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SIZE OF FOREIGN LOAN, ANNUAL REPAYMENT, AND EXCHANGE RATE IN PROGRAMS OF ECONOMIC DEVELOPMENT.

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$$(4) \qquad Y_t^* = L_0 e^{\alpha\alpha\lambda t} \left\{ \frac{S_0 L_0^{\alpha} (1-\beta) e^{(s+\alpha\lambda)^t}}{s+\alpha\lambda} + K_0^{*\,1-\beta} - \frac{S_0 L_0^{\alpha}(1-\beta)}{s+\lambda\alpha} \right\}^{\frac{\beta}{1-\beta}}$$

Now let us suppose that it is required that national income is raised to h times the initial level in t years so that,

$$(5) \qquad Y_t^* = h\, Y_0 = h\, L_0^{\alpha} K_0^{\beta}$$

From (4) and (5) we get

$$(6) \qquad h\, K_0^{\beta} = e^{\alpha\lambda t} \left\{ \frac{S_0 L_0^{\alpha}(1-\beta) e^{(s+\alpha\lambda)^t}}{s+\alpha\lambda} + K_0^{*\,1-\beta} - \frac{S_0 L_0^{\alpha}(1-\beta)}{s+\alpha\lambda} \right\}^{\frac{\beta}{1-\beta}}$$

Given the initial values of the variables and the values of the parameters, (6) can be solved for K_0^* and as $K_0^* = K_0 + K_m$ the value of foreign loan required to raise the income to the desired level by the t_{th} period can be obtained. (6) can be written also as

$$(7) \qquad K_m = -K_0 + \left[\frac{S_0 L_0^{\alpha}(1-\beta)}{s+\alpha\lambda} - \frac{S_0 L_0^{\alpha}(1-\beta) e^{(s+\alpha\lambda)^t}}{s+\alpha\lambda} + (e^{-\alpha\lambda t}\;\; h\, K_0^{\beta})^{\frac{1-\beta}{\beta}} \right]^{\frac{1}{1-\beta}}$$

(7) expresses the required amount of foreign loan explicitely in terms of the initial values of labour and capital and rate of savings and the indices of labour and capital in the production function and those representing the rate of growth of labour and savings. To obtain some numerical values for K_m, we assume the following

$\alpha = 1-\beta = 75 \qquad L_0 = 1,\ K_0/Y_0 = 2.5 \qquad S_0 = .10,\ .15,\ .20$

$\lambda = .01,\ .02,\ .03, \qquad t = 10,\ 20,\ 30,$ and $\quad h = 2.$

Table I gives the values of the size of foreign capital required for the various values of the parameters when the initial value of capital $K_0 = 3.393$ given by $L_0 = 1,\ K_0/Y_0 = 2.5,\ L_0^{\alpha} K_0^{\beta} = Y_0.$

Table I shows that the size of foreign capital required for doubling the national income in t years decreases as the value of t and the value of initial rate of savings S_0, and the rate of growth of labour λ increases. If the initial

Table I

S_0	h	2		
	λ \ t	10	20	30
.10	.01	34.2065	21.2497	11.1486
	.02	23.8770	8.4553	-.6980
	.03	16.1983	1.2970	-4.2052
.15	.01	32.8954	24.4845	7.7086
	.02	22.6206	6.2318	-2.9294
	.03	14.9957	-.5546	-6.9761
.20	.01	30.5938	16.3241	4.5183
	.02	21.3773	4.1298	-4.3662
	.03	13.8124	-2.1368	-10.4999

rate of savings is 15% which increases at the rate of half a per cent per annum and the rate of growth of population is 2% per annum, then in order to double the national income in 20 years the country needs foreign capital equal to 6.2318 when the initial value of capital is 3.393 and the capital output ratio is equal to 2.5. Obviously the country cannot invest all the foreign capital thus required in one year. This will depend on the absorption capacity of the country. The total investment of foreign capital may be spread over 5 to 10 years according to the availability of the technical and managerial skill in the country and the state of the market conditions etc., etc. The table gives a rough idea of the size of foreign capital needed to double the national income of the country in 10 to 30 years.

III

We have roughly estimated the size of foreign capital needed to double the national income in a certain number of years. Of greater interest, however, is the study of the size of foreign capital needed in order to double the per capita national income in a certain number of years. In most of the developing countries, the program of development is couched essentially in terms of the per capita rise in the national income. In this case equation (5) will be replaced by

$$(5') \qquad Y_t^{*} L_t^{-1} = h\, Y_0\, L_0^{-1} = h\, K_0^{\beta} L_0^{\alpha-1}$$

and equation (7) will be replaced by

$$(7') \qquad K_m = -K_0 + \left[\frac{S_0 L_0^{\alpha}(1-\beta)}{s+\alpha\lambda} - \frac{S_0 L_0^{\alpha}(1-\beta)e^{(s+\alpha\lambda)t}}{s+\alpha\lambda} + \left(e^{\beta\lambda t} h\, K_0^{\beta}\right)^{\frac{1-\beta}{\beta}} \right]^{\frac{1}{1-\beta}}$$

It can be seen that the only change that takes place between (7) and (7') is that in the third term within the bracket on the right side α is replaced by $-\beta$.

Table II gives the values of foreign capital required to double the per capital income for the same initial values and values of the parameters as in Table I.

Table II

S_0	h / λ \ t	2: 10	20	30
.10	.01	53.6449	56.4611	59.3595
	.02	59.7207	70.4253	82.8150
	.03	66.4486	87.2347	114.6622
.15	.01	52.1869	53.3234	54.2911
	.02	58.1655	66.8414	76.6093
	.03	64.7954	83.1536	107.0131
.20	.01	50.5710	50.2286	46.7698
	.02	56.6202	63.3053	70.5218
	.03	63.0449	79.1101	99.5210

Table II shows that the value of foreign capital required to double the per capital national income is lower, the higher the initial rate of savings. It is higher the higher the rate of growth of population. It increases as the number of years by which the per capita income is desired to be doubled increases, but not always. When $S_0 = .20$, $\lambda = .01$ foreign capital needed to double the per capita income decreases as the number of years increases. It implies that in this case capital accumulation at a higher rate than the rate at which the population increases. From equation (7'), it is possible to find out the corresponding values of S_0, λ and t, such that K_m decreases as t increases.

Differentiating K_m w.r.t. t we get

$$(8) \qquad \dot{K}_m = \frac{1}{1-\beta}\left[\quad z \quad\right]^{\frac{\beta}{1-\beta}} \cdot \left[- S_0 L_0^{\alpha}(1-\beta)e^{(s+\alpha\lambda)t} + h^{\frac{1-\beta}{\beta}} K_0^{1-\beta} e^{\alpha\lambda t} \cdot \alpha\lambda\right]$$

$$\text{where } z = \left[\frac{S_0 L_0^{\alpha}(1-\beta)}{s+\alpha\lambda} - \frac{S_0 L_0^{\alpha}(1-\beta)e^{(s+\alpha\lambda)t}}{s+\alpha\lambda} + (e^{\beta\lambda t} h\, K_0^{\beta})^{\frac{1-\beta}{\beta}}\right]^{\frac{1}{1-\beta}}$$

From (7') it is clear that for all relevant values of interest, z must be positive. Then $\dot{K}_m$ will be negative when

$$- S_0 L_0^{\alpha}(1-\beta)e^{(s+\alpha\lambda)t} + h^{\frac{1-\beta}{\beta}} K_0^{1-\beta} e^{\alpha\lambda t} \cdot \alpha\lambda < 0$$

or $\quad - S_0\, e^{.005t} + 2^3 \; 3.393^{.75} \cdot \lambda < 0$

putting $L_0 = 1$, $K_0 = 3.393$, $\alpha = .75$, $\alpha + \beta = 1$

or $\quad \frac{S_0}{\lambda} > e^{3.0524} \qquad$ when t = 10

$\frac{S_0}{\lambda} > e^{3.0024} \qquad$ t = 20

$\frac{S_0}{\lambda} > e^{2.9524} \qquad$ t = 30[1)]

Table II lays bare one important fact. It is that for doubling the per capita national income in a country with a rate of population growth of more than 2%, the need for foreign capital is likely to be much higher than that is generally contemplated by the developing economies.

1) For example, for $\frac{S_0}{\lambda} > e^3$, we get $\frac{S_0}{\lambda} > 20$, approximately, which means that if $S_0 = 20$, λ must be less than .01, so that requirement of foreign capital decreases with time for values of t implied in the inequality.

IV

Once the size of the foreign loan has been decided, the question about its repayment arises. The repayment of the loan involves, besides the original sum of the foreign loan, the payment of interest charges and all other costs of servicing the debt. From the point of view of repayment, the service charges on debts stand on a slightly different footing than the interest charges in that not all the former charges are to be met from the earnings of foreign exchange whereas the latter can be only met out of foreign exchange resources. This is because part of the domestic services are rendered by the agencies of the borrowing country. Excluding this part, rest of the service charges will have to be met out of foreign exchange resources. We shall assume that debt charges are in proportion of the loan remaining unpaid, so that we shall include an element of debt services also in the interest payments so as to facilitate calculation below. Or alternatively the expected service charges on debts payable in foreign currency, may be added to the original loan. After the adjustment along one of these lines, we can confine our attention to the annual repayment of the loan in the number of years agreed to by the lending and borrowing countries.

Let us now suppose that after the adjustment mentioned above, the total foreign loan equals K_m to be repaid in t years at a rate of interest 100 r per cent per annum. Let us further suppose that the constant amount of annual payment which will liquidate the loan in t years is equal to E.

Therefore,

$$tE = K_m(1+r)^t - E(1+r)^{t-1} - \ldots\ldots - E(1+r)^{t-2} - E$$

$$= K_m(1+r)^t - E\left(\frac{1-(1+r)^t}{1-(1+r)}\right)$$

$$(9) \quad \therefore \quad \frac{E}{K_m} = \frac{(1+r)^t\, r}{tr+(1+r)^t - 1}$$

(9) gives the value of annual repayment required to liquidate an original loan of K_m at 100 r per cent annual rate of interest repayment in t years. Given the values of t and r, E can be obtained as a ratio of K_m.

Table III gives the values of $\frac{E}{K_m}$ for t = 10, 20 and 30 and r = .02, .04 and .06.

Table III

r \ t	10	20	30
.02	.058	.034	.026
.04	.067	.044	.038
.06	.084	.056	.053

From Table III we can read the annual repayment required to be made, given the values of t and r. For example, if a country borrows a thousand million of some currency at an annual rate of interest including other charges of 4% and the debt to be repaid in 20 equal annual instalments, then the value of repayment will be equal to 44 million.

V

Any payment to a foreign country or to a lending country can be made only through a surplus in the balance of payments. Of course, any payment to a foreign country can be postponed through re-borrowing. But this does not lead to a reduction in the net debt of the developing country; only the form of the debt is changed. The balance of payments can be affected in several ways through tariffs, quotas and licenses, trade agreements, ways subsidies and other export promotion programs, and alterations in the exchange rate. We shall confine our attention to the last item and find out the change which is needed in the exchange rate so as to bring about the required surplus in the balance of payments to meet the annual repayment. Usually the changes of exchange rate have been discussed in the context of bringing about an equilibrium in the balance of payments. The same approach, as far as it goes, can be extended to find the exchange rate that results in the required surplus in the balance of payments. However, the problem has been generally discussed in terms of domestic and foreign demand and supply elasticities with respect to prices. In the present context we need more than developing country due to the rate of increase of its income as compared with that of the lending country or other foreign countries. We should therefore try to find out the required change in the exchange rate taking into account not only the price elasticities, but also the income elasticities. A simple approach[1] to find out the required exchange rate is as follows:

The symbols to be used are:

X = imports in terms of domestic currency
M = exports in terms of domestic currency
p = domestic relative price of exports
p' = foreign relative price of exports
q = domestic relative price of imports
q' = foreign relative price of imports
= exchange rate
d_x = price elasticity of foreign demand for exports
s_x = price elasticity of domestic supply of exports
d_m = price elasticity of domestic demand for imports
s_m = price elasticity of foreign supply of imports

The same elasticities with a prime denote the corresponding income elasticities. Let the domestic supply of exports and the domestic demand for imports be expressed as[2]

$$(10) \qquad X = X_0(p^{s_x} \cdot G^{s'_x}) = X_0((p'\Pi)^{s_x} \cdot G^{s'_x})$$

$$(11) \qquad M = M_0(q^{d_m} \cdot G^{d_m}) = M_0((q'\Pi)^{d_m} \cdot G^{d'_m})$$

where X_0 and M_0 are the initial values of exports and imports respectively, and G is the domestic ratio of growth of national product defined as $\frac{Y+\Delta Y}{Y}$.

Let the foreign demand for imports and the foreign supply of imports be expressed as

1) This is an extension of the approach adopted by W.L. Smith in "Effects of Exchange Rate Adjustments on the Standard of Living", American Economic Review, December 1954, Appendix pp. 823-25.

2) We assume that in the initial period the currency units are defined in such a way that $\Pi = 1$, and we have $p = p' = q = q'$ Cf. W.L. Smith op.cit. p.823-24.

(12) $$X = X_0(p'^{\,d_x} \cdot G'^{\,d'_x})$$

(13) $$M = M_0(q'^{\,s_m} \cdot G'^{\,s'_x})$$

where G' is the ratio of growth of national product in the foreign country. We shall assume that in the initial stage, the country has equilibrium in the balance of payments, so that $X_0 = M_0$. Our aim is to have a surplus balance equal to E, so that

(14) $$E = X - M$$

(15) $$X_0 = M_0$$

and we express E as a ratio of X_0 so that

(16) $$E = \gamma X_0$$

There are five equations from (10) to (14); from these we can eliminate p', q', X, M, to get E in terms of Π, G, G', M_0, X_0 and the various elasticities.

From eqs. (10) and (12) we have

(17) $$X = X_0 \Pi^{\frac{d_x s_x}{d_x - s_x}} \; G'^{\,-\frac{d'_x s_x}{d_x - s_x}} \; G^{\frac{s'_x d_x}{d_x - s_x}}$$

And similarly from (11) and (13) we have

(18) $$M = M_0 \Pi^{\frac{d_m s_m}{s_m - d_m}} \; G'^{\,\frac{s'_m d_m}{s_m - d_m}} \; G^{\frac{d'_m s_m}{s_m - d_m}}$$

Substituting (17) and (18) in (14) and using (15) and (16) we have

(19) $$\Pi^{\frac{d_x s_x}{d_x - s_x}} \; G'^{\,\frac{d'_x s_x}{d_x - s_x}} \; G^{\frac{s'_x d_x}{d_x - s_x}} - \Pi^{\frac{d_m s_m}{d_m - s_m}} \; G'^{\,-\frac{s'_m d_m}{s_m - d_m}} \; G^{\frac{d'_m s_m}{s_m - d_m}} = \gamma$$

Given the values of G, G', γ and the eight elasticities, we can find out the required exchange rate from (19).

VI

The above discussion has proceeded independently of the repercussion on the standard of living of the individuals in the borrowing country. With the annual rates of savings given and fixed, and the rates of annual repayments, to be effected through exchange-rate alterations, being obligatory per lending or borrowing agreements, it is the level of consumption which absorbs all the shocks of the program of economic development through foreign borrowing. The effect on consumption of these programs, however, can easily be gauged.

$$C_t = Y_t - S_t - E.$$

The above equation gives the movement of aggregate consumption over the period during which the repayment continues. Dividing it through the number of workers, we can get the per capita movement of consumption viz.,

$$C_t \, L_t^{-1} = (Y_t - S_t - E)L_t^{-1}.$$

Y_t can be obtained from section II, S_t and L_t is given under assumption and E is obtained from section IV. In most of the underdeveloped countries it is not practicable to suppress consumption below the level which obtains in the beginning of the programs of economic development. No program of borrowing foreign capital will, therefore, be successful if $C_t \, L_t^{-1}$ tends to decrease over time. In many countries, a betterment of the level of standard of life, is urgently required. Hence a program of borrowing foreign capital will be successful only when it allows a steady increase in the per capita consumption level.

Before embarking upon a program of borrowing for economic development, a country is naturally confronted with the willingness of the lending countries to lend and the extent to which they are ready to lend. But besides this, the borrowing country has also to study its absorbing capacity in a period, the foreign loans may remain unutilized if a proper assessment of this capacity is not made before hand and the delay will increase the economic cost of these loans. The absorbing capacity obviously depends upon the availability of managerial, technical and entrepreneurial skills, and that of the cooperant factors and also on the size of the market in some cases. But of crucial importance are the terms of the loans. If they are such as to lead to a reduction in the per capita consumption, over a considerable part of the period of the repayment of the loan it may turn out that these loans do more harm than good.

(A.H.)