# UNITED ARAB REPUBLIC

## THE INSTITUTE OF NATIONAL PLANNING

معهد التخطيط القومى

Memo. No.798

# Input - Output - Tables

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Sept. 1967

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# Input - Output - Tables

Introduction

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## Introduction

Interlacement balances or input-output tables are instruments to safe guard a planned and proportional development of the economy. Furthermore they are important and necessary prerequisits of optimation. Their application is possible and necessary not only in the scope of the entire economy but also in partial systems, such as general organisations, enterprises, etc. They can be used to plan and to follow-up production as well as the expenditures of material, labour, and and all other kinds of costs. The main idea of input-output tables is to crasp the interlacements, the links, between branches, or between the production stages of a certain branch or enterprise. This concerns firstly the supply and demand of raw material or semi-finished products, in order to avoid disproportions , unused capacities, overproduction, and other kinds of losses.

The basic problem of input-output tables is to calculate total and final production of a productive system, including the deliveries of other branches, and considering all the interlacements inside the system. The interlacements of the production flow can be represented by graphs, balances or tables, and mathematic models.

The following memo tries to introduce the terminology, the methods, and the instruments of interlacement balancing to the reader in order to enable him to apply these methods in his practical work. The memo is based upon the textbooks used in the GDR.

1. The basic model

In order to explain the fundamental relations which are to be represented by an interlacement models, we assume a simplified example:
A metallurgical plant may consist of four departments:

furnaces
foundry
steel factory and
rolling mill.

The input-output relations of these departments are characterized by the following figures:

a) The furnaces require 240 units of iron ore, 144 units of coke, and 24 of scrap iron in order to produce 160 units of pig-iron.

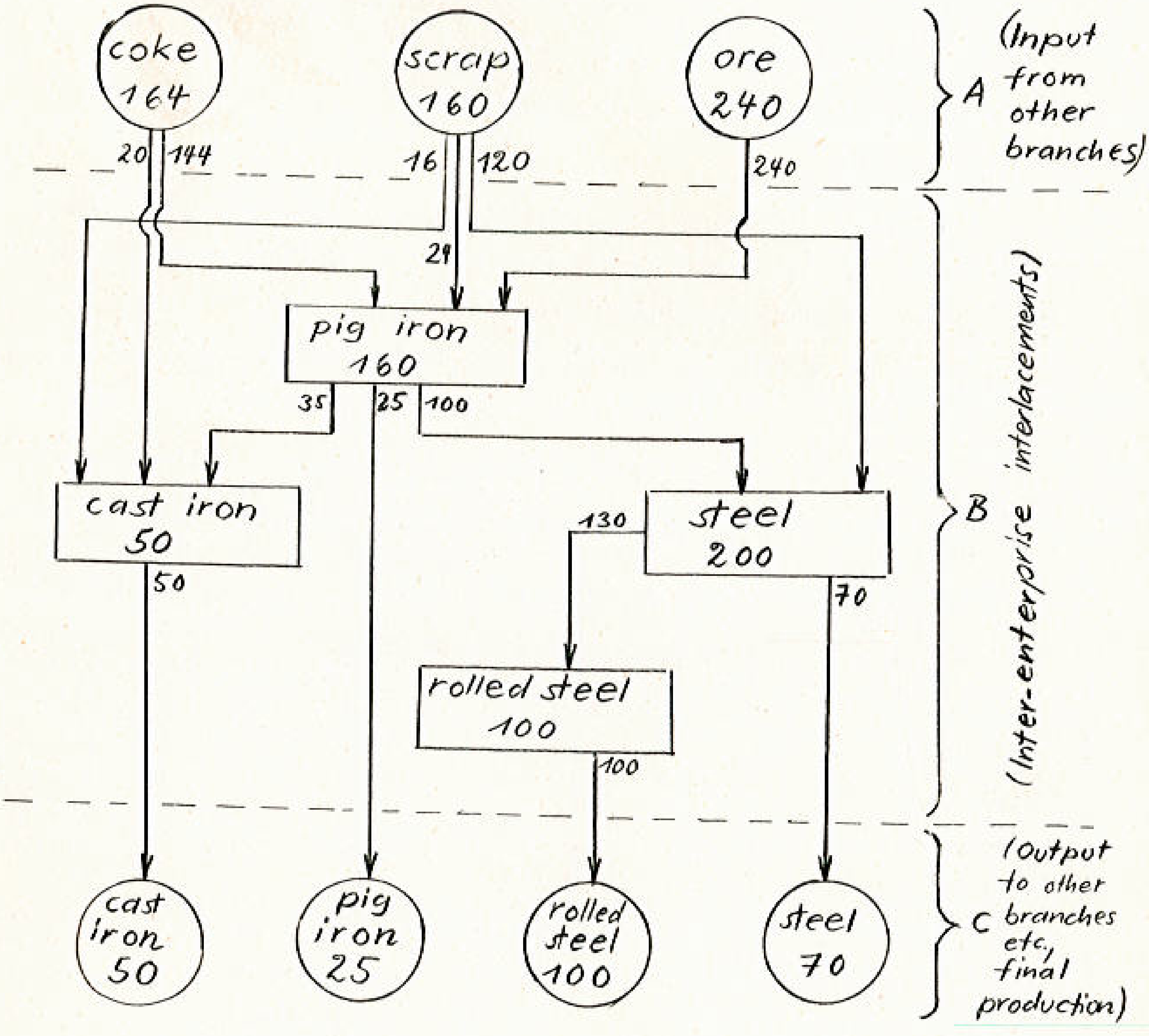
b) The output of the foundry of 50 units cast-iron demands the input of 35 units pig-iron and 16 scrap iron.

c) The steel factory produces 200 units of steel with the aid of 100 pig-iron and 120 scrap-iron units.

d) The rolling mill has a consumption of 130 units steel in order to manufacture 100 units rolled steel.

e) At last the final production of the company amounts to 25 units of pig-iron, 50 cast iron, 70 steel, and 100 rolled steel. This is the market production which the company is able to sell.
The units might be thousand tons.

These relation are represented by the following graph:


coke
164
scrap
160
ore
240
20
144
16
120
240
A
(Input from other branches)
24
pig iron
160
35
25
100
cast iron
50
steel
200
130
50
70
rolled steel
100
100
B
(Inter-enterprise interlacements)
cast iron
50
pig iron
25
rolled steel
100
steel
70
C
(Output to other branches etc, final production)

We are able to recognize three stages:

A: The materials which are to be supplied by other branches,

B: production and self-consumption of the company, the interlacement inside the enterprise;

C: and the final or market production, the output of our enterprise.

On principle similar graphs can be used to represent other kinds of economic partial systems, such as entire branches, the whole economy, or others.

In the following table 1 the same relations are shaped like a balance:

| input of | input for the production of | | | | total input | final production | total production |
|---|---|---|---|---|---|---|---|
| | pig iron | cast iron | steel | rolled steel | | | |
| pig iron | 0 | 35 | 100 | 0 | 135 | 25 | 160 |
| cast iron | 0 | 0 | 0 | 0 | 0 | 50 | 50 |
| steel | 0 | 0 | 0 | 130 | 130 | 70 | 200 |
| rolled steel | 0 | 0 | 0 | 0 | 0 | 100 | 100 |
| coke | 144 | 20 | 0 | 0 | 164 | | |
| scrap | 24 | 16 | 120 | 0 | 160 | | |
| ore | 240 | 0 | 0 | 0 | 240 | | |
| total production | 160 | 50 | 200 | 100 | | | |

In the front column all the products are listed which can be used as raw materials. The heading row contains the products of the enterprise in question. The double-framed quadratic part shows the interlacements in a closer sense,i.e. the internal interlacement inside the enterprise in question. The products, the outputs of the departments of the enterprise are partly consumed by other departments. It is another thing with the three last input items. They are the results of the production process of foreign branches, they are pure inputs which are completely consumed by our factory. In the last three columns the total inputs and outputs are summed up. The total production reduced by the self consumtion of own products gives us the final-or market production, which is ready to be sold to other branches.

It is not necessary to add the total production figures a second time in the last line of our table. We did so only in order to calculate the required coefficients in a more confortable way. For this is our next task: we have to transform the special case shown in table one into a generalized form suitable to every production programme. That requires to substitute the real figures by coefficients. Only the table with the coefficients of the consumed material enables us to plan the further development, to calculate a set of variants. We get the coefficients by dividing the absolute input figures by the corresponding total production figures. That means we calculate the input per unit of production, for example if the production of 200 units steel requires 100 units of pig iron and 120 units of scrap, than the production of one unit steel requires 0.5 units pig iron $\left(\frac{100}{200}\right)$, and 0.6 units of scrap $\left(\frac{120}{200}\right)$. The outcome is the table 2:

| | pig iron | cast iron | steel | rolled steel |
|---|---|---|---|---|
| pig iron | 0 | 0.7 | 0.5 | 0 |
| cast iron | 0 | 0 | 0 | 0 |
| steel | 0 | 0 | 0 | 1.5 |
| rolled steel | 0 | 0 | 0 | 0 |
| coke | 0.9 | 0.4 | 0 | 0 |
| scrap | 0.15 | 0.32 | 0.6 | 0 |
| ore | 1.5 | 0 | 0 | 0 |

Table 2

All the further calculations, all the application of input-output tables is nousense without very exactly determined coefficients. As a rule statistical figures are not sufficient to solve planning problems. We have to determine the coefficients of material consumption for the forthcoming period which is only possible based upon exact plans of technological progress. Therefore we assume that the coefficients registered in table 2 are not expressing the real conditions of the last year but the planned relations of the forth coming plan year. Under this preconditions we can use these figures to calculate the following examples.

a) A numerical example how to solve the first fundamental problem.

The first fundamental problem is the following one: The total production figures are given by central planning institutions as plan figures:

| | | |
|---|---|---|
| pig iron | 200 | units |
| cast iron | 60 | " |
| steel | 220 | " |
| rolled steel | 120 | " |

we have to calculate

a) the final production, and

b) the material consumption, i.e. the required inputs of raw material from other branches.

Multiplying the coefficients by the total production figures and adding the items we get the demand of each material:

| | pig iron | cast iron | steel | rolled steel | total input |
|---|---|---|---|---|---|
| pig iron | 0 x 200 | +0.7 x 60 | +0.5 x 220 | +0 x 120 | = 152 |
| cast iron | 0 x 200 | + 0 x 60 | + 0 x 220 | +0 x 120 | = 0 |
| steel | 0 x 200 | + 0 x 60 | + 0 x 220 | +1.3x120 | = 156 |
| rolled steel | 0 x 200 | + 0 x 60 | + 0 x 220 | +0 x 120 | = 0 |
| coke | 0.9 x 200 | +0.4 x 60 | + 0 x 220 | +0 x 120 | = 204 |
| scrap | 0.15x 200 | +0.32x 60 | +0.6 x 220 | +0 x 120 | = 181.2 |
| ore | 1.5 x 200 | + 0 x 60 | + 0 x 220 | +0 x 120 | = 300 |

Table 3

Shaped as matrices the same table 3 looks like follows:

$$\begin{pmatrix} 0 & 0.7 & 0.5 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.3 \\ 0 & 0 & 0 & 0 \\ & & & \\ 0.9 & 0.4 & 0 & 0 \\ 0.15 & 0.32 & 0.6 & 0 \\ 1.5 & 0 & 0 & 0 \end{pmatrix} \times \begin{pmatrix} 200 \\ 60 \\ 220 \\ 120 \end{pmatrix} = \begin{pmatrix} 152 \\ 0 \\ 156 \\ 0 \\ \\ 204 \\ 181.2 \\ 300 \end{pmatrix}$$

The result vector contains firstly the four figures representing the required materials which are produced in the enterprise in question, the self-consumption, and the second three figures represent the materials which must be delivered from outside the enterprise.

We get the final production by subtracting the self consumption from the total production:

$$\begin{pmatrix} 200 \\ 60 \\ 220 \\ 120 \end{pmatrix} - \begin{pmatrix} 152 \\ 0 \\ 156 \\ 0 \end{pmatrix} = \begin{pmatrix} 48 \\ 60 \\ 64 \\ 120 \end{pmatrix}$$

Under the given conditions the enterprise is able to supply other branches

| | | | |
|---|---|---|---|
| with | 48 | units | pig iron, |
| | 60 | " | cast iron, |
| | 64 | " | steel, and |
| | 120 | " | rolled steel. |

This market production requires

| | | |
|---|---|---|
| 204 | units | coke, |
| 181.2 | " | scrap, and |
| 300 | " | iron are, which must be |

supplied by other branches.

b) A numerical example how to solve the second fundamental problem.

In our practical work we are often confronted with a second problem. Not the total but the final production figures are given :

| | | |
|---|---|---|
| 48 | units | pig iron, |
| 60 | " | cast iron, |
| 64 | " | steel, and |
| 120 | " | rolled steel. |

We have to point out

a) the total production figures, and

b) the required raw material.

The calculation of the material needed to produce the given final production goes in the same way as we know from the first example by multiplying the matrix of the material coefficients by the vector of final production:

$$\begin{pmatrix} 0 & 0.7 & 0.5 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.3 \\ 0 & 0 & 0 & 0 \\ 0.9 & 0.4 & 0 & 0 \\ 0.15 & 0.32 & 0.6 & 0 \\ 1.5 & 0 & 0 & 0 \end{pmatrix} \times \begin{pmatrix} 48 \\ 60 \\ 64 \\ 120 \end{pmatrix} = \begin{pmatrix} 74 \\ 0 \\ 156 \\ 0 \\ 67.2 \\ 64.8 \\ 72 \end{pmatrix}$$

The result means that there are required from outside the branch

| | | |
|---|---|---|
| 67.2 | units | coke, |
| 64.8 | " | scrap, and |
| 72 | " | iron ore. |

Besides these raw material additonal

| | | |
|---|---|---|
| 74 | units | pig iron, and |
| 156 | " | steel must be |

produced in the enterprise. But the production of this amount of pig iron and steel demands additional raw materal,calculated as follows:

$$\begin{pmatrix} 0 & 0.7 & 0.5 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.3 \\ 0 & 0 & 0 & 0 \\ 0.9 & 0.4 & 0 & 0 \\ 0.15 & 0.32 & 0.6 & 0 \\ 1.5 & 0 & 0 & 0 \end{pmatrix} \times \begin{pmatrix} 74 \\ 0 \\ 156 \\ 0 \end{pmatrix} = \begin{pmatrix} 78 \\ 0 \\ 0 \\ 0 \\ 66.6 \\ 104.7 \\ 111 \end{pmatrix}$$

It is obvious, that we need additional 78 units pig iron. So we have to repeat the calculation once again:

$$\begin{pmatrix} 0 & 0.7 & 0.5 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.3 \\ 0 & 0 & 0 & 0 \\ 0.9 & 0.4 & 0 & 0 \\ 0.15 & 0.32 & 0.6 & 0 \\ 1.5 & 0 & 0 & 0 \end{pmatrix} \times \begin{pmatrix} 78 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 70.2 \\ 11.7 \\ 117 \end{pmatrix}$$

This computation shows us, that no additional products of the enterprise in question are required for the purpose of self-consumption in order to produce the demanded final production, i.e. the first four figures of the result vector are equal to zero. At this point we can stop our calculation process and we have only to add the results. By adding the given final production to the calculated internal self consumption we get the total production:

$$\begin{pmatrix} 48 \\ 60 \\ 64 \\ 120 \end{pmatrix} + \begin{pmatrix} 74 \\ 0 \\ 156 \\ 0 \end{pmatrix} + \begin{pmatrix} 78 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 200 \\ 60 \\ 220 \\ 120 \end{pmatrix}}}$$

In the same way we calculate the volume of needed raw material delivered by other branches:

$$\begin{pmatrix} 67.2 \\ 64.8 \\ 72 \end{pmatrix} + \begin{pmatrix} 66.6 \\ 104.7 \\ 111 \end{pmatrix} + \begin{pmatrix} 70.2 \\ 11.7 \\ 117.0 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 204 \\ 181.2 \\ 300 \end{pmatrix}}}$$

In other words , a final production of

| | | |
|---|---|---|
| 48 | units | pig iron, |
| 60 | " | cast iron, |
| 64 | " | steel and |
| 120 | " | rolled steel requires a total pro- |

duction of

| | | |
|---|---|---|
| 200 | units | pig iron, |
| 60 | " | cast iron, |
| 220 | " | steel, and |
| 120 | " | rolled steel, for which reason our |

enterprise has to buy from outside

| | | |
|---|---|---|
| 204 | units | coke, |
| 181.2 | " | scrap, and |
| 300 | " | iron ore. |

Summarizing the results we can state, that the calculation of interlacement balances requires scientific based coefficients expressing the expenditures per unit. The first basic problem is the calculation of the inputs and of the final production under the condition that the total production is given. The second problem is the calculation of the total production and the inputs under the condition that the final production is given.

## 2. The mathematical formulation of the basic model.

On principle the same basic model, the same symbols etc. can be used to compute the input-output relations of an enterprise as well as a branch or the entire economy, i.e., the model is applicable to every partial system of the economy.

Our partial system may consist of n sectors, and we assume at first that each sector (enterprise, department etc.) produces only one article. The sectors and the articles are designated by the figures 1, 2, 3, ..., n, or, any a single article, by i or j. Furthermore we introduce the following symbols:

$X_i$ = total production of article i;

$Y_i$ = final production of article i,

$X_{ij}$ = self-consumption of article i in order to produce article j.

All the figures are summarized to vectors and matrices:

$$\underline{x} = \begin{pmatrix} X_1 \\ X_2 \\ \dots \\ X_n \end{pmatrix} = \begin{pmatrix} 200 \\ 60 \\ 220 \\ 120 \end{pmatrix} \qquad \underline{y} = \begin{pmatrix} Y_1 \\ Y_2 \\ \dots \\ Y_n \end{pmatrix} = \begin{pmatrix} 48 \\ 60 \\ 64 \\ 120 \end{pmatrix}$$

$$\underline{X} = \begin{pmatrix} X_{11} & X_{12} & \dots & X_{1n} \\ X_{21} & X_{22} & \dots & X_{2n} \\ \dots & \dots & \dots & \dots \\ X_{n1} & X_{n2} & \dots & X_{nn} \end{pmatrix} = \begin{pmatrix} 0 & 42 & 110 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 156 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

To make the relations more evident and visible we added the figures out of our numerical examples.

The processing of the articles happens in certain relations which are expressed by the coefficients of material consumption :

$\frac{X_{ij}}{X_j} = a_{ij} =$ consumption of the article i to produce <u>one unit</u> of article j, (self - consumption).

It follows : $X_{ij} = a_{ij} X_j$, in words : The total self consumption of article i to produce article j equals the self consumption per unit multiplyed by the total amount of the production of the article j. There exist always $n^2$ coefficients of internal interlacements, of self consumption, but most of them normally are equal to zero. The interlacement matrix of the coefficients looks like follows:

$$\underline{A} = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix} = \begin{pmatrix} 0 & 0.7 & 0.5 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Besides those products produced and consumed in the enterprise in question other raw materials are needed which must be delivered by other enterprises or branches. Their number may be r (1, 2, ..., r), in general each of them has the designation k. We distinguish between the consumption of those maserials in order to produce a certain article, and the total consumption of k, besides this we formulate the coefficient, i.e. the input of k per unit of an article :

$B_{kj}$ = input of article k to produce article j,
$V_k$ = total input of k,
$\frac{B_{kj}}{X_j} = b_{kj}$ = Input of article k to produce one unit of article j.

Also these figures will be written in the matrix; or vector form :

$$\underline{B}^* = \begin{pmatrix} B_{11} & B_{12} & \dots & B_{1n} \\ \dots & \dots & \dots & \dots \\ B_{r1} & B_{r2} & \dots & B_{rn} \end{pmatrix} = \begin{pmatrix} 180 & 24 & 0 & 0 \\ 30 & 19,2 & 123 & 0 \\ 300 & 0 & 0 & 0 \end{pmatrix}$$

$$\underline{B} = \begin{pmatrix} b_{11} & b_{12} & \dots & b_{1n} \\ \dots & \dots & \dots & \dots \\ b_{r1} & b_{r2} & \dots & b_{rn} \end{pmatrix} = \begin{pmatrix} 0.9 & 0.4 & 0 & 0 \\ 0.15 & 0.32 & 0.6 & 0 \\ 1.5 & 0 & 0 & 0 \end{pmatrix}$$

$$\underline{V} = \begin{pmatrix} V_1 \\ V_2 \\ \dots \\ V_r \end{pmatrix} = \begin{pmatrix} 204 \\ 181.2 \\ 300 \end{pmatrix}$$

a) Mathematical formulation of the first basic problem.

We remember the first fundamental problem was to calculate final production ($\underline{Y}$) and the vector of direct inputs ($\underline{V}$). The matrice of indirect and direct input coefficients ($\underline{A}$ and $\underline{B}$) as well as the vector of total production ($\underline{X}$) are given.

We get the final production by subtracting the entire self consumption from the total production:

$$
\begin{array}{cccccccccc}
X_1 & - & a_{11}X_1 & - & a_{12}X_2 & - & \cdots & - a_{1n}X_n & = & Y_1 \\
X_2 & - & a_{21}X_1 & - & a_{22}X_2 & - & \cdots & - a_{2n}X_n & = & Y_2 \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
X_n & - & a_{n1}X_1 & - & a_{n2}X_2 & - & \cdots & - a_{nn}X_n & = & Y_n
\end{array}
$$

This set of equations can be transformed as follows:

$$
\begin{array}{cccccc}
(1 - a_{11})X_1 & - a_{12}X_2 & - \cdots & - a_{1n}X_n & = & Y_1 \\
- a_{21}X_1 & + (1 - a_{22})X_2 & - \cdots & - a_{2n}X_n & = & Y_2 \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
- a_{n1}X_1 & - a_{n2}X_2 & - \cdots & + (1 - a_{nn})X_n & = & Y_n
\end{array}
$$

Written in the form of matrices:

$$\underline{x} - \underline{A}\underline{x} = \underline{y}, \quad \text{and}$$
$$(\underline{E} - \underline{A})\underline{x} = \underline{y}$$

The internal interlacement of the partial system is now characterized by the matrix:

$$
(\underline{E}-\underline{A}) = \begin{pmatrix} (1-a_{11}) & - a_{12} & \cdots & - a_{1n} \\ -a_{21} & (1- a_{22}) & \cdots & - a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ -a_{n1} & - a_{n2} & \cdots & (1- a_{nn}) \end{pmatrix} = \begin{pmatrix} 1 & -0.7 & -0.5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1.3 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

Each column of the matrix contains the input-output relations of a department of the partial system. For example the last column expresses, that the production (output) of one unit rolled steel requires the input of 1.3 units steel, but no inputs of pig iron and cast iron. The positive elements of the main diagonal stand for the outputs, the negative elements designate the inputs.

Analogous reflexions lead us to the vector of direct inputs from other branches, $\underline{V}$ :

$$\begin{array}{ccccccccc} b_{n}X_1 & + & b_{12}X_2 & + & \cdots & + & b_{1n}X_n & = & V_1 \\ b_{21}X_1 & + & b_{22}X_2 & + & \cdots & + & b_{2n}X_n & = & V_2 \\ \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot \\ b_{n1}X_1 & + & b_{n2}X_2 & + & \cdots & + & b_{nn}X_n & = & V_n \end{array}$$

or, shaped as matrices:

$$\underline{B}\,\underline{x} = \underline{v}$$

It is suitable to form the matrix $(-\underline{B})$, the matrix of direct inputs, so that all input figures are negative :

$$(-\underline{B}) = \begin{pmatrix} -b_{11} & -b_{12} & \cdots & -b_{1n} \\ \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot \\ -b_{rn} & -b_{r2} & \cdots & -b_{rn} \end{pmatrix} = \begin{pmatrix} -0.9 & -0.4 & 0 & 0 \\ -0.15 & -0.32 & -0.6 & 0 \\ -1.5 & 0 & 0 & 0 \end{pmatrix}$$

So the matrices ($\underline{E} - \underline{A}$) and ($- \underline{B}$) have the same number of columns we are able to put both of them together:

$$\begin{pmatrix} \underline{E} - \underline{A} \\ - \underline{B} \end{pmatrix}$$

The composition of the formulas gives us the mathematical model to solve the first fundamental problem :

$$\begin{pmatrix} \underline{E} - \underline{A} \\ - \underline{B} \end{pmatrix} \underline{x} = \begin{pmatrix} \underline{y} \\ -\underline{v} \end{pmatrix}$$

Applicat to our numerical example :

$$\begin{pmatrix} 1 & -0.7 & -0.5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1.3 \\ 0 & 0 & 0 & 1 \\ -0.9 & -0.4 & 0 & 0 \\ -0.15 & -0.32 & -0.6 & 0 \\ -1.5 & 0 & 0 & 0 \end{pmatrix} \times \begin{pmatrix} 200 \\ 60 \\ 220 \\ 120 \end{pmatrix} = \begin{pmatrix} 48 \\ 60 \\ 64 \\ 120 \\ -204 \\ -181.2 \\ -300 \end{pmatrix}$$

b) Mathematical formulation of the second basic **problem.**

The second problem was to determine the was given. That means we know $\underline{y}$ and we want to know $\underline{x}$. In a pure mathematical sense we solve the problem as follows:

We multiply the formula $(\underline{E} - \underline{A})\underline{x} = \underline{y}$. with the inversion of the matrix $(\underline{E} - \underline{A})$ in order to get a formula for $\underline{x}$ :

$$(\underline{E} - \underline{A})^{-1} (\underline{E} - \underline{A}) \underline{x} = (\underline{E} - \underline{A})^{-1} \underline{y}$$

$$\underline{\underline{\underline{x} = (\underline{E} - \underline{A})^{-1} \underline{y}}} \qquad (1)$$

In order to calculate $\underline{v}$, the vector of direct inputs, we start with the formula

$$\underline{B}\,\underline{x} = \underline{v},$$

and substitute $\underline{x}$ by the result of our first step, by $(\underline{E} - \underline{A})^{-1}\,\underline{y}$, in accordance with formula (1) :

$$\underline{B}\,(\underline{E} - \underline{A})^{-1}\,\underline{y} = \underline{v} \qquad (2)$$

The composition of both the formulas (1) and (2) gives us the mathematical model to solve the second problem :

$$\begin{pmatrix} (\underline{E} - \underline{A})^{-1} \\ -\underline{B}\,(\underline{E} - \underline{A})^{-1} \end{pmatrix} \underline{y} = \begin{pmatrix} \underline{x} \\ -\underline{v} \end{pmatrix}$$

Applied to the example :

$$\begin{pmatrix} 1 & 0.7 & 0.5 & 0.65 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1.3 \\ 0 & 0 & 0 & 1 \\ -0.9 & -1.03 & -0.45 & -0.585 \\ -0.15 & -0.425 & -0.675 & -0.878 \\ -1.5 & -1.05 & -0.75 & -0.975 \end{pmatrix} \times \begin{pmatrix} 48 \\ 60 \\ 64 \\ 120 \end{pmatrix} = \begin{pmatrix} 200 \\ 60 \\ 220 \\ 120 \\ -204 \\ -181.2 \\ -300 \end{pmatrix}$$

The most time-consuming task by solving the second fundamental problem is the calculation of the inverse matrix of the coefficients of direct interlacement in order to get the matrix of the coefficients of the total interlacement. The calculation normally requires electronic data processing equipments. In our simplified numerical example we solved the task step by step, we repeated the multiplication of the coefficient matrix by the production vector until no further inputs were required.

From the mathematical point of view the calculation of our example is based on the socalled NEUMANN'S progression, which proves the identity of the following mathematical expressions :

$$(\underline{E} - \underline{A})^{-1} = \underline{E} + \underline{A} + \underline{A}^2 + \underline{A}^3 \; .....$$

This identity is given under certain conditions, which in practice always exist. Though the progression consist of an infinite number of terms, the calculation normally comes to an end very soon, as a rule after three or four steps, because all the elements of the matrix approach to zero. The following example may make it visible :

$$\underline{A} = \begin{pmatrix} 0 & 0.7 & 0.5 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\underline{A}^2 = \begin{pmatrix} 0 & 0 & 0 & 0.65 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\underline{A}^2 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$(\underline{E} - \underline{A})^{-1} = \underline{E} + \underline{A} + \underline{A}^2 = \begin{pmatrix} 1 & 0.7 & 0.5 & 0.65 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1.3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

الجمهورية العربية المتحدة

معهد التخطيط القومي

مذكرة رقم ٨٠٠

# جداول التدفقات القومية واستخدامها في دراسة المشروعات الاستثمارية

الدكتور محمد محمود الامام

سبتمبر سنة ١٩٦٧

القاهرة

٣ شارع محمد مظهر . بالزمالك