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HOW TO PLAN

In The National Panning everything has a price, and Economic Policy Making Means a Compromise among Several Desderation

by

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A c k n o w l e d g e m e n t s

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From our previous many years of contact he knows all the ins and outs of my way of thinking in matters of economic planning, and he has been of invaluable help in preparing the present memorandum.

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Cairo, February 1964.

Ragnar Frisch

C O N T E N T S

Memo 102 - 13 December 1963

To
The Ministry of National Planning
and
The Institute of National Planning
from
Professor Ragnar Frisch

H O W   T O   P L A N

In National Planning everything has its Price, and Economic Policy Making means a Compromise among several Desiderata

The purpose of this Model is to use Egyptian data to illustrate how the basic principle described in the title of this memorandum will work in the Egyptian situation. In working out this Model I will relax the two simplifying - and rather unsatisfactory - assumptions that were underlying Model 1. Cf. the concluding section 5 of Memo 101.

The present Model is designed to shed light on problems connected with the elaboration of the five year plan whose execution is to start in the year 1965/66, i.e. in the year beginning 1 July 1965. In order to preserve generality I shall consider the case where the plan may comprise any number of years. T will be used to denote the number of years in the plan.

1- The variables to be considered

As an example consider the sectorial breakdown for the Egyptian economy used by Dr. Mahmoud El Shafie, Director of His Excellency Abdel Latif El Boghdady's Technical Secretariat. It comprises the following nine sectors:-

1. Agriculture
2. Consumer goods industry
3. Intermediate goods industry

4. Investment goods industry
5. Transport and communication
6. Construction
7. Housing
8. Commercial and financial services
9. Professional and other services

In the subsequent formulae I shall, however, consider the case of any number of sectors. The letter n will be used to denote the number of sectors. The numbering of the sectors will be symbolized by h=1,2 ... n.

For each sector we will consider a corresponding investment channel, that is, a channel of investments through which the domestic capacity of production of the sector in question can be increased. For agriculture we consider two such channels, namely the horizontal and the vertical. The horizontal channel represents investments through which the area of arable land is increased, while the vertical channel represents investments that will in any other way (increase in agricultural machinery, improvement in seed and breeding stock etc.) influence the capacity of production in the sector agriculture. The investment channels will be numbered g=0,1,2.... n, in such a way that g = 0 is the horizontal channel for agriculture and g = 1 its vertical channel, while for all other sectors the channel No. and the sector No. correspond. Thus g = 2 is the channel through which the capacity of production of the sector h = 2 is increased. And similarly for the channels g = 3,4...9. The total number of channels in this model will therefore be n + 1. Out of these only the n channels g=1,2... n will be considered in an explicit way.

The model will include a great number of variables. Many of them will, however, be of such a sort that they can without difficulty be eliminated before we proceed to determining the final programming solution. In other words, it is not needed to consider these eliminated variables explicitly in the analytical procedure by which the solution is found.

A set of variables which is such that in order to proceed to the solution of the model, it is sufficient to include these variables explicitly, will be called an after-elimination set. To determine the number of degrees of freedom in the model it is obviously sufficient to count the number of variables in an after-elimination set and the number of mutually independent equations that exist between these variables (and not the eliminated ones), the set of these equations being such that if they are fulfilled (together with the equations used up during the elimination process), all the features are taken account of that one wants to describe through the model.

The choice of the variables to be included in the after-elimination set, is to some extent conventional. If one wants to eliminate a bounded variable, one must introduce corresponding bounds (lower and/or upper) for the whole expression through which the variable in question is expressed in terms of the after-elimination variables.

Tab.(1.1) describes the set of after-elimination variables to be considered in the present model.

It is not very useful to work out formally a complete list of all the supplementary variables in the model, i.e. those that are introduced in separate parts of the argument but are not retained as after-elimination variables. It is more effective from now on to handle one by one the aspects of the problem that are important from the viewpoint of the substance matter, and in each such part of the analysis to indicate how the aspect in question can be expressed in terms of the after-elimination variables specified in tab.(1.1). Doing this means eliminating whatever supplementary variables we may have found it convenient to use in the discussion of the particular parts of the model.

Tab.(1.1) The set of after-elimination var[illegible] to be considered in the present mod[illegible].

Number of variables of the kind described	Symbols	Description of the variables
n T	X_h^t	Total domestic production in sector h in year t
n T	C_h^t	Private (households) consumption of the h kind of goods in year t
n T	G_h^t	Government use of the h kind of goods in year t for current account operations (not for investment purposes)
n T	A_h^t	Net export (positive, negative or zero) of the h kind of goods in year t
n T	H_g^t	The size of investment startings in channel g in year t. (For the distinction between investment starting and investment sinking see section 4)
n T	D_g^t	Total departmental investment administration in channel g in year t
n T	U_h^t	The part of the existing capacity in sector h which is idle (unused) in the year t
T	E^t	The net foreign creditor position of the country (positive, negative or zero) at the end of the year t. (In previous memoranda denoted E^t_{cum}, where cum stands for the cumulation over time of the changes that occur in the net foreign creditor position in any given year
T	Y^t	National income in year t of the plan
T	L^t	National employment in year t of the plan (L="labour")
1	V	Cumulated (potential) income creation beyond the plan due to all decisional investment startings in the planning period
1	W	Cumulated (potential) employment creation beyond the plan due to all decisional investment startings in the planning period.
2+(7n+3) T	TOTAL	

When we proceed in this way we will automatically be led to a formulation of the final equations, that is, the equations which the after-elimination variables have to satisfy. And we will also be able to indicate how the bounds come into the picture.

The various types of data, e.g. coefficients, that are assumed in the model, and are encountered as the analysis proceeds, are listed in the tables Nos. 1-14 of the Appendix to this memorandum. These tables are intended to be not a simple list but a guide to the data suppliers for presenting the data in a systematic form. The explanations in the headings of these tables are made very explicit in order that the data supplier may have easily accessible a precise definition of what the datum in question stands for. This will, it is hoped, save him a lot of time. He will not need at each instant to read back in the main text of the memorandum.

The time scale used in this model is defined in Append. table 1, and need no special comment.

2- The input coefficients

Let X^t_{hk} be the flow of goods from the delivering sector h to the receiving sector k in year t measured in volume figures, which in practice will mean value figures reckoned at constant prices. The base year for prices in the present model is t = -1, that is 1963/64. This means X^t_{hk} for any of the years t is measured in L.E. at 1963/64 prices. Whether we want to use thousands or millions or some other power of 10 as our unit need not bother us here. This is simply a matter of scaling to be taken account of when the problem is processed for being put into the electronic computer. But it is essential to note that for all the variables which represent volume figures, i.e. L.E. figures at 1963/64 prices, the same unit must be used, say millions.

The complex of two affixes hk on X^t_{hk} may conveniently be read " from h to k".

Further let L^t_k be employment in the receiving sector k in year t, as measured by the L.E. wage-bill in sector k in year t; reckoned at 1963/64 wages. And let B_k^t be <u>complementary import</u>[1].

Therefore (using for typing convenience $ as a summation sign) the expression

(2.1) $$\$_{h=1}^{n} X_{hk}^t + L_k^t + B_k^t \qquad (k = 1,2 \dots n)$$

stands for the total volume of inputs from all sectors and from labour and complementary import into the receiving sector k in year t measured in L.E. at 1963/64 prices and wages. The summation over h in (2.1) runs over h=1,2...n.

We may or may not admit the possibility of inputs from any sector into itself. For the reasoning in the sequel it is of little consequence whether we admit this possibility or not. Since it may be convenient for practical reasons to keep this possibility open, particularly when we work with an input-output table that is obtained by <u>aggregation</u> from a larger table, we assume

(2.2) X_{hh}^t not necessarily equal to 0

The total <u>output</u> from the delivering sector h in year t, as measured in L.E. at 1963/64 prices, we denote X_h^t. This will be an after-elimination variable. Cf.tab.(1.1). To be more specific, this variable is the total <u>domestic</u> production of the h-kind of goods. There may also be another source of supply of this kind of goods, namely competitive imports, but this we will discuss later.

If each of the (n+2) input flows (2.1) into sector k is expressed as a ratio to the total output from sector k, we get the <u>input-coefficients</u>. They fall in three categories, namely

(2.3) the <u>cross-delivery input coefficients</u> $$X_{hk}^{\prime t} = \frac{X_{hk}^t}{X_k^t} \qquad \begin{matrix}(h=1,2\dots n)\\(k=1,2\dots n)\end{matrix} \quad (t = \text{any year})$$

(2.4a) the <u>labour coefficients</u> $$L_k^{\prime t} = \frac{L_k^t}{X_k^t} \qquad (k=1,2\dots n)$$

and

(2.4b) the <u>complementary import coefficients</u> (current account) $$B_k^{\prime t} = \frac{B_k^t}{X_k^t} \qquad (k = 1,2,\dots n)$$

1) Import of <u>kinds</u> of goods that are <u>not</u> produced, and will not be produced by domestic sectors in the planning period. This is in distinction to the <u>competitive</u> imports defined by (3.4). From a purely formal and accounting viewpoint, it would, of course, be possible to classify <u>all</u> imports as belonging to one of the domestic categories h=1,2...n, and hence interpret all imports as competitive. Such a procedure would, however, have very undesirable effects on the imperturbability of coefficients all over the economy. Many of these coefficients would then have to be changed

Alternatively the labour coefficients (2.4a) may be denoted X'^{t}_{Lk} where the letter L is now used as a subscript. For typographical simplicity this is done in Appendix tab.2. Similarly for B'^{t}_{k}. But this notation will not be further exploited in the present memorandum. We will stick to the notation (2.4). From this follows that a summation over h in (2.3) will contain only the affixes 1,2...n, not the affixes L and B. In other words

(2.5) $$\$^{n}_{h=1} X'^{t}_{hk} = X'^{t}_{1k} + X'^{t}_{2k} + \ldots + X'^{t}_{nk} \quad (k = \text{any sector})$$

So far the equations (2.3) and (2.4) are pure definitions and as such unquestionably true (obviously these simple definitions cannot contain any inconsistencies). But if we left the matter at that, we would not be able to make any very useful application of the coefficient concept. We must add some sort of assumption. The fundamental assumption is that the coefficients defined by (2.3) and (2.4) remain the same whatever magnitudes the variables X^{t}_{hk}, L^{t}_{k}, B^{t}_{k} and X^{t}_{k} may assume in our model.[1)] This imposes, of course, a severe restriction on the nature of our model. The model now becomes a model with constant input coefficients.

Constancy in this connection is not constancy over time. Indeed the coefficients (2.3) and (2.4) are explicitly indicated as possibly depending on the year t. What is meant is imperturbability under any sorts of variation that may occur in the model, except, of course, that the coefficients may and usually will change as we go from one delivering sector h to another, or from one receiving sector k to another, or from one year t to another. This is what we mean when we let the coefficients X'^{t}_{hk}, L'^{t}_{k} and B'^{t}_{k} depend on the affixes hk and t, and on nothing else.

We will in the sequel consistently use an apostroph to indicate coefficients that are "constant" or "imperturbable" in this sense.

Continued footnote of p.6

according to whether or not we decide to make certain investments. In other words they would not be stable under that particular kind of variations -variations in the $_{H}S_{g}$ - which we are especially interested in studying.

1) Compatible, of course, with the assumption about the constant coefficients.

In this connection I shall not go into a thorough discussion of how realistic such an assumption is in connection with $X'^{t}_{hk}, L'^{t}_{k}, B'^{t}_{k}$ but only mention that in reality the magnitudes of coefficients may depend on what year we have taken as the base year for prices.[1)]

Multiplying in (2.3) and (2.4) by X^{t}_{k} we get

(2.6) $$X^{t}_{hk} = X'^{t}_{hk} \; X^{t}_{k}$$

(2.7a) $$L^{t}_{k} = L'^{t}_{k} \; X^{t}_{k}$$ (2.7b) $$B^{t}_{k} = B'^{t}_{k} \; X^{t}_{k}$$

These three equations show that when the coefficients $X'^{t}_{hk}, L'^{t}_{k}, B'^{t}_{k}$ are assumed given, cf. Appendix tab.2, the n (n+2) T variables $X^{t}_{hk}, L^{t}_{k}, B^{t}_{k}$ can immediately be eliminated and expressed explicitly in terms of the after-elimination variables X^{t}_{k}. This is convenient/also because we are in the present model not imposing any bounds on the variables X^{t}_{hk}, L^{t}_{k} or B^{t}_{k} taken separately. Since n (n+2) T is of the second order in n, and the total number of variables in tab.(1.1), namely 2+(7n+3)T, is only of the first order in n, the saving regarding number of variables contained in the final equations, will be considerable. In the case of a large n the saving would be enormous.

3- The utilization equation for the sector product.

How is the domestic sector product X^{t}_{h} utilized ?

Any particle of the domestic sector product X^{t}_{h} must belong to one and only one of the following seven categories of utilization

I. It may be used as current account input into sectors of production. [$\sum_{k=1}^{n} X^{t}_{hk}$ = total use of h goods in year t as current account inputs into production sectors.]

II. It may be used for private/current account consumption. [C^{t}_{h} = private consumption of h-goods in year t.]

1) A rather elaborate discussion is given in my paper in the volume published in honour of Professor Harold Hotelling. Chapel Hill, U.S.A. 1960.

III. It may be used for <u>government current account consumption</u> (not investment). [G_h^t = Government current account consumption of h-goods in year t.]

IV. It may be used for (private or government) <u>infra-investments</u>. [I_h^t = amount of h-goods used in year t for infrainvestments, i.e. for investments that aim at changing the coefficients $X_{hk}'^r$, $L_k'^r$, $B_k'^r$ in some years[1] $r \gtreqless t$, or aim at changing some other coefficients.]

V. It may be used for (private or government) <u>capacity investments</u>. [J_h^t = amount of h-goods used in year t for capacity investments, i.e. investments that aim at increasing the capacity of production X_k^r in one or more of the sectors $k=1,2...n$ in some years $r \gtreqless t$.]

VI. It may be added to the country's <u>stock of goods</u>. [K_h^t = net amount of h-goods that are in year t added to the country's stock of h goods.]

VII. It may be <u>exported</u>. [A_h^t = net export of h goods in year t.]

Let us for a moment assume that there are no <u>imports</u> of h goods in the year t, and let us also for a moment assume that there is only a question of <u>adding</u> goods to the country's stock, not a question of taking goods out of stocks.

In this case the above classification must obvious lead to the following utilization equation for the domestic sector product X_h^t

(3.1) $X_h^t = \$_k X_{hk}^t + C_h^t + G_h^t + I_h^t + J_h^t + K_h^t + A_h^t$ (for h=1,2...n) (and any t)

where X_h^t is the <u>domestic</u> sector product that actually emerges in the domestic sector h in year t.

But the <u>same</u> equation with the same <u>domestic</u> interpretation of X_h^t must also apply in the general case

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1) In our model these years will be $r > t$, not $r = t$, but this is a minor point.

where h-goods may be taken out of stock and/or h-goods may be imported. This is easily seen as follows.

Let

(3.2) $$K_h^t = K_h^{in.t} - K_h^{out.t} \qquad \left(\begin{array}{l} h = 1,2 \ldots n \\ t = \text{any year} \end{array} \right)$$

and

(3.3) $$A_h^t = A_h^{exp.t} - A_h^{imp.t} \qquad \left(\begin{array}{l} h = 1,2 \ldots n \\ t = \text{any year} \end{array} \right)$$

where $K_h^{in.t}$ and $K_h^{out.t}$ are the amounts of h goods actually put into stock and actually taken out of stock respectively in year t. Both these amounts are non negative, but K_h^t, the net addition to the stock of h goods, may be positive, negative or zero.

Similarly in (3.3) $A_h^{exp.t}$ and $A_h^{imp.t}$ are the amounts of h goods[1] actually exported and imported in year t. Both these amounts must be non negative, but A_h^t, the net exports of h goods, may be positive, negative or zero. We need not exclude the possibility that re-exports occur. The existence of re-exports simply means that $A_h^{exp.t}$ will include whatever particles that constitute re-exports and $A_h^{imp.t}$ will include whatever particles that might perhaps later be re-exported.

The total availability of h goods in year t will be

(3.4) $$X_h^t + K_h^{out.t} + A_h^{imp.t}$$

The first term here represents the h-goods that are actually produced domestically in year t. The second term represents

1) $A_h^{imp.t}$ is competitive imports because it is imports of the same kind of goods that are produced in the domestic sector h. This in distinction to the complementary (non-competitive) B_k^t. Cf the text before (2.1).

what is in the year t taken from the country's stock of h-goods and the third term represents h-goods that are imported in year t.

The utilization of this total availability of h-goods in year t can be classified in the same seven categories that were mentioned above, the only difference is that category VI must now be interpreted as containing only the actual (gross) inputs into stock, and category VII only the actual (gross) exports.

We consequently have the utilization equation in its total availability form.

(3.5) $$X_h^t + K_h^{out.t} + A_h^{imp.t} = \Sigma_k X_{hk}^t + C_h^t + G_h^t + I_h^t + J_h^t + K_h^{in.t} + A_h^{exp.t} \quad (h=1,2\ldots n) \quad (t=\text{any year})$$

We only have to move the terms $K_h^{out.t} + A_h^{imp.t}$ in (3.5) over into the right member and make use of (3.2) and (3.3), to see that (3.1) applies in the general case where K_h^t stands for net input in the year t into the country's stock of h-goods and A_h^t stands for net export in the year t; while X_h^t - as always - stands for domestic production of h-goods. This domestic interpretation of X_h^t is necessary in order that it shall be realistically founded to build on the input coefficients in (2.6) and (2.7).

Inserting into the first term in the right member of (3.1) the expression for X_{hk}^t from (2.6), we get

(3.6) $$X_h^t = \Sigma_k X_{hk}^{\prime t} X_k^t + C_h^t + G_h^t + I_h^t + J_h^t + K_h^t + A_h^t \quad (h=1,2\ldots n) \quad (t=\text{any year})$$

In the sequel I shall disregard the infra investments I_h^t and the change K_h^t in the country's stock of h goods. The simplification that consists in disregarding the variable I_h^t is particularly regrettable, but I have to make it here in order to keep the model linear.