UNITED ARAB REPUBLIC

THE INSTITUTE OF NATIONAL PLANNING

التخطيط القومي

Memo No 891

ON THE MACHINE INTERFERENCE PROBLEM

BY

ALWALID ELSHAFEI
OPERATIONS RESEARCH GROUP

MAY 1969

CAIRO
3, MOHAMED MAZHAR - ZAMALEK

"Opinions Expressed and Positions Taken by Authors are Entirely Their Own and do not Necessarily Reflect the Views of the Institute of National Planning".

1. THE PROBLEM:

In one of the Job-shops, there are about 20-30 machines in operation. Due to fatigue and aging, those machines are subject to breakdown. Some repairmen are maintained on the regular payroll in order to restore the machines to operation.

THE PROBLEM is how many repairmen should be kept
THE OBJECTIVE is minimum cost
THE STRATEGIES are the various number of repairmen who might be hired
THE STATES OF NATURE are:
- The various rates at which the machines may breakdown
- The various rates at which repair takes place.

2. COST ANALYSIS

Concerning cost elements, there are two conflicting elements. One which increases as the number of repairmen increases, which is the cost of their idle time; while the other decreases as their number increases, which is the cost of idle machines due to the unavailability of repairmen for service.

As an example, suppose that we have a shop with the following characteristics:
- 20 machines are in operation
- The cost of one machine being out of operation for one hour is estimated to be £ 60.
- Repairmen capable for this job arepaid £ 7 per hour.
- The probability distribution for the breakdown (on an hourly basis) is as follows:

BREAKDOWNS PER HOUR	PROBABILITY	CUMULATIVE PROBABILITY
0	0.613	0.613
1	0.281	0.894
2	0.106	1.000

TABLE 1

In which case the mean of the distribution of machine breakdown is calculated by:

$0(0.613) + 1(0.281) + 2(0.106) = 0.493$ machines/ hour

In other words, on the average, one machine breaks down every 2.029 hours (1/0.493 = 2.029)

- The probability distribution for the repairtime is as follows:

HOURS TO REPAIR	PROBABILITY	CUMULATIVE PROBABILITY
1	0.491	0.391
2	0.217	0.708
3	0.194	0.902
4	0.062	0.964
5	0.036	1.000

TABLE II

The average repair time is calculated as follows:
1(0.491) + 2(0.217) + 3(0.194) + 4(0.062) + 5(0.036) = 1.915

3. NEED FOR OPTIMIZATION:

It might appear that there is really no problem here at all. On the basis of the available information we have been given it seems that, on the average, a machine will break down every 2.029 hours and it will take the repairman, again on the average, 1.915 hour to repair it. Thus, one repairman can easily handle the necessary repairs and remain idle for a period of, on the average) 0.114 hour between consecutive repairs. Associated with each breakdown there is a loss of 1.915 x 60 = £114.90. Considering four breakdowns during an 8-hours day, the loss will be £459.60.

This analysis, misfortunately, is too aggresive because it completely overlooks the outstanding characteristic of this kind of problem. We have based the fallacious argument of the analysis on the assumption that we can calculate the cost of the machine's being out of operation by simply multiplying the average time during which the machine remains idle by the cost of an idle hour. This is based on the assumption that there is a repairman available to start work once the breakdown takes place, but what happens if he is not. Suppose that he is busy repairing another machine. In such event the machine will have to wait, and the time is has to wait costs £ 60 per hour. We have to study the characteristics of such idle time. There is a possibility that more than one machine are idle and waiting for service. Those machines have to QUEUE UP for service forming what is called a QUEUE or a WAITING LINE.

Unfortunately, the mathematics involved in the analysis of waiting lines and waiting times is too complex and behind the scope of this article. Instead, Monte Carlo Simulation Techniques" can help us to get some insight into the problem.

4. USE OF SIMULATION :

The idea behind simulation is the use of RANDOM SAMPLING to construct a version to simulate the process being analyzed. By this means we can actually see what happens rather than having to calculate it from mathematical equations.

In the present cases we want to see what happens if we have machines that break down according to the given pattern and a single repairman who repairs the breakdowns according to another given pattern (refer to tables I&II)

4-1- STEPS OF SIMULATION PROCEDURE :

1- Decide upon the number of hours to be simulated (lenght of experiment, or sample size)
2- Calculate failures in each simulated hour as follows:
   a- Genarate a 3 - digit uniform random number, transfer it to a deviate (between 0-------1)
   b- Compare it with the numbers in the last column in Table 1, ascendingly till we arrive at a stage where the generated deviate is found to be less than or equal to the corresponding cumulative probability figure. The corresponding no.of breakdowns per hour are read as well (either 0 or 1 or 2)
3- For each failure, calculate the repair time, using 3 - digits uniform random numbers in conjunction with table II. It is obvious that hours with no failure (or 0 failure) are exculded from our analysis heretoafter.
4- Proceed with the analysis to determine : TERMINATION TIME (or time to terminate repair). If at the hour the breakdown occurs, the repairman is vacant; he can handle the error right at once. In this case the terminating time is simply the product of adding therepair time(as found in step 3) to the hour of failure.

   In another cas, the repairman might be busy repairing another machine. The machine has to wait, idle, till he finishes the one he is dealing with

---

* This procedure of "RANDOM PICKING" is well documented in Memo 842 of the NPIC.

Then the terminating time would be the repair time of this machine added to the terminating time of the one that has been repaired immediately before it.
WAITING TIME : In the previous case, one machine had to WAIT till the repairman sets another one to the working conditions. The waiting time is the result of subtracting the time of the ith failure from the time of terminating the repair of the (i-1)th failure.
IDLE TIME : If the repairman remains idle between two repairs this time is to be recorded. It is the product of subtracting the terminating time of the repair of the (i-1) th failure from the occurance of the ith failure. One important restriction is that this product is positive.
QUEUE LENGTH : Whenever a failure arrives we investigate the no. of machines waiting (or quening up) for repair.
This procedure is to be followed for each failure.
Also comparisons and accumulations are to be setup for NECESSARY STATISTICS such as :
Maximum queue length
Cumulative service time
Cumulative queue length
Cumulative waiting time (idle time for machine)
Cumulative inactive time
Machines' idleness factor
Worker's idleness factor

4-2- PROGRAMMING FOR THE IBM-1620 :

The same steps, as mentioned before, were translated into a FORTRAN II coded program. It has to be noticed that :

1- Read statements were included to allow for :
   - Setting up characteristics of Random Number Generator
   - Reading Failure Pattern
   - Reading Repair Time
2- There are three SUBROUTINES
   - SUBROUTINE to generate Random Numbers ☆
   - SUBROUTINE to calculate Queue Length
   - SUBROUTINE to calculate Necessary Statistics and Terminating Conditions.
3- There is a part of the program for the case of 2 - REPAIRMEN for further analysis and comparative analysis (as will be shown later on). The program is applicable for either case (one or two repairmen) using Sense Switch 2

☆ I apologize for not being able to include this subroutine.

4- Due to IBM-1620 limited strage capacity. The program can be used to simulate 100hours (at a maximum) as it can be seen from the DIMENSION statement. In such simulation experiments, we should be able to simulate a much longer period (in the order of 2000-3000hours). Misfortunately we are completely disabled to do this. The least we can do is show the methodology and don't rely much on the simulated values. In the near future, if I can perform a longer run on a biggar computer, I will publish resuts in a Part II of this memo.
The results of the 100 simulated hours period are also included.

4-3 ANALYSIS OF OBTAINED RESULTS :

Examining the results, we can see that :

- Cumulative waiting time (m/c's idle time) = 119 hours
- There were 45 breakdowns in 100 hours. This compares well with the mean of the distribution of machine breakdowns (as found in section 2) as found to be 0.493. In other words our sample, through very small in size, is very much representing original population.
- Cumulative service time is 81 hours to prepare 45 breakdowns On the average of 81/45 = 1.80 hour/breakdown. In section 2, the average repair time was calculated to 1.915 hour/ breakdown. This is another evidence that our sample is quite representative of the original data.
- The total waiting time, as found to be 119 hours, can be averaged as 119/100 = 1.19 hours for each of the 100hours represented in the sample. At the stated cost of £ 60 per hour for a machine out of order, this will cause £ 71.4 loss due to waiting time per hour.
  Since an additional repairman would cost only £ 7 per hour, it appears worth investigating whether an additional repairman would save more than £ 7 in waiting time . This is why I have included the part of TWO REPAIRMAN in the program. Results for the simulation experiment in this case are also shown.

5- COMPARATIVE ANALYSIS FOR TWO REPAIRMEN :

- The total waiting time in this case = 9 hours
  This represents an average waiting time of 0.09 hours /hour
  At a £ 60 cost of an idle machine per hour, the cost of idleness in this case = £ 5,4
  ∴ The additional repairman has decreased the average hourly cost of idleness by an amount = 71,4 - 5,4 = £ 66
  This decrease has been achieved at the cost of an additional repairman's salary of £ 7.
  ∴ The net saving = 66 - 7 = £ 59
  ∴ An additional repairman should be definitely added.
  This conclusion is surprising because it seems that the idle time for the case of one repairman (being 30.61%) should preclude the idea of adding another repairman. However, results from minimizing total costs show the contrary.

6- DISCUSSIONS AND CONCLUSION :

From the results previously obtained, an obvious conflict between the results obtained and the ones we should normally expect. This is due to the fact that the obvious averages are not good measures of effectiveness in a process like this.

We have proceeded in the following terms :

ON THE AVERAGE , a machine breaks down every 2.029 hours
ON THE AVERAGE , 1.915 hours are needed to prepare the machine
So : ON THE AVERAGE , the repairman will be able to repair breakdowns
So : ON THE AVERAGE , why should we need another repairman ?

The fault in this reasoning is that it does not take account of another kind of average, which is :

ON THE AVERAGE , machines will not break down in intervals nicely spread to allow the repairman to handle them all. Rather , the breakdowns will cluster in the way they have done in the simulation experiment.This clustering of breakdowns accounts for the waiting time which is not considered in the ON THE AVERAGE reasonning traced above.

List of Abbreviations,

EPS	=	Precision Factor For Comparison
NO	=	Full Length For Random Number Generator
MO	=	Characteristic For Random Number Generator
M3	=	Number of Simulated Hours
IHB(I)	=	Intervals For Failure Pattern
IBP(I)	=	Frequencies For Intervals Of Failure Pattern (Three Digits)
IDT(I)	=	Intervals For Repair Pattern
ICT(I)	=	Frequencies For Intervals Of Repair (Three Digits)
NR	=	Generated Random Uniform Number (three Digits)
IA(K)	=	Arrival Time For Event K
IRT(L1)	=	Repair Time For Event L1
TW(I2)	=	Waiting Time For Event I2
TINA(I2)	=	Inactive Time For Worker Associated With Arrival Of I2
STW	=	Cumulative Waiting Time
STINA	=	Cumulative Inactive Time
ITER(I2)	=	Terminating Time For Repair Of Failure I2
IQL(I2)	=	Queue Length When Event I2 Arrives
IQL1	=	Maximum Queue Length
ISTS	=	Cumulative Servie Time
ISQL	=	Cumulative Queue Length
PINAM	=	Machines IdlenessFactor
PINAW	=	Worker's Indleness Factor

```
*0608
      CIMENSION NRC(1CC),NRT(1CC),ENB(100),TW(100),TINA1(100)
      CIMENSIONIA(1CC),IHB(5),IBP(5),IDT(15),ICT(15),B(20)
      CIMENSION ITER(1CC),IRT(1CC),IZ(1CC),ICL(100),IX(100)
 1000 REAC11,EPS
C     READ R.N. GENERATOR CHARACTERS AND NUMBER CF ARRIVALS
      READ21,NC,MC,M3
C
C     READ FAILURE PATTERN
      READ21,MM
  500 CO5I=1,MM
    5 READ2,IHB(I),IBP(I)
C
C     READ REPAIR PATTERN
      READ21,NN
      CO10I=1,NN
   10 READ2,IDT(I),ICT(I)
C
      MM=0
      NN=0
C
C     PREPARATION OF R.N. GENERATOR DATA
      CO15I=1,NC
   15 B(I)=1
      I=0
      J=0
      K=0
   18 PRINT1
    1 FORMAT(47HSENSE SWITCH 1 CN FOR REPAIR TIME , PRESS START)
      PAUSE
C
   20 CALL RNGEN(B,NC,MC,RN)
C
      IF(SENSE SWITCH 1)105,390
C     CALCULATE FAILURE TIME
C
  390 IF(J-M3)40,45,45
   40 NRC(I)=NR
      GOTO20
   45 NRC(I)=NR
C
      K=0
      M=0
      MM=0
   50 CO72I=1,M3
      NRI=NRC(I)
      CO60J1=1,3
      J2=J1
      IF(NRI-IBP(J1))65,65,60
   60 CONTINUE
   65 ENB(I)=IHB(J2)
      IF(ENB(I)-EPS)72,72,70
   70 MM=MM+1
      IZ(MM)=I
   72 CONTINUE
      CO85IL=1,MM
      K=K+1
      L=K+1
      IK=IZ(IL)
      IF(ENB(IK)-2.)75,80,80
   75 IA(K)=IK
      M=K
      GOTO85
   80 IA(K)=IK
      IA(L)=IK
      M=L
```

```
      K=L
   85 CONTINUE
C     DETERMINATION OF REPAIRTIME
      I=C
      J=0
      GCTO18
  1C5 IF(J-M)11C,112,115
  110 NRT(I)=NR
      CO TO 2C
  112 NRT(I)=NR
  115 CO13CL1=1,M
      NRJ=NRT(L1)
      CO12C I2=1,15
      I3=I2
      IF(NRJ-ICT(I2))125,125,12C
  12C CONTINUE
  125 IRT(L1)=ICT(I3)
  130 CONTINUE
  555 PRINT3
      PAUSE
    3 FORMAT(48HSENSE SWITCH 2 CN FCR TWC REPAIRMEN, PRESS START)
      IF(SENSE SWITCH 2)444,333
C
C     CASE OF ONE REPAIRMAN
C
C     CALCULATION OF TERMINATING TIME , WAITING TIME , ICLE TIME
  333 ITER(1)=IA(1)+IRT(1)
      TW(1)=C
      TINA1(1)=IA(1)
      STINA=C
      STW=0
      STINA=STINA+TINA1(1)
      IK2=1
      CO 145 I2=2,M
      IK2=IK2+1
      C=IA(I2)-ITER(I2-1)
      IF(C)14C,135,135
  135 TW(I2)=C
      ITER(I2)=IA(I2)+IRT(I2)
      STW=STW+TW(I2)
      TINA1(I2)=C
      STINA=STINA+TINA1(I2)
      GCTO145
  140 TW(I2)=-C
      STW=STW+TW(I2)
      TINA1(I2)=C
      ITER(I2)=ITER(I2-1)+IRT(I2)
      STINA=STINA+TINA1(I2)
  145 CONTINUE
      AA=IA(M)
      PINAM=(STW/AA)*1CC.
      PINAW=(STINA/AA)*1CC.
C
C     CALCULATE QUEUE LENGTH
      CALL QULEN(IA,ITER,ICL,M)
C
C     PUNCH SIMULATION TABLE
      PUNCH4
      PUNCH6
      PUNCH7
      PUNCH8
      CO 18C I2=1,M
  180 PUNCH2,I2,IA(I2),IRT(I2),ITER(I2),TW(I2),TINA1(I2),ICL(I2)
      PUNCH8
C     END OF SIMULATION TABLE
C
```

```
C     CALCULATE NECESSARY STATISTICS
      CALL TERM(IRT,IQL,IQL1,ISTS,ISCL,M)
C
C     DOCUMENT  NECESSARY STATISTICS
      PUNCH31
      PUNCH32,IQL1
      PUNCH33,ISTS
      PUNCH34,ISCL
      PUNCH36,STINA
      PUNCH38,STW
      PUNCH39,PINAM
      PUNCH42,PINAW
      PUNCH43
C
      PAUSE
      GOTO555
C
C
C     CASE OF TWO REPAIRMEN
C
C     CALCULATION OF TERMINATING TIME , WAITING TIME
  444 STW=0
      DO200L=1,M
      ITER(L)=0
      TW(L)=0
  200 IQL(L)=0
      TW(1)=0
      TW(2)=0
      ITER(1)=IA(1)+IRT(1)
      ITER(2)=IA(2)+IRT(2)
      DO290 I2=3,M
      I1=I2-2
      L=0
      K=0
      DO215J=1,I1
      I3=I2-J
      I4=I2-J-1
      IC=ITER(I3)-ITER(I4)
      IF(IC-IRT(I3))210,205,210
  205 L=L+1
      IX(L)=I2-J+1
      GOTO215
  210 K=K+1
  215 CONTINUE
      IF(K-I1)220,250,220
  220 IF(L-2)240,225,225
  225 DO235IL=2,L
      IL1=IL-1
      X1=IX(IL)-IX(IL1)
      IF(X1-1.)235,230,235
  230 J1=I2-IX(IL)+1
      GO TO 245
  235 CONTINUE
      GO TO 245
  240 J1=I2-IX(L)+1
  245 TW(I2)=IA(I2)-ITER(J1)
      ITER(I2)=ITER(J1)+IRT(I2)
  250 D1=IA(I2)-ITER(I2-2)
      IF(D1)260,255,255
  255 TW(I2)=0
      ITER(I2)=IA(I2)+IRT(I2)
      GO TO 290
  260 D2=IA(I2)-ITER(I2-1)
      IF(D2)270,265,265
  265 TW(I2)=0
      ITER(I2)=IA(I2)+IRT(I2)
```

```
      GO TO 290
  270 IF(D1-D2)275,280,280
  275 TW(I2)=-D2
      ITER(I2)=ITER(I2-1)+IRT(I2)
      GO TO 285
  280 TW(I2)=-D1
      ITER(I2)=ITER(I2-2)+IRT(I2)
  285 STW=STW+TW(I2)
  290 CONTINUE
      AA=IA(M)
      PINAM=(STW/AA)*100.
      PINAW=(STINA/AA)*100.
      PAUSE
C
C     CALCULATE QUEUE LENGTH
      CALL QULEN(IA,ITER,IQL,M)
C
C     PUNCH SIMULATION TABLE
      PUNCH41
      PUNCH6
      PUNCH7
      PUNCH8
      DO295I2=1,M
  295 PUNCH12,I2,IA(I2),IRT(I2),ITER(I2),TW(I2),IQL(I2)
      PUNCH8
C     END OF SIMULATION TABLE
C
C     CALCULATE NECESSARY STATISTICS
      CALL TERM(IRT,IQL,IQL1,ISTS,ISQL,M)
C
C     DOCUMENT NECESSARY STATISTICS
      PUNCH31
      PUNCH32,IQL1
      PUNCH33,ISTS
      PUNCH34,ISQL
      PUNCH38,STW
      PUNCH39,PINAM
      PUNCH42,PINAW
      PUNCH43
C
      GOTO1000
    2 FORMAT(10X,4(I5,5X),2(F7.2,3X),I5)
    4 FORMAT(8X,64HRESULTS FOR THE M/C INTERFERENCE PROBLEM , CASE OF ON
     XE REPAIRMAN)
    6 FORMAT(80H========================================================
     X========================)
    7 FORMAT(12X,6HARIVAL,2X,10H ARIV TME ,10H SERV TME ,10H TERM TME ,1
     X0H WAIT TME ,10H INAC TME ,4H Q.L)
    8 FORMAT(80H--------------------------------------------------------
     X------------------------)
   21 FORMAT(10X,3I4)
   11 FORMAT(F5.3)
   12 FORMAT(10X,4(I5,5X),F7.2,13X,I5)
   22 FORMAT(10X,3(I5,5X),2(F7.2,3X))
   31 FORMAT(10X,21HNECESSARY STATISTICS,)
   32 FORMAT(20X,25HMAXIMUM QUEUE LENGTH     =,I6)
   33 FORMAT(20X,25HCUMULATIVE SERVICE TIME =,I6)
   34 FORMAT(20X,25HCUMULATIVE QUEUE LENGTH =,I6)
   36 FORMAT(20X,25HCUMULATIVE INACTIV TIME =,F9.2)
   38 FORMAT(20X,25HCUMULATIVE WAITING TIME =,F9.2)
   39 FORMAT(20X,25HM/C IDLENESS FACTOR      =,F9.2)
   42 FORMAT(20X,25HMAN IDLENESS FACTOR      =,F9.2)
   43 FORMAT(/)
   41 FORMAT(8X,64HRESULTS FOR THE M/C INTERFERENCE PROBLEM , CASE OF TW
     XO REPAIRMEN)
      END
```