u>", and how ϵ is placed in order to complete M+N-1.

After this, the same steps will be repeated many times until all the elements in the R-S-Costs table are zero or positive like in (22).

<u>Remark</u>: It is very important to say that these results took no more than 2 minutes only by the computer, so the reader can expect how powerfull it is.

1. THE LEAST COST METHOD.

C(2, 1) = 2Ø.ØØØØØØØØ
X(2, 1) = 4.ØØØØØØØØ
C(2, 2) = 8Ø.ØØØØØØØØ
X(2, 2) = 3.ØØØØØØØØ
C(3, 2) = 5Ø.ØØØØØØØØ
X(3, 2) = 2.ØØØØØØØØ
C(3, 3) = 4Ø.ØØØØØØØØ
X(3, 3) = 1.ØØØØØØØØ
C(1, 3) = 12Ø.ØØØØØØØØ
X(1, 3) = 2.ØØØØØØØØ

TOTAL COSTS = 7ØØ.ØØØØØØØØ (2)

3. X(I,J)

Ø.ØØØØØØØØ	Ø.ØØØØØØØØ	2.ØØØØØØØØ	4.ØØØØØØØØ	3.ØØØØØØØØ
Ø.ØØØØØØØØ	Ø.ØØØØØØØØ	2.ØØØØØØØØ	1.ØØØØØØØØ	

SUM OF X(I) = 5.ØØØØØØØ (M+N-1) = 5.ØØØØØØØØ (4)

5. X(I,J)

Ø.ØØØØØØØØ	Ø.ØØØØØØØØ	2.ØØØØØØØØ	4.ØØØØØØØØ	3.ØØØØØØØØ
Ø.ØØØØØØØØ	Ø.ØØØØØØØØ	2.ØØØØØØØØ	1.ØØØØØØØØ	

6.a U(I)

7Ø.ØØØØØØØØ 2Ø.ØØØØØØØØ -1Ø.ØØØØØØØ

6.b V(J)

Ø.ØØØØØØØØ 6Ø.ØØØØØØØØ 5Ø.ØØØØØØØØ

7. THE SHADOW TABLE.

7Ø.ØØØØØØØØ	13Ø.ØØØØØØØØ	12Ø.ØØØØØØØØ	2Ø.ØØØØØØØØ	8Ø.ØØØØØØØØ
7Ø.ØØØØØØØØ	-1Ø.ØØØØØØØØ	5Ø.ØØØØØØØØ	4Ø.ØØØØØØØØ	

8. THE REAL SHADOW COSTS TABLE.

-2Ø.ØØØØØØØØ	-3Ø.ØØØØØØØØ	Ø.ØØØØØØØØ	Ø.ØØØØØØØØ	Ø.ØØØØØØØØ
8Ø.ØØØØØØØØ	11Ø.ØØØØØØØØ	Ø.ØØØØØØØØ	Ø.ØØØØØØØØ	

TOTAL COSTS = 64Ø.ØØØØØØØØ (9)

10. X(I,J)

Ø.ØØØØØØØØ	2.ØØØØØØØØ	Ø.ØØØØØØØØ	4.ØØØØØØØØ	3.ØØØØØØØØ
Ø.ØØØØØØØØ	Ø.ØØØØØØØØ	Ø.ØØØØØØØØ	3.ØØØØØØØØ	

SUM OF X(I) = 4.ØØØØØØØØ (M+N-1) = 5.ØØØØØØØØ (11)

12. X(I,J)

Ø.ØØØØØØØØ	2.ØØØØØØØØ	Ø.ØØØØØØØØ	4.ØØØØØØØØ	3.ØØØØØØØØ
.ØØØØ1ØØ	Ø.ØØØØØØØØ	Ø.ØØØØØØØØ	3.ØØØØØØØØ	

U(I)
****

40.00000000 20.00000000 -90.00000000

V(J)
****

0.00000000 60.00000000 130.00000000

THE SHADOW COSTS TABLE.
****

40.00000000	100.00000000	170.00000000	20.00000000	80.00000000
150.00000000	-90.00000000	-30.00000000	40.00000000	

THE REAL SHADOW COSTS TABLE
****

10.00000000	0.00000000	-50.00000000	0.00000000	0.00000000
0.00000000	190.00000000	80.00000000	0.00000000	

TOTAL COSTS = 640.00010000

X(I,J)
****

0.00000000	1.99999900	.00000100	4.00000000	3.00000100
0.00000000	0.00000000	0.00000000	3.00000000	

SUM OF X(I) = 5.ØØØØØØØØ (M+N-1) = 5.ØØØØØØØØ

X(I,J)

Ø.ØØØØØØØØ 1.999999ØØ .ØØØØØ1ØØ 4.ØØØØØØØØ 3.ØØØØØ1ØØ
[illegible].ØØØØØØØØ Ø.ØØØØØØØØ Ø.ØØØØØØØØ 3.ØØØØØØØØ

U(I)

1Ø.ØØØØØØØØ 2Ø.ØØØØØØØØ -4Ø.ØØØØØØØØ

V(J)

Ø.ØØØØØØØØ 6Ø.ØØØØØØØØ 8Ø.ØØØØØØØØ

THE SHADOW COSTS TABLE.

4Ø.ØØØØØØØØ 1ØØ.ØØØØØØØØ 12Ø.ØØØØØØØØ 2Ø.ØØØØØØØØ 8Ø.ØØØØØØØØ
1ØØ.ØØØØØØØØ -4Ø.ØØØØØØØØ 2Ø.ØØØØØØØØ 4Ø.ØØØØØØØØ

THE REAL SHADOW COSTS TABLE

1Ø.ØØØØØØØØ Ø.ØØØØØØØØ Ø.ØØØØØØØØ Ø.ØØØØØØØØ Ø.ØØØØØØØØ
5Ø.ØØØØØØØØ 14Ø.ØØØØØØØØ 3Ø.ØØØØØØØØ Ø.ØØØØØØØØ

TOTAL COSTS = 64Ø.ØØØ1ØØØØ

FINISHED WITH SUCCESS..GOOD LUCK.

CHAPTER 3

IRON -ORE PROBLEM.

3.1. Introduction:-

In this chapter, we are going to apply the classical transportation methods, in solving the problem of "Iron-Ore" industry. This problem deals with the following:

In "EL WAHAT EL BAHAREIA", it was discovered by our Egyptien-Engineers that this place contains iron-ore with great quantities. They discovered many mines:-

NASER, ORABY, EL-GEDIDA, and EL HARA mines.
They are distributed in different spots, separated by distances varying between 1-20 Kilometers.

Now; the iron-ore produced have a large size, makes it impossible to be placed into the furnce. So "a Crushing and Loading-Centres" must be established. These Centres must have a capacity able to store the production of all these mines per day.

Our problem now;

1. How many Centres must be put into existance, and where will be their location??

2. Determination of routes between mines and these Centres which give minimum costs.

3. Determination of costs of shipment per unit, through each of these routes.

4. Determination of the number of units shipped through each of these routes.

The problem now, becomes very complicated, because in determining the routes, we must put into consideration the topography of the site such as mountains, valleys, quider sand, and the obstacles, we are going to face.

Secondly, we must know the production of every mine perday on the average, in order to decide how many centres must be installate, This leads us to know the set-up costs, running costs, and service life of centres and routes per day.

So our problem is a "Decision problem" first, and a "transportation problem" secondly. So, we will deal first in the following what is meant by decision, and then we will try to solve our transportation problem numerically.

3.2. Types of Decision:-

There are many types of decision, each depending upon the amount of information we are able to obtain.

According to this amount, they are classified into:

1. decision under certainty.
2. decision under risk.
3. decisions under uncertainty.

The first type involve situations in which the decision maker has complete information about exactly what would happen if he chooses any given course of action.

The other types involve a troublesome shortage of information, as we can not know the future.

These last two types are not important for our case, because we have approximately all the information required, and we will attempt to make a sure decision. Therefore, our problem is decision under certainty in which there are a certain number of alternatives with all the costs and service life of each alternative known in advance.

In the following, we shall not be able to present all the tools for treating decisions under certainty. Instead, we shall present parts of the important tool of Engineering Economics only which will assist us in solving our problem.

3.3. Engineering Economic Concepts:-

In the following, we will use these symbols;

i = the interest rate per interest period.

n = the number of interest periods (service life).

P = present sum of money that can be borrowed with a promise to make a specified series of furture payments.

R = an end-of-period payment in a uniform series continuing for n interest periods, the entire series of such payments being equivelent to P at interest rate i.

S = a furture sum of money, which after n interest periods, and at an interest rate i equivelent to P.

Now:

if P is invested at interest i

at $n = 1$ $\quad S = P + iP = P(1+i)$

$n = 2$ $\quad S = P(1+i)+i[p(1+i)] = p(1+i)^2$

$n = 3$ $\quad S = P(1+i)^3$

$\dots n$ $\quad S = P(1+i)^n \qquad (1)$

Therefore

if R is invested at the end of each year of n-years, the amount of S, at the end of n years will be the sum of the compound amounts of the indivdual investments.

Each earn of R pounds will earn interest for (n-1) years.

at the end of the first year

$$S = R$$

at the end of the second year

$$S = R + R(1+i)$$

is at the end of the nth years

$$S = R(i+(1+i) + (1+i)^2 + \dots\dots + (1+i)^{n-1})$$

multiply both sides of equation by (1+i)

$$(1+i)\ S = R\left[(1+i) + (1+i)^2 + (1+i)^3 + \ldots + (1+i)^n\right]$$

Subtracting the original equation from the second equation:

$$iS = R\left[(1+i)^n - 1\right]$$

Then $R = S\left[\frac{i}{(1+i)^n-1}\right]$ (2)

Substituting the values of S from equation (1) into equation(2)

$$R = S\left[\frac{i}{(1+i)^n-1}\right] = P(1+i)^n\left[\frac{i}{(1+i)^n-1}\right]$$

$$R = P\left[\frac{i(1+i)^n}{(1+i)^n-1}\right] \qquad (3)$$

The expression $\left[\frac{i(1+i)^n}{(1+i)^n-1}\right]$ is called "Capital recoveryfacotr" abbreviated (USCRF) i.e. Uniform Series Capital Recovery factor. The (USCRF) when multyplied by a present debt P, gives a uniform annual end-of-year payment R necessary to repay the debt.

By use of this interest formula, and the compound interest factor derived from it, it is possible to convert any given single amount of money, or series of amounts to amounts or series of amounts at any desired future time that are equivalent at a specified rate of compound interest. These amounts may be investments, yearly expenses of various kinds, or any other sums of money that are involved.

3.4. Mathematical Model of the Problem:-

In order to simplify our model, we will deal first with the information required, so assume the following:-

Set up costs of routes = C_1

Service life of routes = n_1

Set up costs of centres= C_2

Service life of centres= n_2

Set up costs of cars used in this project = number of cars X cet up cost of each car = c_3

Service life of cars = n_3

Running Costs of routes: (maintenance)/day = $\grave{C}_1$

Running Costs of Contres: (laboure's salarly+maintenance + Inventory cost)/ day = $\grave{C}_2$

Running Costs of Cars: (maintenance+fuel+Salary of deriver)/ day = $\grave{C}_3$

Production of Naser-mine/Year = a_1 Ton/Year.

Production of Oraby-mine/Year = a_2 Ton/Year.

Production of El-Gedida-mine/Year =a_3 Ton/Year.

Capacity of each centre = b Ton/day.

Distance between (Naser, Oraby) mines is = d_1 Kilometers

Distance between (Naser, Elgedida) mines is=d_2 Kilometers

Distance between (Oraby, Elgedida)mines is =d_3 Kilometers

Now our Model will be as follows:

First: for routes:-

P = present sum of money = C_1

i = 6% or 7%

n = number of interest periods = n_1

$$R_1 = P\left[\frac{i(1+i)^n}{(1+i)^n-1}\right] = C_1 \left[USCRF\right]^{i=6\%}_{n_1} \quad /Year$$

$$R_1' = R_1/365 \quad /day$$

$\therefore$ total costs of routes = $C_1' + R_1'$ (1)

Secondly; for centres:-

P = present sum of money = C_2

i = 6% or 7%

n = number of interest periods = n_2

$$\text{In } R_2 = P\left[\frac{i(1+i)^n}{(1+i)^n-1}\right] = C_2 \left[USCRF\right]^{i=6\%}_{n_2} \quad / Year$$

$$R_2' = R_2/365 \quad /day$$

total casts of centres = $C_2' + R_2'$ (2)

Thirdly; for cars:-

P = present sum of money = C_3

i = 6% or 7%

n = number of interest periods = n_3

$$R_3 = P\left[\frac{i(1+i)^n}{(1+i)^n-1}\right] = C_3 \left[USCRF\right]^{i=6\%}_{n_3} \quad / Year$$

$$R_3' = R_3/365 \quad /day$$

total costs of cars = $C_3' + R_3'$

Now; if we add (1,2,3), this will gives us the total costs/ day in order to establish (routes, centres).

we will do the same operations, but instead of routes, it will be railway lines.

Then, we will compare the total costs, the less of the two will be chosen, this will be our decision.

Now, our problem became a general transportation problem, in which we have N-mines, M-crushing and Loading centres.

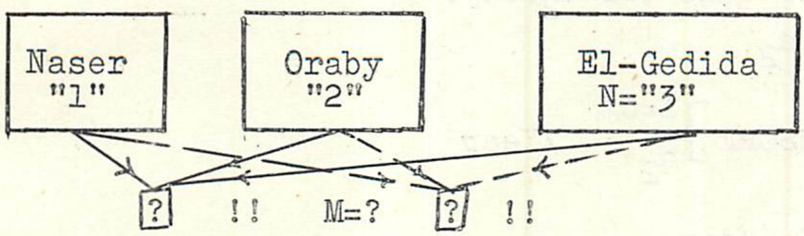


Fig 3-4-1. Net-work representation of iron ore problem.

Note:- The reader may notice that I neglect the fourth well (El-Hara), and this is because our information about its production is unknown to now.

Now; how to get the costs of shipping a unit along each of these routes, this will be deduced as follow's:-

First, we will consider that 1 unit = the charge of 1 car.

assume that:-

cet up cost of route 1 = d_1 / period.
running costs of route 1 = d_1'/day
service life = n_1

$$R_1 = d_1 \left[USCRF \right]_{n_1}^{i=6\%} \quad /Year$$

$$R_1' = R_1/365 \quad / day$$

total costs of this route = $R_1' + d_1'$. /day

now for 1 car only:-

cet up cost of 1 car = d_2 / period
running costs of this car=d_2'/day.
service life = n_2

$$R_2 = d_2 \left[USCRF \right]_{n_2}^{i=6\%} \quad /Year$$

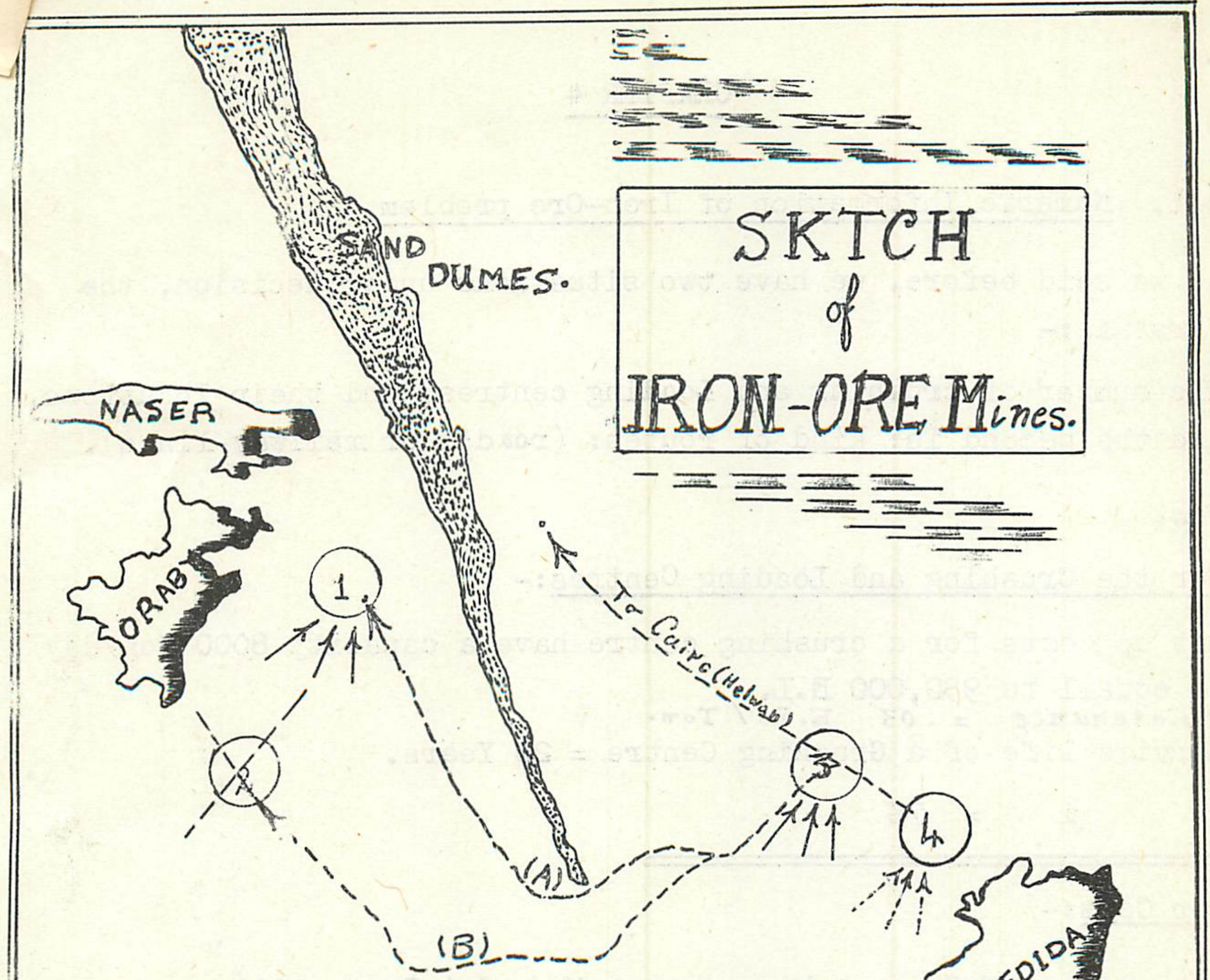
$$R_2' = R_2/365 \quad /day$$

total costs of this car = $R_2' + d_2'$ /day/unit.

for the crushing centre 1, the same as before $\therefore$ total costs $= R_3' + C_3'$. / day.

the cost of shipping one unit/day = $C_{11} = (R_1' + d_1') + (R_2' + d_2') + (R_3' + C_3')$

All the costs of shipping 1 unit/unit time (day) can be deduced by the same method.


SKTCH
of
IRON-ORE Mines.
SAND
DUMES.
NASER
ORAB
1.
2
3
4
To Cairo (Helwan)
(A)
(B)
EL GEDIDA
EL HARA.

CHAPTER 4

4.1. Numeric Information of Iron-Ore problem

As we said before, we have two situations under decision, the first is:-

The number of crushing and Loading centres, and their locations, and the second is: kind of routes: (road's or railway-lines).

Now;

for the Crushing and Loading Centres:-

Cet up costs for a crushing centre have a capacity 8000 Ton/day is equall to 950,000 E.L.
Maintenance = .03 E.L. / Ton.
Service life of a Grushing Centre = 20 Years.

i = 7%

---

for Cars:-

Cet up costs of a car have a capacity of 1-Ton = 1000 E.L. We will consider the unit on the average = 30 Ton. For every car, 3 runs daily (go and back = 1 run) with velocity 40 km/hour.

Maintenance = .05 E.L. / Ton.

---

for roads:-

Cet up costs of roads = 6000 E.L. /km.

Service life = 20 Years

Maintenance = .48 E.L. / Km / Year.

<u>for railway lines :-</u>

Cet up costs of railway lines = 25,000 E.L. /Km.

Service life = 30 Years.

Maintenance = .48 E.L./Km./Year.

The costs of shipping 1-Ton by railway with sufficient loading equall to 4 million Ton = .80 E.L. /Ton.

Distance between Naser, and Ghoraby mines = approximately one Kilometre.

Distance between Naser, and El-Gedida mines = 14 Km.
Distance between Oraby, and El-Gedida mines = 14 Km.

Production of Naser-mine = 1. million Ton/Year.
Production of Oraby-mine = 1. million Ton/Year.
Production of El-Gedida-mine = 2. million Ton/Year.

These are the Information, I got thankly from Eng. Fikri, what will be our decision??

This is what we will do in the next pages.

So, to the following pages .... !!

4.2. Final Soltuion of Iron-Ore Problem.

1. For the Crushing and Loading Centres:-

Since, the production of the 3-mines = 4×10^6 Ton/Year.

total production/day on the average = $4 \times 10^6 / 365$

= 10,960 Ton / day

We will assume that the production is = 12,000 Ton/day in order to avoid the probability of the increase in production daily. So, our decision will be as follows:

The number of crushing-centres = 2

Each have capacity of 6000 Ton / Day

A bout their Locations:-

The first will be in the situation (4) on the Sketch of Iron-Ore mines.

the second will be either in situation (1) or (2), and this decision will depends upon our decision of routes/

The reader may ask me, why did you choose these specially (1 or 2) and 4 situations or locations..!!

The answer will be; in order to avoid the impossibility of shipping the production of the mines for large distances (because, in this case, the iron-ore will be in great size).

So, the reader now can understand why my choice for these situations was specially nearer to mines.

Now,

the cet-up costs for each one of the crushing centres =6000 x 950,000 / 8000 = 712,500 E.L.

$$R = P \ (USCRF)^{i=7\%}_{n=20}$$

$$= 712,500 \times (.094515) = 67,342 \text{ E.L /Year.}$$

$$R' = 67342 / 365 = 184.5 \text{ E.L. / day.}$$

Maintenance / 6000 Ton ::(C') = 6000 x .03 = 180 E.L/day

Total costs = R̀ + C̀ = 184.5 + 180 = 364.5 E.L/day

2. <u>For routes:-</u>

Of course, the shipment from mines to centres will be with cars because of the very small distance between them.

If we look for a moment in the Sketch, we will observe two routes have been determined A,B; the problem now will be;

Which one of these two, will be chosen ??

And this one, will be railway line or road. ?? This will depend upon the daily costs of each.

So, to the following computations ..

<u>First for the route (A) :- (1→4)</u> :-

The length = 13.25 K.m.

<u>Second for the route B:- (2 ——→ 4)</u> :-

The length = 14.25 K.m

So, the route (A) have many advantages:-

a. nearer to the mines Naser, Oraby.

b. its length is smaller than B.

So; the route will be (A), which combines the centres 1,4.

For Railway line:-

Since,

The cet-up costs for 1 Km = 25,000 E.L.

the cet up costs for A = 13.25 x 25,000 = 331,250 E.L.

Service life of this route = 30 Years.

after 30 years, the whole route will be changed.

So, after the 30th Year, (331,250 E.L) will be paid, .. etc..
My computations will be made for (60) years; why ... ??
This will described later.
So, we will consider that the other amount of money which will be paid after 30 years is S_2 (future sum of money, after 30 Years), this S_2 must be converted to P_2 (present Sum of money) at the beginning of the first year. P_2 will be added to P_1 (= 331, 250 E.L) which will be paid at the first year, to get total P, then we will get R_1 for (60) Years.

$$S_2 = P_2 (1 + i)^n$$

$$P_2 = S_2 \left(\frac{1}{(1+i)^n} \right)_{i=7\%}^{n=30}$$

$$P_2 = 331,250 \text{ (SPPWF)} = 331,250 \text{ (13675)}$$

$$= 45298.4375 \text{ E.L.}$$

$$P = P_1 + P_2$$

$$= 331,250 + 45298.4375 = 45629.6875$$

$$= 45630 \text{(E.L) approximately}$$

$$R_1 = P \text{ (U.SCRF)}_{i=7\%}^{n=60}$$

$$= 45630 \text{ (.07225)} = 3297.49 = 3300 \text{ (E.L) approximatel}$$

$$\grave{R}_1 = 3300 / 365 = 9 \text{ E.L./day approximately.}$$

Remark:-

Since the maintenance for roads and railways is similar, So, we will neglect it in our computations.

For roads (A sphalt):-

Since,

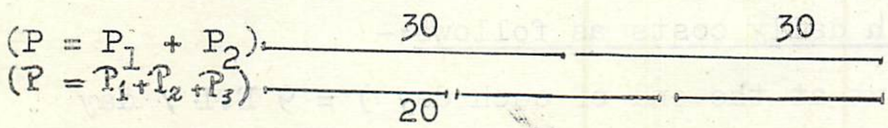
the cet-up costs for 1 km = 6,000 E.L.

the cet-up costs for A = 13.23 x 6,000 = 79,500 E.L.

Service life of this route = 20 years.

after every 20 years, the whole route will be changed So, after the 20th Year, (79,500 E.L) will be paid, and after the 40 th year, (79,500 E.L) also will be paid, Now, my computations also will be made for (60) years. why ?? ... I can tell you now .... !!

In order to get good decisions, the period must be similar in both situations under decision, So, I choose (60) because 30 x 2 = 60 , 20 x 3 = 60 also ... !! which is shown in the following diagramm:-



Now to the computations:-

$$P_2 = S_2 \left(\frac{1}{(1+i)^n} \right)^{n=20} = 79{,}500(.26315) = 20920.425 \quad E.L$$

$$P_3 = S_3 \left(\frac{1}{(1+i)^n} \right)^{n=40} = 79{,}500(.0716) = 5692.2 \quad E.L$$

$$P = P_1 + P_2 + P_3$$

$$= 79.500 + 20920.423 + 5692.2 = 106412.625 \quad E.L$$

$$R_2 = 106412.625 \, (USCRF)^{n=60}_{i=7\%}$$

R_2= 106412.625 (.07225) = 7688 E.L appro.,

R_2'= 7688 / 365 = 21 E.L / day appro.,

Comparing R_1' , R_2' , obviousely :-

R_1' is smaller than R_2'

∴ our decision will be:

Establishing Railway line of length 13.25 between centres (1;4) , with daily costs as follows:-

R_1' (investment at the end of each day) = 9 E.L / day

Costs of shipping 1 Ton = .80 E.L / Ton

total costs or daily costs of shipping all the quantity produced = 12,000 x .08 = 9600 E.L./day

Maintenance/Year = 13.25 x .48 = 6.36 E.L/Year.

Maintenance/day = 6.36/365 = .018 E.L/Year.

Remark:-

There are many decisions about the location of Crushing Centres, and for example, I think that we will get better results if we replace (3) instead of (4) for many reasons, some of these are the following:-

a. I think that it is easier for the train to go directly to Helwan passing through C.C.3 (not to 4, and then back again for the same distance to 3, and then continuing to Helwan). So, we will save the costs of shipping from 3 to 4 and vise-versa.

b. Also, we will save in establishing railway lines about 2 K.m.'s from 3 to 4.

So, the reader can expect how this decision is very important. The same procedure will be performed but instead of length 13.25, the new length will be = 11.25 K.m.

"Final Results"

We have 2 decisions:-

1. Railway line between C.C's (1,4) have a length = 13.25 K.m.
2. " " " C.C's (1,3) " " " = 11.25 K.m.

I think that the 2nd is the best one.

This is what I think, but of course, the final decision will be left to manager's and engineers of Iron-Ore Industry.

Now, we are in face of Transportation problem deal with the following: we have 3 sources (mines) (S_1, S_2, S_3) :

S_1 (Naser) = 3000 Ton/day approximately,

S_2 (Oraby) = 3000 Ton/day " , and

S_3 (El-Gedida)=6000 Ton/day " ,

and we have 2 crushing an Loading Centres (C_1 , C_2) :

C_1 (Crushing centre 1) = 6000 Ton/day, and

C_2 (Crushing centre 3 or 4) = 6000 Ton/day.

In order to solve the classical transportation problem, costs of shipping one unit through road must be deduced, and this is a huge costs-problem, because it needs many information about:-

(Labourer -costs/day + No. of cars, Salary paid for derivers, fuel, ... etc.) and these are not available now, so I must stop here, leaving to workers in Iron-Ore Industry to continue solution when the data are availabe.(They must follow the same steps of solution of a classical transportation problem as in chapter 1).

-:-.-.-.-.-.-.-.-.-.-.-

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