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The transportation problem on the 1620 Digital computer

By

Dr. Yousif Nasr El-Din

Eng. Abd El-Hady Nabih.

Operations Research Center

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CAIRO

3. MOHAMED MAZHAR - ZAMALEK

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The Transportation or distribution Problem

1.1. Introduction:

One of the earlist puplished accounts of a scientific method for formulating and solving distribution or transportation problems appeared in 1941. World war II subsequently saw considerable development of the techniques of linear programming and operations research. In 1951 considerable advance was given to linear programming by the cowles comission, and particularly to the solution of distribution problems by G.B. Dantzig and T.C. Koopmans.

At this stage, however, the methods were still diffcult for a non mathematician to under stand. The first really simplified approach,"the North west corner method" was presented in 1953 by W.W. Cooper and A. Charnes..

Work since 1955 by individuals such as H.W. Kuhn, M.M. Flood, p.s. Dayer has contributed to and advanced other techniques for solving problems of distribution. Dr. B.A. Galler has done some work in Solving multidimensional distribution problems, and had developed a very suitable approximate method which is superior, in terms of solution time and cost, to the more general simplex method.

The transportation method was developed to solve problem of minimizing the cost of distributing a product from a number of sources or origins to a number of destinations.

The use of transportation method is not restiricted to transportation Problems alone; it is also applied to that class of linear programming problem in which the requirements and resources are stated in terms of only one kind of unit, e.g., Product, boxcars, tracks etc.

Any transportation method invovles the following steps:

(1) Define the Problem, including the objective funtion to be optimized and the restrains imposed on the solution;

(2) Develop an intial solution;

(3) Evaluate alternatives to the existing solution;

(4) Change the solution;

(5) Repeat steps (3) and (4) untill an optimal solution is founds

1.2. Statement of the Problem:

Let us consider a manufacturing firm that owns (m) factories (sources), and (n) warehouses (destinations) in different geographical locations. We will consider a fixed period of time, and assume that output of each factory (a_i, i=1,2, ... , m) and the requirements of each warehouse (b_j, j=1,2, ... , n) are known.

The Problem in its tabular form:

First of all; Let us have the following table for the cost of transporting a unit of product from the sources to the destinations.

Table (1)

Source \ Destinations	D_1	D_2	...	D_j	...	D_n
S_1	C_{11}	C_{12}	...	C_{1j}	...	C_{1n}
S_2	C_{21}	C_{22}	...	C_{2j}	...	C_{2n}
⋮	⋮	⋮	⋮	⋮	⋮	⋮
S_i	C_{i1}	C_{i2}	...	C_{ij}	...	C_{in}
.	.	.	...	.	...	.
S_m	C_{m1}	C_{m2}	...	C_{mj}	...	C_{mn}

Where C_{ij} denotes the cost of transporting a unit of product from source i to destination j secondly following table exhibits the quantities x_{ij} to be shipped from each source (i) to any destination (j) and also the demand for each abstination b_j the supply for each source a_i

Table (2)

Distinations / Source	D_1	D_2	...	D_j	...	D_n	Supply
S_1	x_{11}	x_{12}	...	x_{1j}	...	x_{1n}	a_1
S_2	x_{21}	x_{22}	...	x_{2j}	...	x_{2n}	a_2
.	.	.	...	.	...	.	.
S_i	x_{i1}	x_{i2}	...	x_{ij}	...	x_{in}	a_i
.	.	.	...	.	...	.	.
.	.	.	...	.	...	.	.
S_m	x_{m1}	x_{m2}	...	x_{mj}	...	x_{mn}	a_m
Demand	b_1	b_2	...	b_j	...	b_n	

Where x_{ij} denotes the quantity to be shipped from source i to the distination j.

The problem is, what quanties of goods to ship and from what factories to what wherehouses at minimum total transportation cost.

The Mathematical formulation of the problem:

From tables (1) and (2) the problem is to find mxn array of non-negative numbers, x_{ij}, which minimize the objective function

$$Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

i.e. minimize the total cost

subject to

$$\sum_{j=1}^{n} x_{ij} \leqslant a_i \qquad i=1,\ldots,m$$

$$\sum_{i=1}^{m} x_{ij} \geqslant b_j \qquad j=1,\ldots,n$$

Remarks:

(1) In this model to find a solution for a problem it must be assumed that $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$

(i.e. supply equal demand)

(2) If we have $\sum_{i=1}^{m} a_i \geqslant \sum_{j=1}^{n} b_j$ to solve problem we can add a fictitions destination n+1 where $c_{i,n+1} = 0$ for all i with requirement

$$b_{n+1} = \sum_{i=1}^{m} a_i - \sum_{j=1}^{n} b_j$$

(3) If $\sum a_i \leqslant b_i$, there is no feasible solution

Example: To be more specific let us take a particular example represented in following table.

There are 3 sources and 5 destinations; transportation cost per unit of product are shown in the table. For instance, it cost 3 £.E per unit of product to ship from source 1 to destination 4.

Table -3- also introduces the total output of the sources are 100,100 and 120 units; whereas the demand in the destinations are 40,40,80,80 and 80 units.

Let us also consider a mathematical notation that c_{ij} denotes the transportation costs per unit of product to be shipped from source i to destination j.

Talbe(2-a)lists in North East corner of the cells such costs. The problem is to obtain the values of X_{ij} (i=1,2,3; j=1,2,3,4,5) of table 2 such that they:

1. Satisfy the given stipulated demand
2. minimize the total cost of so doing.

(Table 2-a)

Destinations / Source	D_1	D_2	D_3	D_4	D_5	Supply
S_1	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	100
S_2	x_{21}	x_{22}	x_{23}	x_{24}	x_{25}	100
S_3	x_{31}	x_{32}	x_{33}	x_{34}	x_{35}	120
Demand	40	40	80	80	80	320

1.5. Obtaining feasible Solution:

The first step in using the transportation technique is to exhibit a feasible solution, namely one which satisfies the requirements. (if a feasible solution also minimzes the total cost beside satisfing the requirements, it is then called an optimal feasible solution.)

The computational procedure to solve the transportation problems can be in the form of either of the following different three methods,

- The Northwest corner method;
- The Vogel's method;
- The so called U-V method. (simplex multipliers)

2. North-West Corner Algorithm

The following method of solving a transportation problem is the northwest corner, its rules may be stated as follows:

1. Start in the northwest corner of the requirements table (table 1) and compare the amount available at S_1 with the amount demanded at D_1.

   a. If $D_1 < S_1$, i.e., if the amount needed at D_1 is less than the number of units available at S_1, set X_{11} equal to D_1 and proceed to X_{12} (i.e. proceed horizontly)

   b. if $D_1 = S_1$, set X_{11} equal to D_1 and proceed to X_{22} (i.e. proceed diagonally).

   c. if $D_1 > S_1$, set X_{11} equal to S_1 and proceed to X_{21} (i.e. proceed vestically).

2. Continue in this manner, step by step, away from the north west corner untill, finally, a value is reached in the South east corner. Thus, let us return to 3 by 5 Problem shown in tables 1,2 and try to find some sort of solution to the problem, even if this will not the cheapest solution. One proceeds as follows:

   1. Set X_{11} equal to 40, namely the smaller of the amount available at S_1 (100) and that at destination D_1(40), and

   2. Proceed to X_{12} $\xrightarrow{H.}$ (rale a). Compare the number of units still available at S_1(namely 60 units) with the amount required at D_2(40) and, accordingly, set $X_{12} = 40$

3. Proceed to X_{13} (rule a) where, here there is no more than 20 units left at S_1 while 80 units are required at D_3. Thus set X_{13} = 20 and then

4. Proceed to $X_{23}\downarrow$ (rule c). Here X_{23} = 60

5. Continuing, X_{24} = 40, X_{34} = 40, and, finally, in the southeast corner X_{35} = 80.

The feasible solution obtained by this northwest corner rule is shown in table 3 - by the circled values of the X_{ij}.

That this set of values is a feasible solution is easely varified by checking the respective row and column requirements.

First feasible solution
(Totalecost of 520)

Destinations / Sources	D_1	D_2	D_3	D_4	D_5	Total Supply
S_1	(40)	(40)	(20)			100
S_2			(60)	(40)		100
S_3				(40)	(80)	120
Total Demand	40	40	80	80	80	320

(Table 3)

Remark: the circled cells are known as Basic variable cells.

The corresponding total cost of this solution is obtained by multipling each circled X_{ij} in table (3) by its correspondis c_{ij} in table 2 and summing the products. That is

$$\text{Total Cost} = \sum_{j=1}^{5} \sum_{i=1}^{3} c_{ij}\, x_{ij} = \sum_{i=1}^{3} \sum_{j=1}^{5} c_{ij}\, x_{ij}$$

The total cost associated with the first feasible solution is computed as follows

$$T.C. = x_{11}c_1 + x_{12}x_{12} + x_{13}c_{13} + x_{23}c_{23} + x_{24}c_{24} + x_{34}c_{34} + x_{35}c_{35}$$

$$= 40x2 + 40x1 + 20x2 + 60x2 + 40x1 + 40x1 + 80x2 = 520$$

4.2. Evaluation of alternatives:-

Once we have a feasible solution, the next problem is that ofdeciding whether or not the solution we have found is optimum.

In other words, is there is any other program that, when we put into operation, will result in a decrease in the total cost? and if there the question we pose is how to change this allocation of transportation so that the total shipping cost will decrease?

To answer the first question we must evaluate the alternatives available to us. In this case the alternatives are the cells that are not presently used, i.e. having uncirded variables.

It is apparent that any unused cell will not be considered for inclusion in a new solution if the change in cost is positive. If a cell is evaluated favourally so that the change in cost is negative, the cell will be considered for inclusion in a new solution.