UNITED ARAB REPUBLIC

التخطيط القومى

THE INSTITUTE OF NATIONAL PLANNING

Memo. No. 405

Cumulative Marginal Productivity, Life of Plants and the Optimal Technology.

by
Dr. A. Qayum

March 1964.

CAIRO

Cumulative Marginal Productivity, Life of Plants and the Optimal Technology*

1. Introduction

In the post World War II period, an ever growing concern has been accorded to economic development in general and to that of the underdeveloped countries in particular. The economic development of a country mostly depends on how best the resources are utilised. The rate of growth of the country can be maximised only when the optimal allocation of resources is obtained. And the latter can be achieved only when the optimal techniques of production are introduced. The choice of the optimal technology is one of the most essential elements in the maximisation of economic growth. During the last decade a good deal of attention has been given to the choice of technology and several criteria have been put forward. We shall briefly examine these criteria first and then develop a more comprehensive approach which contains some essential considerations that have so far been neglected. The criteria which have been hitherto advanced can be broadly classified into two categories; one is based on the principle of the minimisation of real cost and the other on that of the maximisation of savings.

II. The Principle of the Minimisation of Real Cost

The criteria included in the first category are more

* I am grateful to the participants for their comments in a seminar discussion held on this paper in the Planning Division of the Indian Statistical Institute, Calcutta. My thanks are also due to Mr.A. Salim who has read the manuscript.

or less of static nature. Under this category come the contributions of such authors as Polak, Buchanan, Kahn, Lewis, Chenery and others[1]. The basic idea behind the criteria suggested by these writers is that those techniques are the best which combine the factors of production in such a proportion that the cost per unit of output is the least, the cost being reckoned, in most cases, at the opportunity cost of factors[2]. Almost all these writers take an oversimplified picture of an economy with one scarce factor, capital, and with an abundant supply of labour (and possibly other factors). The abundance of labour implies that its real cost or the opportunity cost is zero and hence according to these criteria, those techniques are the best which combine the least amount of the scarce factor, capital. Moreover, all these writers assume the relative scarcity of factors and their opportunity costs constant and equal to those obtaining in the period in which the plants or projects are going to be started. This approach can be regarded

---

1. J.J.Polak, 'Balance of Payments Problems of Countries Reconstructing with the Help of Foreign Loans', Q.J.E. LVI (Feb. 1943) N.S.Buchanan, 'International Investment and Domestic Welfare' (New York, 1945); A.E.Kham, 'Investment Criteria in Development Programmes', Q.J. E., LXV (Feb.1951); W.E.Lewis, 'The Theory of Economic Growth', (London and Homewood),(Ill., 1955); H.B.Chenery, 'The Application of Investment Criteria' Q.J.E., LXVIII (Feb. 1953).

2. In any broad classification, it is not possible to do full justice to the individual authors. This is still more true here, for not all the references cited are directly concerned with the choice of technology. Moreover, each of them has various other points of refinement and extensions which cannot various other points of refinement and extensions which cannot be considered here. This also applies to the authors included in the other category.

satisfactory only when the relative scarcity of factors and hence their opportunity costs remain constant or the plants last for only one period i.e. the period in which they are installed. As neither of these two are likely to hold true, these criteria are quite inadequate and unsatisfactory. Furthermore, these criteria do not give any consideration to the long run economic development which largely depends on the rate of saving and investment.

III. The Principle of Maximisation of Savings

The other category of criteria suggested by writers like Galenson and Leibenstein, Bettelheim, Eckstein, Dobb, Sen and others takes a longer period in view and is based on the reasoning that output is to be maximised not only in the present but also in the future periods[1]. This leads to the suggestion that the techniques which are the best are not those which yield the maximum output-cost ratio, but those which yield the maximum rate of savings, implying that the greater the addition to the stock of capital, the larger the volume of output per period in future. As savings are made after consumption needs have been satisfied, this set of criteria finds those techniques best which involve minimum consumption, consequently minimum use of labour power and hence maximum captial labour ratio. The longer the period taken into view, the higher the savings

1. W. Galenson and H. Leibenstein, 'Investment Criteria, Productivity, and Economic Development', Q.J.E. LXIX (Aug. 1955), Ch. Bettelheim 'Studies in the Theory of Plannin' (New York, Asia Pub. House, 1959), Otto Eckstein, 'Investment Criteria for Economic Development and the Theory of Intertemporal Welfare Economics', Q.J.E. LXXVI (Feb.1957); Maurice Dobb, 'An Essay on Economic Growth and Planning' (London, 1960), A.K. Sen, 'Choice of Capital Intensity Further Considered, Q.J.E.LXXII (Aug.1959).

rate that will ultimately maximise the volume of output. Carried to the extreme, i.e. when a period tending to infinity is taken into view, this line of argument will lead to the conclusion that all income should be saved, even if all members of the community are starved to death. In order to avoid this eventual situation the exponents of this set of criteria have been compelled to introduce a certain time limit by which the national product has to be maximised. The introduction of some device of this type, in fact, appears to be inescapable as the quest for optimal rate of savings has not yet met with any appreciable success[2] Moreover, the criteria in this category do not furnish a method for the choice of the most efficient techniques from the point of the maximisation of output from given resources which must be the essential consideration in the choice of the optimal technology. A technique may yield as high a rate of savings as is desired, and yet it may not be combining the factors in a proportion which conforms to the maximisation of output in the economy. The maximisation of savings is not the sole and not even the main problem in the choice of technology although it is an essential element in the long-term economic development and it must be included in any choice of the optimal technology in the appropriate way as has been done later in this paper.

IV. The Shortcomings of the above Criteria

The brief summary of the literature on the choice of technology given above makes the following remarks in order:

---

2. Prof. J. Tinbergen's article 'The Optimum Rate of Savings' in the E.J. (Dec. 1956) has been only followed by his disclaimer in Econometrica (April, 1960).

1. Both types of criteria are inadequate. The first type takes only one period (the present) into consideration. The second has to introduce an arbitrary time horizon.

2. No consideration is given to the life-span of the plants and projects, as no attempt is made to relate the choice of the optimal combination of factors (i.e. the optimal technology) to their life-span.

3. The problem of rigidity and immobility of factors employed in the already existing plants is not adequately dealt with.

4. Most of the writers abstain from introducing explicitly a set of production functions or a technology matrix. But without it a concrete method for finding the optimal choice of technology for the whole economy is hardly possible.

5. Though a number of the writers have claimed that their approach can be actually applied in practice, this has been rarely possible due to the arbitrariness, inadequacy and procedural complexity involved in the methods suggested by them[1].

In what follows an attempt is made to develop a fresh approach towards seeking a more appropiate solution of the problems connected with the choice of the optimal technology.

---

1. Cf. Hans Neisser, 'Investment Criteria, Productivity, and Economic Development: Comment', Q.J.E. LXX (Nov. 1956); John Moes, 'Investment Criteria, Productivity and Economic Development', Q.J.E. LXXI (Feb. 1957); H.H. Villard, 'Investment Criteria, Productivity, and Economic Development Comment', Q.J.E. LXXI (Aug. 1957); A.Q. Hirschman and G. Sirkin, 'Investment Criteria and Capital Intensity Once Again': Q.J.E. LXXII (Aug. 1958).

V. Technological Equilibrium Under the Ideal Conditions of Perfect Competition.

We start with the well-established economic principle of competitive equilibrium. Let F be a production function characterizing the productive activity in general. Let L denote total labour, ${}^{i}L$ labour in industry i and ${}^{i}L_j$ labour in plant j of industry i. Similarly for capital K and output O. Then in competitive equilibrium which ensures maximisation of output, we have:

$$\frac{\partial F(L,K)}{\partial L} = \frac{\partial F({}^{i}L, {}^{i}K)}{\partial {}^{i}L} = \frac{\partial F({}^{i}L_j, {}^{i}K_j)}{\partial {}^{i}L_j}$$

and similarly for K.

$${}^{i}O_j = {}^{i}L_j \frac{\partial F({}^{i}L_j, {}^{i}K_j)}{\partial {}^{i}L_j} + {}^{i}K_j \frac{\partial F({}^{i}L_j, {}^{i}K_j)}{\partial {}^{i}K_j}$$

$$= {}^{i}L_j \frac{\partial F(L,K)}{\partial L} + {}^{i}K_j \frac{\partial F(L,K)}{\partial K}$$

$$O = \sum_i {}^{i}O = \sum_j \sum_i {}^{i}O_j .$$

We shall use the above in finding the optimal techniques. The above formulae could be directly applicable if (a) plants lasted for one period only, i.e., the current period in which they are started and (b) all the amounts of factors were freely mobile and perfectly fluid. Unfortunately neither of these conditions holds; few plants last one period and the amounts of factors combined with the already existing plants are rigid and immobile.

VI. The Introduction of Essential Considerations

The optimal technology is the set of techniques which when an economy is stabilised on it, can lead to the maximisation of output over time, given the availability of factors and the community's preference schedule. In a certain period, however, not all the amounts of factors available can be combined with alternative techniques; for the amounts of factors that are utilised by the already existing plants are specific to them and have become rigid and immobile. In this paper we shall consider two factors of production, labour and capital. We define mobile labour as that part of labour which can be combined with alternative techniques; such labour force will include un-and under-employed workers plus the workers released from plants (or parts there of) that have depreciated. Similalry we define fluid capital as that part of capital which can be combined with alternative techniques; such capital will include gross savings or net savings plus the depreciation allowances[1]. Foreign loans and assistance minus service charges on such items can also be included in fluid capital, but we shall not make any explicit reference to foreign transactions here. The assumption that the labour and capital utilised by the existing plants are immobile and rigid is not far removed from reality. A capital equipment created to produce a certain commodity can seldom be used to produce another commodity. In most cases, if it ceases to produce the commodity for which it is meant, it can be used only as junk. Similarly the employed labour force in one line

1. The term 'depreciation allowance' used here is equivalent to replacement costs. What is meant, of course, is that capital item of any form or design can be created corresponding to what is included in fluid capital.

of production can seldom be expected to change over to (completely) other lines of production. Experience shows that even unskilled workers generally look for and mostly succeed in getting the jobs which they have been doing in the past.. This is truer in underdeveloped countries where the workers can hardly afford to learn to perform other jobs. However the argument is not materially affected if a small portion of employed labour and concretised capital can be used in alternative ways. In that case this portion will have to be included in the mobile and fluid amounts of factors.

VII. Illustrating with a Plant Lasting Two Periods

In order to illustrate, we shall first consider the choice of techniques in a plant lasting two periods. To start with a simple case, we shall also assume the amounts of mobile labour and fluid capital given in the two periods. We denote them by L(1), L(2), and K(1) and K(2) respectively. As the amounts of factors utilised by the already existing plants are rigid and immobile, the real marginal productivity of labour in period 1 is

$$\frac{\partial F(L(1),\ K(1))}{\partial K(1)} \quad \text{and in period 2} \quad \frac{\partial F(L(2),\ K(2))}{\partial K(2)} .$$

Now let plant j to be started in industry i in period 1 employ ${}^{i}L_{j}$ of labour and ${}^{i}K_{j}$ of capital to produce ${}^{i}O_{j}$ of net output in each period. This implies a very realistic assumption that after a plant has been started, it produces the same net output in each period over its life-span. This assumption is valid even when we allow for depreciation. What is implied is that the plant runs at its full capacity with due repairs and servicing etc........, till the end of its life-time when it is replaced by a new plant out of the accumulated depreciation allowances or replacement fund. Our problem in the reverse form is, given ${}^{i}Oj$, to find the most economical or efficient combination of labour and capital. Apparently the following seems to be the solution:

$$
\begin{aligned}
{}^{i}O_{j} &= {}^{i}L_{j}\,\frac{\partial F(L(1),K(1))}{\partial L(1)} + {}^{i}K_{j}\,\frac{\partial F(L(1),K(1))}{\partial K(1)} \\
{}^{i}O_{j} &= {}^{i}L_{j}\,\frac{\partial F(L(2),K(2))}{\partial L(2)} + {}^{i}K_{j}\,\frac{\partial F(L(2),K(2))}{\partial K(2)}\,.
\end{aligned}
\qquad (1)
$$

The MP's being known and ${}^{i}Oj$ given, (1) gives the values of ${}^{i}Lj$ and ${}^{i}Kj$. This, however, is not the most efficient solution. For the quantities of factors ${}^{i}Lj$ and ${}^{i}Kj$, thus arrived at, are in equilibrium with the MP's of factors derived with respect to their mobile and fluid amounts available in period 1, i.e., L(1) and K(1) and then those in period 2, i.e., L(2) separately; but in period 2, they are not in equilibrium with the MP's derived with respect to the sum of the mobile and fluid amounts of factors available in period 1 and 2, viz., with respect to L(1) + L(2) or $\int_{1}^{2} L(t)$ and K(1) + K(2) or $\int_{1}^{2} K(t)$. It means that the techniques chosen according to (1) are not in equilibrium with the fluid and mobile amounts of factors accumulated in period 2 from period 1. We call the MP of a factor derived with respect to its mobile or fluid amounts accumulated in the period from the period in which a plant is started the cumulative marginal productivity of the factor relating to that plant, or, CMP, for short. Here we make two assertions:

(1) It is possible to choose techniques in the starting period

of a plant which are in equilibrium with the CMP's of factors in each period during the life of the plant. In case of a plant lasting two periods we can choose techniques in period 1, that are in equilibrium with MP's derived with respect to L(1) and K(1) in period 1, and with respect to $\int_1^2 L(t)$ and $\int_1^2 K(t)$ in period 2, (2) the techniques chosen this way are the optimal.[1)]

The first assertion holds true, if we can find the values of $^{i}L_j$ and $^{i}K_j$ such that

$$
\begin{aligned}
{}^{i}O_j &= {}^{i}L_j \frac{\partial F(L(1),K(1))}{\partial L(1)} + {}^{i}K_j \frac{\partial F(L(1),K(1))}{\partial K(1)} \\
{}^{i}O_j &= {}^{i}L_j \frac{\partial F(\int_1^2 L(t), \int_1^2 K(t))}{\partial \int_1^2 L(t)} + {}^{i}K_j \frac{\partial F(\int_1^2 L(t) \int_1^2 K(t))}{\partial \int_1^2 K(t)}
\end{aligned}
\qquad (2)
$$

In the special case of two factors and a plant lasting two periods, the system is exactly soluble, for we have two unknowns and two equations. The second assertion is valid because the larger the amount of factors compatible with the equilibrium techniques in successive periods, the higher the production. The techniques arrived at from (2) will, thus, raise the production higher than those from (1), because in (2) all the mobile and fluid factors in period 2 accumulated from period 1, are in equilibrium with the techniques, where as in (1) only the mobile and fluid amounts of factors available in period 2 are in equilibrium with the techniques.

VIII. Generalisation of the Approach

We can now generalise the procedure. Let us consider a

1) The proof of this will be given in a forthcoming memo.)

plant lasting t periods. If t is equal to the number of factors (that enter the productive activities of the economy) then a system of equations as in (2) can be set up and this system will be exactly soluble. But if t is greater than the number of factors, the system will be over-determinate, and if t is less than the number of factors, the system will be indeterminate.

In order to illustrate the case, let us take a project which lasts three periods; then we should have

$$
\begin{aligned}
{}^{i}O_{j} &= {}^{i}L_{j}\,\frac{\partial F(L(1),K(1))}{\partial L(1)} + {}^{i}K_{j}\,\frac{\partial F(L(1),K(1))}{\partial K(1)} \\
{}^{i}O_{j} &= {}^{i}L_{j}\,\frac{\partial F(\int_1^2 L(t), \int_1^2 K(t))}{\partial \int_1^2 L(t)} + {}^{i}K_{j}\,\frac{\partial F(\int_1^2 L(t), \int_1^2 K(t)}{\partial \int_1^2 K(t)} \qquad (3) \\
{}^{i}O_{j} &= {}^{i}L_{j}\,\frac{\partial F(\int_1^3 L(t), \int_1^3 L(t))}{\partial \int_1^3 L(t)} + {}^{i}K_{j}\,\frac{\partial F(\int_1^3 L(y), \int_1^3 K(t))}{\partial \int_1^3 K(t)}
\end{aligned}
$$

In (3) we notice that there are three equations but only two unknowns, i.e., ${}^{i}Lj$ and ${}^{i}Kj$. This means that the system is over-determinate. In order to get over this situation, we modify our approach a little without reducing its efficiency[1].

---

1. This modification does not need more than our dodging the hitherto unsettled question of discounting the goods available in coming periods. However, we need to neglect the consideration of discounting just over the life of an average plant which in any case is likely to be immaterial and offsetting, firstly because the output of the plant is assumed to be constant in each period during its life and secondly it is not clear to see which period during the life of the plant should be taken as the vantage point. For an interesting discussion on these points see A.K. Sen: 'Income Optimising Rate of Saving', Economic Journal, vol. LXXI, Sept. 1961.

Instead of requiring that the value of the output of the plant in each period must equal the sum of the amounts of labour and capital utilised by the plant multiplied by their CMP's in the corresponding periods respectively, we require that the value of the output produced by the plant in each period multiplied by the number of periods it lasts must equal the sum of the amount of labour and capital multiplied by the sums of their respective CMP's for all the periods over which the plant runs. That is instead of having three equalities in (3), we should have one equality by summing up the values on the left and the right hand sides. Thus (3) can be replaced by the following:

$$3 \times {}^{i}O_{j} = {}^{i}L_{j}\left\{\frac{\partial F(L(1),K(1))}{\partial L(1)} + \frac{\partial F\left(\int_{1}^{2} L(t), \int_{1}^{2} K(t)\right)}{\partial \int_{1}^{2} L(t)} + \frac{\partial F\left(\int_{1}^{3} L(t), \int_{1}^{3} K(t)\right)}{\partial \int_{1}^{3} L(t)}\right\} \tag{4}$$

$$+ {}^{i}K_{j}\left\{\frac{\partial F(L(1),K(1))}{\partial K(1)} + \frac{\partial F\left(\int_{1}^{2} L(t), \int_{1}^{2} K(t)\right)}{\partial \int_{1}^{2} K(t)} + \frac{\partial F\left(\int_{1}^{3} L(t), \int_{1}^{3} K(t)\right)}{\partial \int_{1}^{3} K(t)}\right\}.$$

From (4) it is clear that when the number of factors employed by a plant is less than the number of periods over which it runs, there is no unique combination of factors that will conform to the optimal allocation, where the latter is defined as the maximisation of output produced by a plant, given the availability of resources in the economy. So far as our argument has proceeded, a high capital labour ratio as well as a low one both may be compatible with the optimal techniques depending on the values of these ratios, the CMP's of the factors and the number of periods over which the plant lasts. We can generalise (4) for a plant lasting t periods as given below:

(13)

$$t \times {}^{i}O_{j} = {}^{i}L_{j} \left\{ \frac{\partial F(L(1),K(1))}{\partial L(1)} + \frac{\partial F(\int_1^2 L(t), \int_1^2 K(t))}{\partial \int_1^2 L} + \dots + \frac{\partial F(\int_1^t L(t), \int_1^t K(t)}{\partial \int_1^t K(t)} \right.$$

$$\left. + {}^{i}K_{j} \left\{ \frac{\partial F(L(1),K(1))}{\partial K(1)} + \frac{\partial F(\int_1^2 L(t), \int_1^2 K(t))}{\partial \int_1^2 K(t)} + \dots + \frac{\partial F(\int_1^t L(t), \int_1^t K(t))}{\partial \int_1^t K(t)} \right\} \right. .$$

The procedure outlined so far will be sufficient if we were concerned only with the once-over-all optimal utilisation of the given amounts of resources, irrespective of their availability in the future. But every economy is concerned not only with the maximisation of the present output but also with its maximisation over time, or what may be called the maximisation of the rate of economic growth. As the latter depends on the supplies or the availability of resources in the future the procedure discussed so far is not sufficient.

IX. Consideration of the Rate of Savings

Out of the two resources, labour and capital, the first is mostly determined by exogenous factors. Moreover, most of the underdeveloped countries which need very urgently the maximisation of the rate of growth do not suffer from any shortage of labour supply. Therefore, capital is the resource whose supply can be and has to be maximised.

While introducing the consideration of the rate of savings in the choice of techniques, at least three approaches may be adopted: (1) the rate of savings should be maximised, (2) a given rate of saving should be attained in the aggregate and (3) a given rate of saving should be effected in case of each individual plant.

We know equation (5) does not give a unique solution, more