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On the Critical Path Method (C.P.M.) and its Application in the G.D.R.

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1. Problem and matters in principle

Network models and methods are used above all for flow and time planning of complex processes. In each case the preparation of decision is based upon certain problems of sequence.

On principle always such processes or systems are concerned in which partial processes flow simultaneously or successively. That means on the one hand that a simultaneous flow of different partial processes (parallel flow) is given and on the other hand that partial processes may only begin, if proceding ones are completed (flow in series).

Therefore different partial processes united by conditions for their corresponding arrangement (reciprocal dependency) and flowing partially successively and simultaneously must be examined as to the most favourable temporal series according to fixed points of view. The target is to consider the dependencies between the single partial processes and of the co-operation to determine the duration of the total process and thus its finishing data or to observe or to underbid a given date by showing possibilities of abbreviating the duration and of the most favourable coordination of partial processes as well as by corresponding controlling measures connected with it.

The problem is further to establish whether there are determinated or stochastic quantities. Depending on this, different methods of network technics must be employed. If determinated quantities are given (e.g. with most of the building projects) the determinated model and the C.P.M. method or another similar method may be used. If this condition is not met and one may work only with quantities distributed according to probability (e.g. mostly when planning research works) the stochastic model and among others the PERT method must be used.

The methods of network technics are based upon the theory of graphs. As a rule a graph consists of a number of points some of which are combined with each other as pairs by ares (cp. the following figures).

A network is a graphically represented model of a complex composed of partial processes dependent on one another the dependency of the partial processes being explained, i.e. their necessary sequence or their parallel accomplishment and by this the temporal order of the events belonging to them (start and finish).

The "decomposition" of the total complex with the aid of the network makes it possible

- to obtain a qualitative survey of the flow of the total process,
- to quantitatively fix the expenditure of time and possibly wanted capacities, cost a.o.,
- to realize the share of each partial process in the total project,
- to determine the critical path and by this the critical partial processes,
- to compute the floats (or buffer times) for the non-critical partial processes,
- to realize failures in time when accomplishing single partial processes and their influence on the total project as well as to take up measures of keeping the section, and
- if necessary to determine optimal times of accomplishment the total project considering further factors (minimizing of cost, continuous use of capacities, continuity in the production process a.o.)

2. Flow Planning

When planning the flow of a complex it is necessary to subdivide the total process into partial activities, e.g. particular sections of work. On the basis of these small partial sections flow planning and resulting from it flow time planning are established.

A moment at which one or several partial activities begin or end is called "event". Such events are e.g. the dates for material supply, for the acceptance of technical and similar facilities, for the completion of certain works and so on. These events are combined by the partial processes calling for a space of time.
An event

- suggests a certain point in the flow,
- is start or finish of a partial process,
- requires neither working-hours nor means of work.

These partial processes are called "activities". Here it is the question of e.g. the accomplishment of works, the production of parts, the scheduled terms of delivery, the instruction of manpower, the coming to a decision and so on.
An activity

- corresponds to the completion of a partial process,
- represents a time section (expenditure of time);
- requires manpower and (or) means of work.

The elements "activity" and "event" may be interpreted in a different way by the oriented are (arrow) or the node. Correspondingly two manners of representation for networks are distinguished, that is event-oriented as well as activity-oriented networks. By this it is already expressed that there is a difference in the allocation of natural numbers to events or activities.

Here only event-oriented networks are discussed. In such a network each activity is marked as consequence of the event-orientation by two numbers (one for the start event and one for the end event).

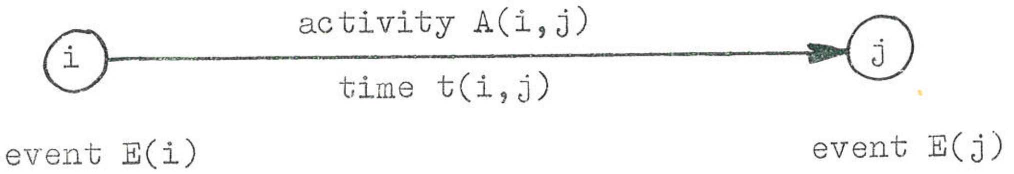

Fig. 1

The start point, start event of the network, gets the 0 and the following events get the natural numbers 1, 2, ..., n; therefore n is defined as end event of the network. The number of events of a total project is $n + 1$. The events are numbered according to the order beginning from the lower number. With this the condition $i < j$ is combined.

When logically reflecting on the whole flow and when fixing the correlations in the networks it is reasonable to answer the following questions for each activity :

- Which partial processes immediately precede the activity and must be terminated before its start?
- Which partial processes may flow simultaneously (parallelly) with the activity?
- Which partial processes must immediately follow the activity ?

If two or more activities begin and end with the same events, a fictive activity, a dummy activity, is inserted for the exact representation - especially for the time planning which will be discussed later. It is marked by a dashed line (cp. Fig. 3) .

Two activities a and b begin e.g. with the event 3. They end at the same moment. In this case the end should be marked by two different events, 4 and 5. Accordingly it is objectionable to represent this flow in the network as follows :

Fig. 2

In order to obtain correct planning documents the following form of representation must be chosen :

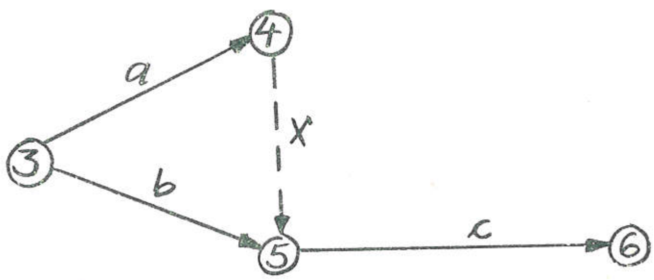

Fig. 3

In this case x is the fictive activity with the duration of time 0. In any case it should be avoided by using fictive activities that parallelly flowing partial processes have start or end events in common.

Comprehensive objects may be subdivided into partial projects and by this may be represented more clearly. In such a case the single partial projects are represented in the main project by collective activity the single partial processes may only be seen from the network of the partial project which is separately designed. Therefore one speaks of a detailed and a concentrated network.

Instead of the single representation of the activities b to f in Fig. 4 in Fig. 5 the collective activity x is to be found.

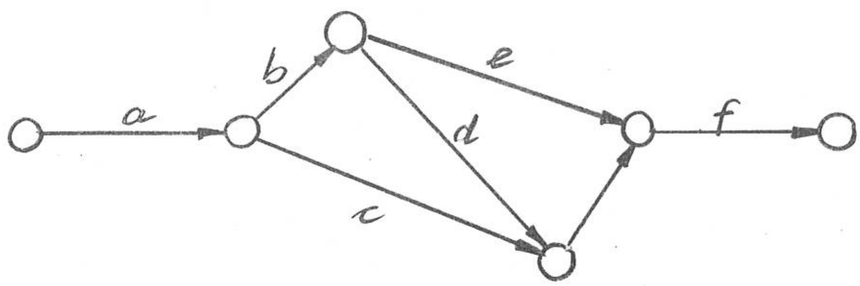

Fig. 4

Fig. 5

Further it should be emphasized that a tail event at which several activities are ending has only occurred, if all these activities are terminated. The critical path method shall be illustrated below by an example.

First a list of all activities is required (table 1.) It is the question of a rough planning. Only collective activities are joined in the network. The single activities are logically arranged - according to their flow. The resulting network is represented in Fig. 6.

Table 1

E(i)	E(j)	A(i,j)	t(i,j) normal (weeks)	t(i,j) minimal (weeks)
0	1	earthworks	10	6
0	3	terms of delivery for technical facility	48	30
1	2	rough brickwork	22	18
1	3	road approach	18	10
2	5	roof	8	4
3	4	completion	4	4
3	6	outer plants	20	8
4	6	assembly of the technical facilities	16	10
5	6	aerial works	20	10

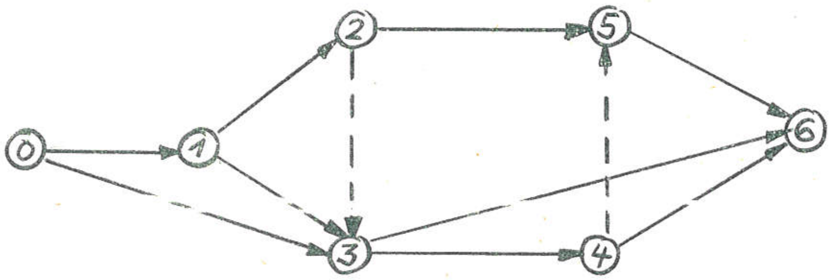

Fig. 6

Summing up it may be said to the flow planning that it consists of the following important tasks :

- establishing the complete list of all partial processes;
- determining the dependencies between the partial processes, it being necessary to determine for every activity which partial processes must precede, which must follow immediately, and which may run parallelly;
- arranging of the partial processes corresponding to the logic flow, and establishing of a network and, possibly, additionally of an arranged list considering the assigned processing algorithm.

3. Time planning

Before explaining the time planning by an example in detail the following symbols are introduced to this end :

E(i) = start event for an activity
E(j) = end event for an activity.

It should be noted that each event - except the start and end point of the whole project - must appear at least once as E(i) and once as E(j).

O = start event of the network
n = end event of the network
G = total project, total process with n + 1 events
Ta start date (beginning, start moment) for the total project G
Te = finish date (end, final moment) for the total project G
A(i,j)= activity (partial process, section of work) running from the event E(i) till the event E(j)
t(i,j)= duration of the activity A(i,j)

fTa(i,j) = earliest start date (beginning, start moment) for the A(i,j) i.e. earliest possible moment of the happening of E(i) referred to the start point of the total project with the start moment 0, being $f\,Ta(i,j) = \max_{i'} \left[fTa(i',i) + t(i',i) \right]$,

i' running through all line indicess for which in the matrix the i-th column is occupied ($i > i' > 0$).

sTa(i,j) = latest start date (beginning, start moment) for the A(i,j),

$sTa(i,j) = sTe(i,j) - t(i,j)$

fTe(i,j) = earliest finish date (end moment) of the A(i,j), $fTe(i,j) = fTa(i,j) + t(i,j)$

sTe(i,j) = latest finish date (end moment) for the A(i,j), i.e. la test moment of the happening of E(j), referred to the beginning of the total project at the moment 0, being
$sTe(i,j) = \min_{j'} \left[sTe(j,j') - t(j,j') \right]$
j' running through all column indices for which the j-th line is occupied in the matrix ($n \geqq j' > j$).

From table 1 the t(i,j) are transferred to table 2.

Table 2

E(i) \ E(j)	0	1	2	3	4	5	6	fTa(i,j)
0		10		48				0
1			22	18				10
2				0		8		32
3					4		20	48
4						0	16	52
5							20	52
6								72
sTe(i,j)	0	22	44	48	52	52	72	
sTe(i,j) −fTa(i,j)	0	12	12	0	0	0	0	

Corresponding to the mentioned formula the earliest possible date of the start of the single activities -fTa(i,j) - must be calculated. Here it should be taken into account that in each case the maximum must be inserted. In the example a total duration of 72 weeks appears. Starting from this value the latest finish date sTe(i,j) is determined in a retrograde manner. Subsequently the difference sTe(i,j) - fTa(i,j) is to be formed. If follows

sTe(i,j) - fTa(i,j) = 0

this will be equivalent to the fact that the earliest start date for an activity equals the latest finish date of the proceding longest activity. Those results for which in the last line resulted 0 are lying on the critical path. If only one critical path exists through the network, this will be sufficent to mark it. In the example it touches the following events : 0 - 3 - 4 - 5 - 6 . This corresponds to a 72 weeks duration of construction (cp. Fig. 7).

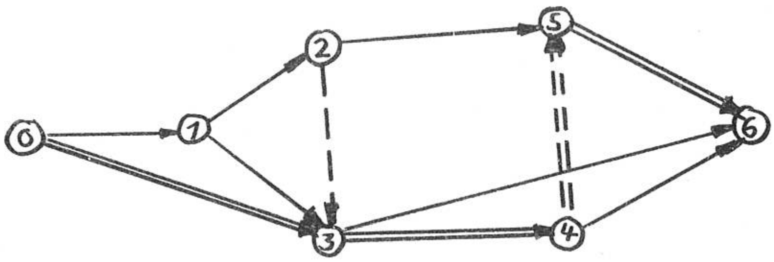

Fig. 7

If it is not possible to unequivocally conclude from the network by marking the events lying on the critical path, which activities are critical, and if thus it is impossible to fix the critical path or paths (there may be several), an additional calculation must be made. The critical activities are established not by graphic representation but by calculation. In the example discussed till now this does not occure, however. But as a rule this is the case, if several critical paths are existing.

In this calculation it is assumed that beside the qualitative criterion used here a quantitative one exists, too. The qualitative criterion indicates that at least all those activities are critical which lie between two critical events - but for 0 and n. For the quantitative criterion is decisive

$$fTa(i,j) + t(i,j) = sTe(i,j)$$

If this condition is fulfilled, i.e. if there is no float, the corresponding activity is critical.

The floats shall only be mentioned, but not be calculated for the example.

tf(i,j) = total float for A(i,j) = sTe(i,j)-fTa(i,j) - t(i,j)

ff(i,j) = free float for A(i,j) = fTa(j,j') - fTe(i,j), j' > j and fTa(i,j') being the earliest start of the activity following (i,j).

if(i,j) = interferring float for A(i,j) = sTa(j,j') - fTe(i,j), being j' > j.

uf(i,j) = independent float for A(i;j) = max [fTa(j,j') - - sTe(j,j') - t(i,j)], being uf(i,j) $\geqq$ 0 and j' > j.

Summarizing it can be stated that the time planning passes in following stages:

- Fixing the temporal values for the single activities,
- Calculating dates and data for the events and activities as well as determining the critical path,
- Establishing the necessary floats.

4. Shortening of the time required for the total project

The 72 weeks time of construction is too long. The edifice
shall be ready within a year (= 52 weeks). If the critical path
of a total process has been established, the point is <u>to check</u>
the single concerned <u>activities</u> with their events in the sense if
it is <u>possible to shorten</u> the provided time by measures of dif-
ferent kind. According to considerations related to operational
economy those variants are chosen requiring, as to the target to
be reached, the least expenditure, but have the greatest effici-
ency. By the aid of the initiative of the working people all or-
ganizational, technological, technical, and other possibilities
of shortening the total flow must be used.

For the example it is supposed that after checking the
periods of activities (o,3) are shortened by 16 weeks and of (5,6)
by 8 weeks. Thus table 3 arises. It is calculated in the same
manner as table 2.

Table 3

E(j) / E(i)	0	1	2	3	4	5	6	fTa(i,j)
0		10		32				0
1			22	18				10
2				0		8		32
3					4		20	32
4						0	16	36
5							12	40
6								52
sTe(i,j)	0	10	32	32	36	40	52	
sTe(i,j) -fTa(i,j)	0	0	0	0	0	0	0	

In this case several paths are critical. They cannot be exactly determined by means of the critical events to be seen from table 3. Therefore for each activity it must be checked, if

$$fTa(i,j) + t(i,j) = sTe(i,j).$$

If this condition is fulfilled, the corresponding activity is critical.

This checking proves that in the network only two activities are not critical, they are the activity (1,3) and the dummy activity (4,5). Several critical paths run through the network. The target, a 52 weeks time of structure, is reached by this plan.

This time corresponds to the duration of the total project G. The paths showing this duration are critical paths. The events and activities lying on these paths are the critical events and the critical activities resp. All other paths through the network as well as the other events and activities are considered as slack paths.