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DEVELOPMENT PLANNING: THE SECTOR PHASE, WITH DIFFERENT GESTATION PERIODS .

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1. The Setting of the Problem and the Assumptions Made.

In this paper a set of formule and an example are offered for the sector phase of development planning, with special reference to the complications arising from the existance of different gestation periods in the various sectors. The paper heavily draws on what other authors in the field have already developed; its main additional attempt is to arrive at the simplest formulae conceivable without neglecting vital features. The author hopes it will be useful for practical planning purposes and mangeable for the staff usually available in planning units.

We assume that planning mainly consists of estimating the most desirable development over time of the volumes of production in a number of sectors of the economy. We conceive of the planning process as a stepwise process, starting with a macro-step. During this phase a rough idea has been obtained of the most desirable volume of investment and the most desirable rate of growth of income, the two being interrelated. The task of the sector phase is to refine this development by splitting up the economy into sectors with different capital-output ratios and different gestation periods.

Constant input-output coefficients will be assumed for the additions to production; for part of the economy these coefficients may have to be derived from project data. Contrary to the habit prevailing in many applications of the input-output method and of linear programming models we will not use as our central variables the volumes v of production of the various sectors, but their contribution y (in constant prices chosen equal to one throughout) to national income. The two variables are related by the formula

$$y^h = \varphi^{oh} v^h \qquad (1)$$

where h indicates the sector and

$$\varphi^{oh} = 1 - \sum_{h} \varphi^{h'h}$$

$\varphi^{h'h}$ being the input coefficients from sector h' into sector h. The symbols v and y indicate <u>increases over the previous year;</u> time will be indicated by a lower index; hence y_t^h stands for the increase of income from sector h in year t over year t-1. Absolute values will be indicated by a bar: $\bar{y}$.

A major instrument of analysis will be the distinction between national and international sectors. The products of the former cannot be imported or exported since their transportation costs are prohibitive. The national sectors therefore must produce the total demand for their products prevailing in the country; the international sectors do not have such an obligation, surpluses can be exported and deficits imported. International sectors are only chosen because of their income earning capacity. The most important national sectors are building, the operation of buildings, energy, inland transportation, personal services, retail and a considerable portion of wholesale trade, education of most types, government services. Of these, building represents an investment activity (we may once also consider education as such as activity); it will be indicated by $h = 1$.

Our choice of sectors and their production volume will be based on a <u>minimization of investments</u> in order to obtain a given increase in national income, as estimated in the macro phase. One of the advantages of this version of the optimization process consists of the possibility to introduce capital-output ratios which depend on the increases in production. Other advantages will become clear in the regional phase of planning, when our income target is multiple (all regional incomes may be targets) and it is simpler then to consider these as given and to have only one minimand, total investment.

We will give a special shape to the method by assuming <u>given</u> the <u>income earned in the non-building sectors.</u> We assume that the portion of investment represented by building is approximately known for the economy as a whole, If also the volume of investment is given, at least approximately, it follows that the volume of building is more or less given and hence also the difference between total and building income.

We are going to introduce the <u>gestation periods</u> by assuming that investments j_t^h needed for any sector in any year depend on the income increases enisaged for this sector in a number of successive years, starting with the year t itself. This assumption may be written as

$$j_t^h = \sum_1^{G^h} g \, x_g^h \, y_{t+g-1}^h \qquad (3)$$

Where $G^h - 1$ is the gestation period for sector h. Future investments will be discounted by multiplying them by p^g for period g.

As announced we are going to use the input-output method by assuming that production volumes are equal to final demand plus intersectoral deliveries. It will appear that we <u>only need the input-output or balance equations for the national sectors.</u> These sectors do not have export surpluses (positive or negative); their final demand is only internal (national) demand. This enables us to transform the balance equations into equations of the form:

$$y_t^{h'} = \sum_{h \neq h'} n^{h'h} y^h \qquad (4)$$

where h' indicates a national sector, h indicates all sectors and the term with $y_t^{h'}$ on the right-hand side has already been brought to the left-hand side. The coefficients n are a composite of coefficients discussed in the Appendix (section 4).

2. A Simple Example.

Before trying to formulate in a more general way the set of restrictions and the optimum problem to be solved we are going to illustrate its features by a simple example, where all the complications are showing up in the simplest from conceivable.

This example will be characterized by the following features. We have four industries; h=1 represents building, as already announced. Moreover, h - 2 is the only other national industry and h = 3 and h = 4 are two international industries. There must always be at least two international activities to choose between, since the production volume of the national sectors depend on those of the international ones and cannot be freely chosen therefore. We assume $G^1 = G^2 = G^4 = 2$ and $G^3 = 1$.

The restrictions will be:

(i) the target equations:

$$y_t^2 = y_t^2 + y_t^3 + y_t^4 \qquad (5)$$

where y_t are target values of development.

(ii) the balance (input-output) equations:

$$y_t^2 = n^{21} y_t^1 + n^{23} y_t^3 + n^{24} y_t^4 \qquad (6)$$

The restrictions enable us to express y_t^3 in terms of the other y_t^h : from them we deduce:

$$y_t^3 = \frac{y_t - n^{21} y_t^1 - (1 + n^{24}) y_t^4}{1 + n^{23}} \qquad (7)$$

Combining (6) and (7) we get:

$$y_t^2 = n^{21} y_t^1 + n^{23} \left(\frac{y_t - n^{21} y_t^1 - (1 + n^{24}) y_t^4}{1 + n^{23}} \right) + n^{24} y_t^4$$

or

$$y_t^2 = \theta^2 y_t + \theta^{21} y_t^1 + \theta^{24} y_t^4 \qquad (8)$$

where

$$\theta^2 = \frac{n^{23}}{1+n^{23}}; \; \theta^{21} = \frac{n^{21}}{1+n^{23}} \; ; \; \theta^{24} = \frac{n^{24} - n^{23}}{1+n^{23}} \qquad (9)$$

The investment activity j_t needed in any period t will be:

$$\bar{j}_t = x_1^1 y_t^1 + x_2^1 y_{t+1}^1 + x_1^2 y_t^2 + x_2^2 y_{t+1}^2 + x_1^3 y_t^3 + x_1^4 y_t^4 + x_2^4 y_{t+1}^4 \qquad (10)$$

where the term with y_{t+1}^3 is lacking since $x_2^3 = 0$.

Assuming proportionality between total investments and those taken care of by the building industry we have

$$\bar{j}_t = a\, y_t^{-1} \qquad \text{with } a > 1 \qquad (11)$$

and hence

$$a\bar{y}^1_t = x^1_1 y^1_t + x^1_2 y^1_{t+1} + x^2_1 (\theta^2 y_t + \theta^{21} y^1_t + \theta^{24} y^4_t) +$$
$$+ x^2 (\theta^2 y + \theta^{21} y^1 + \theta^{24} y^4) + x^3 \left(\frac{y_t - x^{21} y^1_t - (1+x^{24}) y^4_t}{1 + x^{23}} \right)$$
$$+ x^4_1 y^4_t + x^4_2 y^4_{t+2} \qquad (12)$$

Using our formulae (9) we may write this as

$$a\bar{y}^1_t = K^1_1 y^1_t + k^1_2 y^1_{t+1} + C_t + K^4_1 y^4_t + K^4_2 y^4_{t+1} \qquad (13)$$

where

$$K^1_1 = x^1_1 + x^2_1 \frac{n^{21} - x^3_1 n^{21}}{1+n^{23}} \qquad (14)$$

$$k^1_2 = x^1_2 + x^2_2 \frac{n^{21}}{1+n^{23}} \qquad (15)$$

$$C_t = (x^2_1 \frac{n^{24}}{1+n^{23}} + x^3_1) y_t + x^2_2 \frac{n^{23}}{1+n^{23}} y_{t+1} \qquad (16)$$

$$K^4_1 = x^2_1 \frac{n^{24} - n^{23}}{1+n^{23}} - x^3_1 \frac{1 + n^{24}}{1+n^{23}} + x^4_1 \qquad (17)$$

$$K^4_2 = x^2_2 \frac{n^{24} - n^{23}}{1+n^{23}} + x^4_2 \qquad (18)$$

Appling (13) to t = 0, 1, 2, ... successively we obtain:

$$\left.\begin{aligned}
a\bar{y}^{1}_{o} &= k^1_1 y^1_o + k^1_2 y^1_1 + C_o + K^4_1 y^4_o + K^4_2 y^4_1 \\
a(\bar{y}^1_o + y^1_1) &= K^1_1 y^1_1 + K^1_2 y^1_2 + C_1 + K^4_1 y^4_1 + K^4_2 y^4_2 \\
a(\bar{y}^1_o + y^1_1 + y^1_2) &= K^1_1 y^1_2 + K^1_2 y^1_3 + C_2 + K^4_1 y^4_2 + k^4_2 y^4_3 \\
2(\bar{y}^1_o + y^1_1 + \bar{y}^1_2 + y^1_3) &= K^1_1 y^1_3 + k^1_2 y^1_4 + C_3 + K^4_1 y^4_3 + K^4_2 y^4_4
\end{aligned}\right\} \quad (19)$$

etc.

We now propose to consider as the <u>minimand of our program</u> the expression:

$$a(\bar{y}^1_o + p\bar{y}^1_1 + p^2\bar{y}^1_2 + p^3\bar{y}^1_3 + \dots\dots\dots)$$

being all investment from period $t = 0$ on, duly discounted. From formulae we derive :

$$a\frac{\bar{y}^1_o}{1-p} + \frac{a\,p y^1_1}{1-p} + \frac{a\,p^2 y^1_2}{1-p} \dots = K^1_1 y^1_o + y^1_1(K^1_2 + pK^1_1) + py^1_2(K^1_2 + pK^1_1)$$

$$+ \dots + C_o + pC_1 + p^2C_2 + \dots + K^4_1 y^4_o +$$

$$+ (K^4_2 + pK^4_1)(y^4_1 + py^4_2 + p^2y^4_3 + \dots) \quad (20)$$

or

$$\frac{a}{1-p}(\bar{y}^1_o + pY^1) - (K^1_2 + pK^1_1)\,Y^1 = C + K^1_1 y^1_o + K^4_1 y^4_o + (K^4_2 + pK^4_1)y^4 \quad (21)$$

where we have transferred all the y^1- terms to the left-hand side and where

$$\left.\begin{array}{l} Y^1 = y_1^1 + py_2^1 + p^2y_3^1 + \ldots\ldots \\ C = C_o + pC_1 + p^2C_2 + \ldots\ldots \\ Y^4 = y_1^4 + py_2^4 + p^2y_3^4 + \ldots\ldots \end{array}\right\} \qquad (22)$$

From (21) we see that minimizing Y^1 will at the same time minimize total future investment as defind by equation (20).

The left-hand expression in (21) may be written :

$$\frac{a\bar{y}_o^1}{1-p} + \left(\frac{a\ p}{1-p} - K_2^1 - pK_1^1 \right) Y^1 \qquad (23)$$

The coefficient in front of Y_1^1 can be assumed to be positive, since 1-p is small, $a > 1$ and, as a rule K_2^1 and K_1^1 rather small too. In order to minimize (23) we must minimize or maximize Y^4 whenever the "criterion" $K^4 = K_2^4 + pK_1^4 >$ or < 0. Written in terms of the original technical coefficients the criterion runs:

$$K^4 = x_2^4 + px_1^4 - px_1^3 \frac{1+n^{24}}{1+n^{23}} + \frac{n^{24} - n^{23}}{1+n^{23}} \left(x_2^2 + px_1^2 \right) \ldots \quad (24)$$

We will first consider the case where the <u>n are constants.</u> In order to make Y^1 a minimum we must take $y_t^4 = 0$ whenever $K^4 > 0$ and y_t^4 equal to its maximum (folowing from (5) whenever $K^4 < 0$; in this case we will have $y_t^3 = 0$. It is the criterion K^4 which decides on the choice between sectors 3 and 4. For an interpretation of our result let us first assume that $n^{23} = n^{24}$ and $p = 1$. The criterion then reads:

$$K^4 = x_2^4 + x_1^4 - x_1^3 \qquad (24')$$

Since in our problem $x_2^4 + x_1^4$ represents the capital-output ratio of industry 4 and x_1^3 the one of industry 3, we have the familiar result that the sector with the lowest capital-output ratio should be chosen. (This is only so if our only target is to increase income and our only scarce factor is capital, as is implied in our setup.

For $p < 1$, or a positive time discount, we find that industry 3 is relatively more favoured than industry 4: instead of x_1^3 we get px_1^3 and instead of $x^4 = x_2^4 + x_1^4$ we get $x_2^4 + px_1^4 > px^4$. This preference for industry 3 - within limits - is due to its shorter gestation period. The criterion now reads:

$$K^4 = x_2^4 + px_1^4 - px_1^3 \qquad (24'')$$

and industry 3 will be preferred as soon as $px_1^3 < x_2^4 + px_1^4$.

Finally let us consider the most general case where $n^{24} \neq n^{23}$. In formula (24) we observe two opposite influences of, for instance a positive difference $n^{24} - n^{23}$, meaning that activity 4 requires more inputs from the national industry 2. According to the last term in (24) this will diminish the preference for industry 4. The intensity of this influence is proportional to the capital-output ratio of industry 2: $x_2^2 + px_1^2$. This is the influence dealt with in the semi-input-output method. It reflects the necessity to invest not only in the international industry chosen (4 or 3), but also in the national industries whose inputs will be needed.

There is a counteracting influence, however, to be found in the term with x_1^3. The coefficient $\frac{1+n^{24}}{1+n^{23}}$ occurrring in this term may be said to represent the substitution rate between industries 3 and 4 needed in order to meet the target y_t and can be found back in formula (7): for a unit more of y_t^4 we must give up $\frac{1+n^{24}}{1+n^{23}}$ units of y_t^3, if we want to attain the same value of y_t. This is due to the fact that this unit of y_t^4 will entail a larger necessary production of good 2 than a unit of y_t^3 needs and this production of good 2 also is considered as a contribution to the target.

Let us now consider the case where th <u>x's are not constant.</u> They may depend on the corresponding y's. The optimum problem no longer remains a linear programming problem, but becomes one of at least quadratic programming (if we assume x_2^4 or x_1^4 to be linearly dependent on y_t^4). Depending on the numerical data we may now either have a flat minimum or one on a boundary. Flat minima open up the possibility of <u>diversification,</u> excluded (as far as the international sectors are concerned) in the linear case, except by coincidence (equality of two capital-output ratios).

3. Some Remarks on Generalizations.

The simple example of section 2 may be seen as an intermediary approach where the sectors are grouped into building, other national sectors and two groups of international sectors. It is better of course to generalize it by considering a larger number of sectors. Whereas this can be done without too much trouble, it does of course complicate the calculations and increase the number of degrees of freedom.

More equations of type (6) will result of the introduction of a larger number of national sectors. A larger number of international sectors will only raise the number of terms of all equations and of the minimand (21). It remains possible to eliminate the y_t of one international industry from (5) and to eliminate all the y_t of the national sectors from (6) etc. We are then left in (21) with y's of possibly more international industries.

It stands to reason also that criterion (24) becomes more complicated if we have more than one national sector besides building. The inputs of these various national sectors and their capital-output ratios become relevant then.

So far we have assumed that all coefficients used do not change over time. As soon as we admit that possibility we will find that preferences for some industry may change into preferences for another sector.

The reader will be able, we trust, to write down the equations needed for all these more complicated situations. Because of their increasing complexity it does not make much sense to write them down without having to solve a concrete numerical case.

4. Appendix.

The usual presentation of the balance equations of the input-output method runs as follows:

$$v^h = c^h + j^h + e^h + \sum_{h'} \phi^{hh'} v^{h'} \qquad (25)$$

where v^h is the increase in volume of production, c^h the increase in consumption, j^h the increase in investment, e^h the increase in export surplus of good h and $\phi^{hh'}$ the input coefficient of good h into sector h'.

For national non-investment sectors $j^h = e^h = 0$. Consumption increase will be dependent on the increase in national product $y = \sum_h \phi^{oh} v^h$ the coefficient y^h may be the marginal propensity to consume good h, making

$$c^h = \gamma^h \sum_{h'} \varphi^{oh'} v^{h'} \tag{26}$$

substitution of (26) into (25) leads to:

$$v^h = \sum_{h'} (\varphi^{hh'} + \gamma^h \varphi^{oh'}) v^{h'} \tag{27}$$

Using now equation (1) we may switch from v's to y's:

$$\frac{y^h}{\varphi^{oh}} = \sum_{h'} (\varphi^{hh'} + \gamma^h \varphi^{oh'}) \frac{y^{h'}}{\varphi^{oh'}}$$

or

$$y^h = \sum_{h'} (\varphi^{hh'} \frac{\varphi^{oh}}{\varphi^{oh'}} + \gamma^h \varphi^{oh}) y^{h'} \tag{28}$$

In order to arrive at (4) we have to transfer the term with y^h on the right-hand side to the left-hand side:

$$y^h (1 - \varphi^{hh} - \gamma^h \varphi^{oh}) = \sum_{h' \neq h} (\varphi^{hh'} \frac{\varphi^{oh}}{\varphi^{oh'}} + \gamma^h \varphi^{oh}) y^{h'} \tag{29}$$

or, by interchanging h' and h:

$$y^{h'} = \frac{\varphi^{h'h} \frac{\varphi^{oh'}}{\varphi^{oh}} + \gamma^{h'} \varphi^{oh'}}{1 - \varphi^{h'h'} - \gamma^{h'} \varphi^{oh'}} \tag{30}$$

where h' now refers to some national sector and h to all sectors. We observe that we only need to know the $\varphi^{h'h}$ from the input-output equations for the national sectors; for the international sectors we only must know the φ^{oh} as defined in equation (2).

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