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Memo. No. 564

PROGRAM EVALUATION AND REVIEW TECHNIQUE (PERT) WITH COMPUTER APPLICATION

II. PERT/COST

By

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Introduction:

In the previous memo. (PERT/TIME), the PERT method was introduced and it was shown how the critical path method (CPM) could be used in preparing the time schedule of the project, in pinpointing the areas in the project that needs special attention, and in determining the man-power and equipment necessary to excute the plan. This memo. is a continuation of the above work where additional features are added to incorporate the cost variable in addition to the time variable. The purpose of introducing the cost variable is to develop a plan of action for cost expenditures by applying the necessary cost estimating technique and to act as a monitor in determining where the actual costs are different from estimated costs.

It is necessary, however, before presenting the new technique that the reader should review the memo. on PERT/TIME as a necessary background for the material presented here.

Cost-Time Concept:

After the arrow diagram has been completed as is indicated in the previous memo., the cost-time relationship is determined for each activity. This relationship indicates the variation of the costs of executing the job (activity) with time, and takes into account the manpower, money, and methods used. This information is then used to obtain the optimum way to complete the project so that any deviation from this way causes increases in cost. Let us take as an example a job which requires two men to perform it most efficiently in five days and assume that the nature of this job will not allow any more effort on it. A number of possibilities exist:

1. Two men may work five eight-hours day shifts.
2. Four men may work two shifts and complete--in three day - three day shifts and two night shifts. Shift pnremiums will make the cost higher than in the first instance.

3. Six men may work three shifts and finish - in two days - two day shifts two second shifts, and one third shift. Additional premiums will furthur increas the cost.
4. If more than six men work, the cost will sky rocket and two days will still be required to do the job.
5. One man will drag out the job for more than ten days and will increase costs because of inefficience.

The time cost curve for this job is shown in Figure (1).

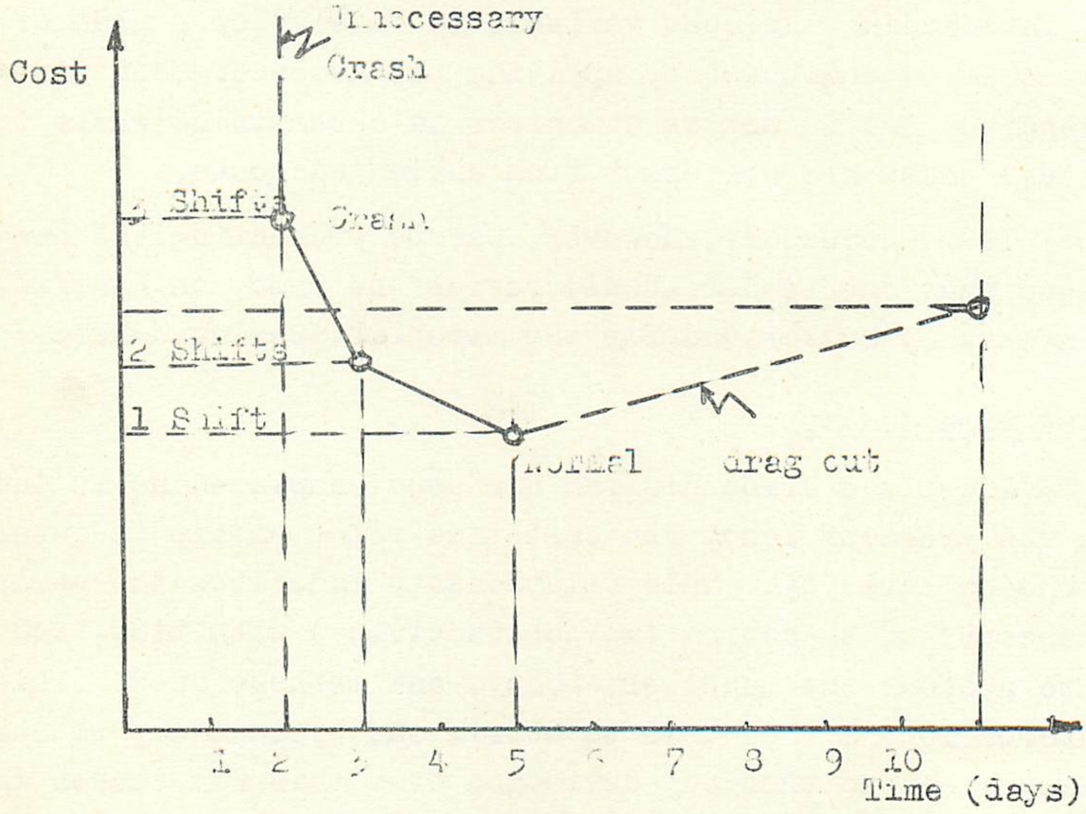


Figure (1)

<u>Use of the time-cost curve in CPM:</u>

After the time-cost curve for each activity is estimated, the following steps are used in order to obtain the required cost information about the project:

1. Using CPM, one determines the critical Path of the project when all activities are at normal costs. This will naturally give the lowest cost of the project.

2. Select the activity on the critical path having the <u>smallest</u> rate of increase in costs and "crash" it to its minimum time and then compute the new cost of the project and its new critical path.
3. Repeat step (2) until the project duration can no longer be compressed.

It is noted from the above discussion that at each iteration, the cost corresponding to any project duration are minimum costs possible since in proceeding from one iteration to the next, we select the job having the smallest increase in costs. The end result of the above computations will be a time - total direct cost curve as shown in Figure (2). It is noted from this

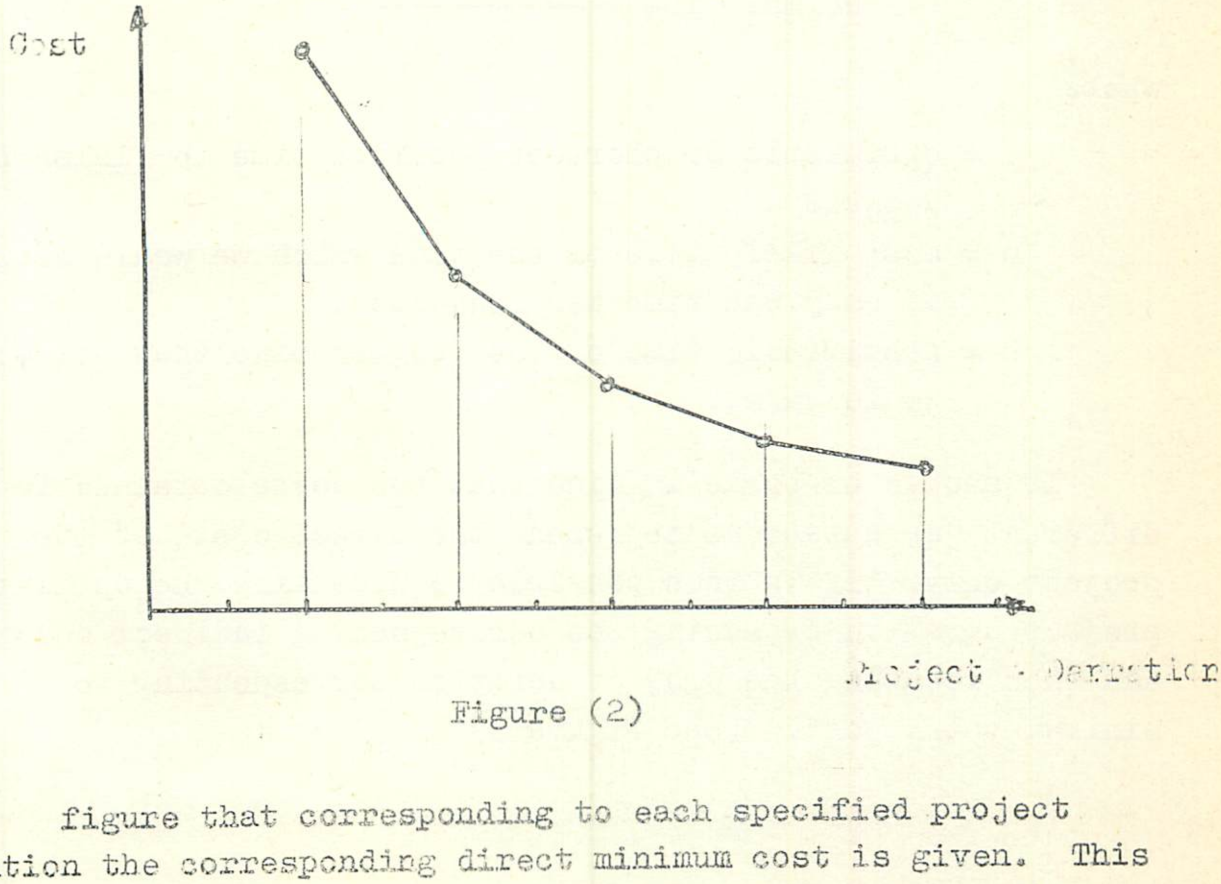


Figure (2)

figure that corresponding to each specified project duration the corresponding direct minimum cost is given. This can be used in drawing the most suitable plan for executing the project.

It should be noted however, that for the purpose of CPM, the time - cost curve for each activity need not contain all the information given in Figure (1) above. This is so since sufficient data are generally not available. Rather we only estimate the "normal" and "crash" points. A linear relationship between these two points is generally sufficient to produce acceptable results comparable to the accuracy of the data input. The slope of the cost curve in this case will then be used as the basis for selecting the "critical" job to be crashed:

The estimation of the "normal" time for an activity is usually based on three time estimates by the use of the following formula:-

$$\text{Normal time} = \frac{a + 4m + 6}{6}$$

where

a = optimistic or shortest possible time to finish the activity.

m = most likely time or the time which we would estimate if only one time was requested.

b = pessimistic time or the longest time that the activity would take.

It should be borne in mind that the costs obtained for different job durations represent the direct costs of the project only. If is thus possible to determine the optimum project duration by adding the corresponding indirect costs and then locating the project duration corresponding to minimum total cost. (see Figure 3)

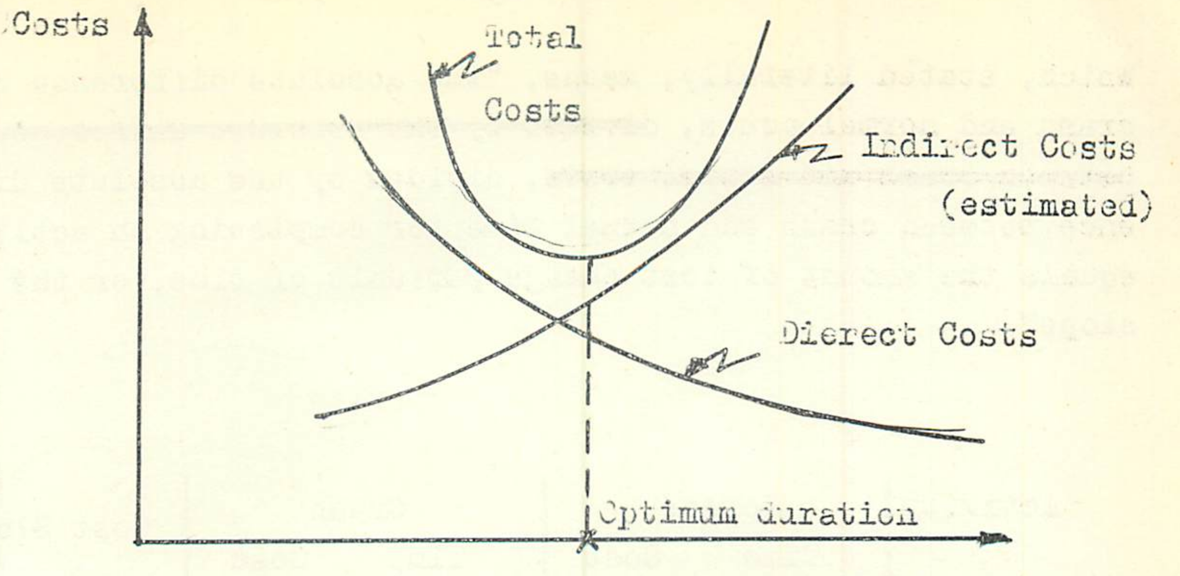


Figure (3)

Example of PERT/COST computations:

Keeping in mind the conventions for arrow diagramming, consider the following hypothetical diagram and arbitrary time and direct cost values assigned in Table 1.

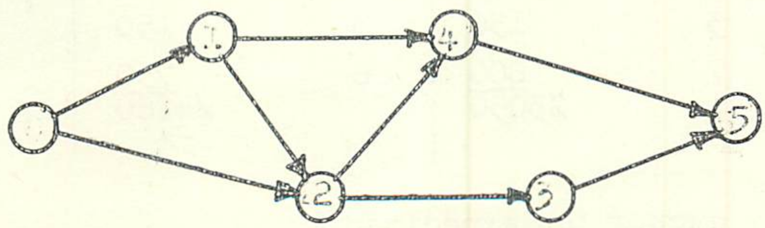


Figure (4)

Note also the additional concept of cost slope (last column of Table 1.) Cost slope, a useful factor in determining optimum time and cost points per activity, is defined as the rate of change in cost per unit of change in activity duration. It may be readily calculated from the formula.

$$\frac{\text{crash cost} - \text{normal cost}}{\text{crash time} - \text{normal time}} = \text{cost slope (dollars per unit of time)}$$

Which, stated literally, means, "the absolute difference between crash and normal costs, ~~divided by the absolute difference between crash and normal costs~~, divided by the absolute difference between crash and normal time for completing an activity, equals the amount of cost change per unit of time, or the cost slope".

Activity	Normal		Crash		Cost Slope
	Time	Cost	Time	Cost	
(0,1)	4 days	$ 210	3 days	$ 280	$ 70
(0,2)	8	400	6	560	80
(1,2)	6	500	4	600	50
(1,4)	9	540	7	600	30
(2,3)	4	500	1	1100	200
(2,4)	5	150	4	240	90
(3,5)	3	150	3	150	-*
(4,5)	7	600	6	750	150
		$3050		$4280	

*This activity cannot be expedited.

Table 1

The first observation that can be made from Table 1 is that, if all jobs are performed in the normal duration, the total direct cost will be $3050. Secondly, if all activities are performed on a crash basis, the total direct cost will be $4280.

The next information to be determined is the time required to complete the project on an all-normal basis and on an all-crash basis.

1. <u>All-Normal Solution:</u>

Considering first the all-normal solution, the normal durations are entered from Table 1, and the earliest and

and latest occurrence times and the critical path are determined:

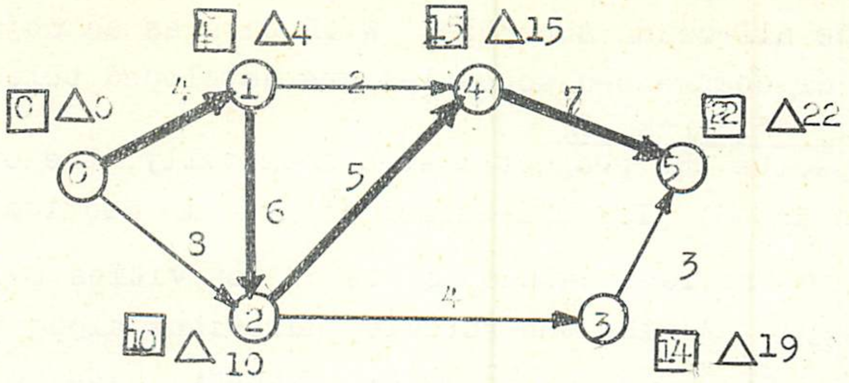


Figure (5)

The above information is readily derived by applying the principles described before. Earliest and latest occurrence times are calculated and entered in the proper squares and triangles, and each activity is tested to determine if it lies on the critical path. It is found that the all-normal schedule provides a project duration of 22 days at the previously established cost of $3050 (Table 1).

2. <u>All-Crash Solution:</u>

<u>Using the all-crash</u> durations for each activity and again calculating the earliest and latest occurrence times and again testing each to see if it is critical, the following solution is achieved:

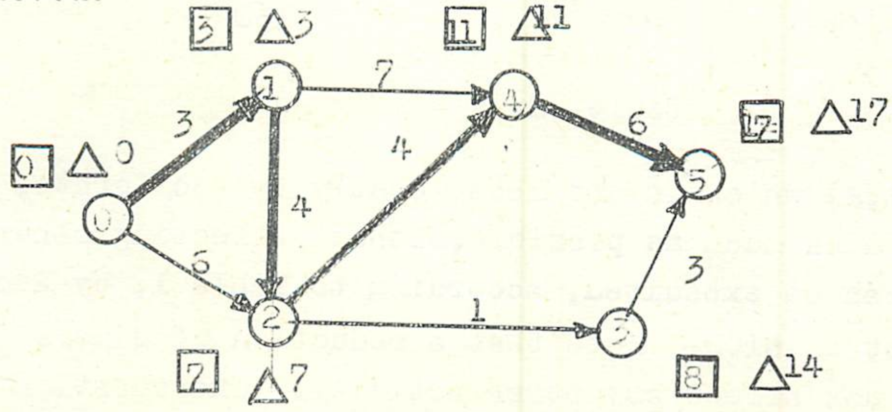


Figure (6)

As noted in the all-crash solution, above, the all-crash schedule will permit the project to be completed in 17 days, but at a cost of $4280 (an increase of $1230 over the all-normal schedule).

By selective compression of activities, it is possible that the project can be completed in the same minimum time, but at less cost than the all-crash schedule. With this as an objective, accelerated or compressed schedules are developed below.

3. Schedule Compression

To expedite the project most economically, the critical path must be known. The all-normal diagram in Section (1) shows that the critical path consists of activities (0,1),(1,2), (2,4) and (4,5). Adding the activity durations along this path gives the project duration, 22 days. Because those activities are critical, it is apparent that project duration can be shortended only by reducing one or more of these activities.

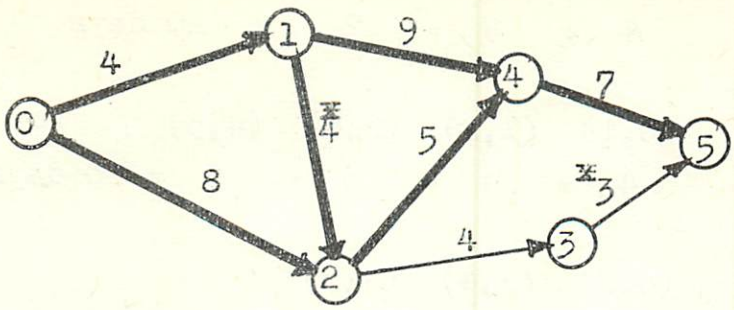
(a) Expediting a Single Activity

The cost of accelerating an activity is a function of the cost slope for that activity. An examination of the costs slopes for the critical activities as listed in Table 1, Shows the expediting costs per day to be as follows:

Activity	Cost Slope
(0,1)	$ 70
(1,2)	50
(2,4)	90
(4,5)	150

Table 2

Activity (1,2) which is the least costly to accelerate, should be expedited as much as possible,without affecting other activities (1,2) can be expedited, according to Table 1, by 2 days at a total cost of $100. Note that a reduction of 2 days in activity (1,2) does not affect any other activity. The duration of activity (0,1) plus the duration of activity (1,2) equals 8 days, as does the duration of activity (0,2). The times on the new arrow diagram then become



* indicates activities reduced to their limits.

Project Duration: 20 days

Project Cost : $ 3150

1st Expedited Schedule

Figure (7)

Time and costs now appear as follows:

Activity	Time	Cost
(0,1)	4 days	$210
(0,2)	8	400
(1,2)	4	600
(1,4)	9	540
(2,3)	4	500
(2,4)	5	150
(3,5)	3	150
(4,5)	7	600
	Total Cost	$3150

Table 3

The new project duration is 20 days (a two-day reduction) and the new project cost is $3150 (an increase of $100). However, the number of critical paths has increased to three. The total project duration of 20 days is obtained by the following connected sequences of activities.

Path 1 = (0,1) (1,4) (4,5)
4 + 9 + 7 = 20 days

Path 2 = (0,1) (1,2) (2,4) (4,5)
4 + 4 + 5 + 7 = 20 days

Path 3 = (0,2) (2,4) (4,5)
8 + 5 + 7 = 20 days

The cost slopes for these critical activities are shown in Table 4.

Path 1	Cost Slope
(0,1)	$ 70
(1,4)	30
(4,5)	150

Path 2	Cost Slope
(0,1)	$ 70
(1,2)*	50
(2,4)	90
(4,5)	150

Path 3	Cost Slope
(0,2)	$ 80
(2,4)	90
(4,5)	150

*At its crash limit

Table 4

To reduce the schedule duration ~~duration~~ further each critical path must be reduced by the same amount.

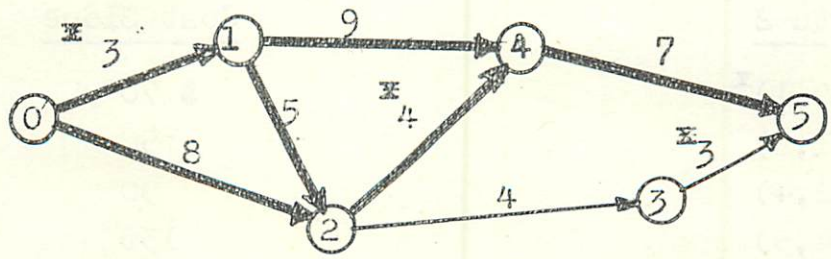
(b) Multi-Path Expediting

In path 1, activity (1,4) can be reduced one day at a cost of $30. In paths 2 and 3, activity (2,4) can be reduced one day at a cost of $90. Therefore, one day could be reduced from the schedule at a cost of $120. But, is this the best that can be done?

Sometimes the acceleration of some activities permits the relaxation of other without adverse effect on project duration. Consider the following alternative:

Suppose activities (0,1) at $70 per day and (2,4) at $90 per day were reduced one day each. The total increase in cost would be $160. However, activity (1,2), which was previously crashed in duration, can now be relaxed one day at a savings of $50. Thus, the net cost of this one day reduction would be only $110.

Reflecting the above compression of activities (0,1) and (2,4), the times on the diagram become.



x indicates activities reduced to their limits.

Project Duration: 19 days

Project Cost : $3260

2nd Expedited Schedule.

Figure (8)

The resulting times and costs are shown in Table 5.

Activity	Time	Cost
(0,1)	3	$280
(0,2)	8	400
(1,2)	5	550
(1,4)	9	540
(2,3)	4	500
(2,4)	4	240
(3,5)	3	150
(4,5)	7	600
Total Cost		$3260

Table 5

The project duration is reduced to 19 days and the project cost has increased to $3260. The critical paths have not been altered, so once again three paths must be evaluated. (Table 6)

Path 1	Cost Slope
(0,1)*	$ 70
(1,4)	30
(4,5)	150

Path 2	Cost Slope
(0,1)*	$ 70
(1,2)	50
(2,4)	90
(4,5)	150

Path 3	Cost Slope
(0,2)	$ 80
(2,4)*	90
(4,5)	150

* At its crash limit

Table 6

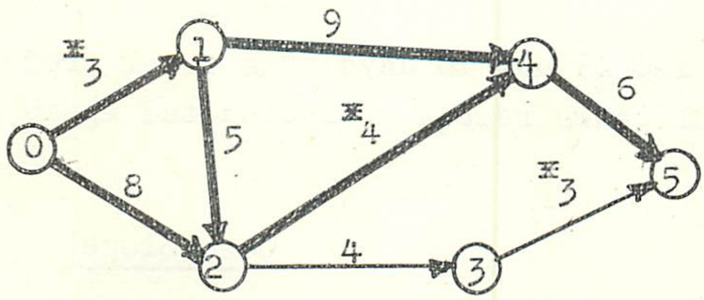
(c) Effects of Further Expediting

Bacause certain activities (noted by an asterisk in Table 6) are now at their crash limit, only two possible ways remain in which the total project duration can be reduced.

One possibility is to reduce activity (1,4) at a cost of $30; activity (1,2) at a cost of $50; and activity (0,2) at a cost of $80. The total cost of a one day reduction would be $160.

The other alternative is to reduce activity (4,5) at a cost of $150.

Choosing the second alternative results in the following activity durations:



* indicates activities reduced to their limits.

Project Duration: 18 days
Project Cost : $3410

3rd Expedited Schedule

Figure (9)

The activity times and costs become as shown in Table 7.