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THE TWO - LEVEL PLANNING MODEL

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Introduction

Economic planning is a task that can be approached by different method and idea. As an example by the use of input-output table. But such input-output tables are not suited for optimiziation and is merely designed to achieve correct proportions between sectors.
That mean that optimization methods are used only for project analysis but not for the whole national economy some proposals and trouls were made in the direction of optimization to optimize the whole economy such as those done by Ragnar Frish,Johonson Rodalf and others, but those trails were complicated to be applied and solved, due to the large compution needed for such approaches. Now we have a big computers and highly information systems.

The solution of optimization problem can now by done without no diffcults.
The planning processe can be dealt as a single linear programming problem. called (main problem or center problem).
This is the global problem or the overwhole problem of national economy.
After that (the problem of formulating the whole econommy) one must move to other level of which is the sectoral level of industry.

The sectors of production thus are connected to the center by some sort of constraints. Thus it can be said that planning as an optimization task is done on two-levels, which are the center level (center model) and the sector level(sector model) i.e. two types of model are needed for two-level planning process. This paper will deal then with two-level planning models.

Two-level Planning Model(General):

The two-level planning model provides some organizational principles concering the division of functions between the center and sectors, and with that the flow of information between the two levels.

The optumum allocation of resource can be achieved only if the information sent up by sectors is frank and objective, because this will effect the objective function and the constraints of both levels.

The two-level planning model contain elements of competition, such as the home production and import are compting with one anther, also there is competition between direct satisfaction of domestic demand by production and its indirect satisfaction by imports paid for by export. Also the center of the two-level planning put and study the critera to reallocate the resources.

At each level, all units will have to construct a mathematical programming model embarcing their activities and constraint prescribed. In the two-level planning models, the results and information output of calculation carried out with one model provides the basic data, the information input for one or more of the other models. These informations will be as

final output obligations-material quatas-resource quatas-shadow pricesplan estimates-etc. On the basis of information, recived by the unit will continuously correct the individual parameters of their models and carry out improvment of their plan.

Level I.

The General Model:

This model gives certain defenitions and provides a basis for the economic application of the idea of two-level planning. In that level the planning board formulate the planning proposal for the plan targets and figures for the sector due to the general information about the sectors. The center due to its information about the sectors begin to distribute its avialable resources (material, manpower, etc) among sectors as a frist run, meanwhile demand from the sectors some of output target are required from the sectors.

The sectors begins after reciving the proposal from the center to propose changes to the center due to their ability. Accordingly and on theat base the center planning board begins to modifies its original targets and again sends this new proposals to the sectors.

Now to formulate the center model and the sector model, a briet elaboration about primal and dual linear programming problems, since the two-level planning will be derived us linear programming.

In such treatment one must take into account that the original linear programming problem (center in formation problem) which is decomposed into sectorial problems which is related to the center.

The mathematical formulation of the overall problem is given by:

Given a vector X which represents a primal variable for the center program then the primal and dual problem will be.

primal problem	dual problem
$C'X$ --- max	$y'b$ --- mim
subject to	
$AX \leqslant b$	$y'A \leqslant C$....(I)
$X \geqslant 0$	$y \geqslant 0$

The dual problem will be the center shadow price system.

An optimal solution for such system will be

$$\max C'X = \min y'b = C'X^* = y^{*\prime}b =$$

where $x^* \in X$, $y^* \in Y$

which means that a feasible solution of the primal and dual exist. Now taking the above illustration in mind the center model will be formulated as follows

$$\sum_{\substack{j=1 \\ j \neq i}}^{n} Z_{jit} + d_{it} = V_{it} \leqq V_{it} \qquad (1)$$

$$\sum_{i=1}^{n} W_{it} = W_t \qquad (2)$$

$$V_{it} \geqslant 0$$

$$Z_{jit} \geqslant 0$$

$$W_{it} \geqslant 0$$

For $i=1,\ldots n,$; $t=1,\ldots,T$

where.

V_{it} = is what to be produced by the sector i in the interval t (supply task).

Z_{ijt} = is the quantity to be produced from the sector i of the project j in the interval to (material quota).

W_{it} = is the available manpower for the sector i in the internal t (manpower quota).

d_{it} = the consumption from product i in the time interval t.

all the above information is given to the sector i by the center.

Level II

The primal sector model:

First we deal with the general linear programming problem(I).

If $A=\left[A_1, A_2,\ldots A_n\right]$

i.e. the Matrix A in the general model is divided to sub-matrices

$(A_1, A_2,\ldots A_n)$

then problem (I) will be

Primal problem

$C_1'X_1 + C_2'X_2 + \ldots\ldots + C_n'X_n \ldots\ldots$ max !

$A_1X_1 + \ldots\ldots + AnX_n \leqslant b$

$X_1 \geqslant 0$

$X_2 \geqslant 0$

$X_n \geqslant 0$

dual problem

$y'\,b$ - - - min !

$y'A \geqslant C_1'$

$y'A_n \geqslant 0$

$y \geqslant 0$

Now let a vector u be used such that

$$u_1 + u_2 + \ldots\ldots u_n = b$$

i.e. u have the some size as the constraint vector b

it follows that

$$U = \left[u_1', \ldots, u_n' \right]$$

u will be centeral program and u_i will be the sector componant. i.e. the problem can be formulated.

The primal problem

$C_i'\,X_1$ --- max !

$A_iX_i \leqslant u_i$

$X_i \geqslant 0$

and $i = 1,2,\ldots\ldots,n$

The dual problem

$y_i'\,u_i$ --- min !

$y_i'\,A_i \geqslant C_i'$

$y_i \geqslant 0$

Mathematical Formulation of the Sector Model:-

In order to get a formulation of this model we frist define the following:

X_{ikt} = denote the k $\underline{th}$ output of the sector i during the period t (t = 1,....,T).

X_{ik} = the volume of investment activity in the i^{th} sector

X_{ikt} = the k^{th} export activity of the i^{th} product in the t^{th} period ,
k = exp (t = 1,...,T)

X_{ikt} = the volum of the k^{th} bounded import activity, importing the i^{th} product in the t^{th} period , k = impo (t = 1,...,T).

The above variables are given according to their economic nature The formulation of the primal problem will be.

Under the following set of constraints

The frist set of contrants

$$V_{it} \leq \sum_{k \neq imp} f_{ikt} X_{ikt} + \sum_{k=inv} f_{ikt} X_{ikt} \leq V_{it}$$

k = pro,ex (t=1,...T)
imp,o

where

where

f_{ikt} = is the output cofficient

$$= \begin{cases} = 1 & \text{for production} \\ \geqslant 0 & \text{for investment} \\ = -1 & \text{for export} \\ = \ 1 & \text{for import} \end{cases}$$

k in the frist part is for production ; for export, for import only, and k in the second part for investment the above set of constraint are of the type which shows that the sector complies with the plan figuers received from the center.

The second set of constraints are

$$\sum_{\substack{k=all \\ k \neq inv}} g_{ijkt}\, x_{ikt} + \sum_{k=inv} g_{ijkt}\, x_{ikt} \leq Z_{ik}$$

$$j = 1,\dots,n \quad , \quad j \neq i \quad , \quad t = 1,\dots,T$$

where

g_{ijkt} = material input coefficient

$$= \begin{cases} \geqslant 0 & \text{for production and also for investment} \\ = 0 & \text{for foreign trad activities} \end{cases}$$

The thrid set of constraints

$$\sum_{\substack{k = all \\ k \neq inv}} h_{ikt}\, x_{ikt} + \sum_{k=inv} h_{ikt}\, x_{ik} \qquad W_{it}$$

where

h_{ikt} = labour force coefficient

$$= \begin{cases} >0 & \text{for production} \\ \geq 0 & \text{for investment} \\ = 0 & \text{for foreign trade} \end{cases}$$

those above three types of constraints are related directly to the center.

Also some types of constraints can be set up for special sectors which have certain characteristic. These types of constraints may be written in the general form.

$$\sum_{t=1}^{T} \sum_{\substack{k = all \\ k \neq inv}} a^{0}_{ilkt}\, x_{ikt} + \sum a^{0}_{ilkt}\, x_{ik} \leq b^{0}_{il}$$

where

b^{0}_{il} is the upper limit of production in the sector

The objective function of level two will be

$$\sum_{t=1}^{T} \sum_{\substack{k=all,o \\ k\neq inv}} S_{ikt}\, x_{ikt} + \sum_{k=inv} S_{ik}\, X_{ik} \ldots \quad \max !$$

where S_{ikt} , S_{ik} are the fore gn currency return.

It is also assumed that

$$\max_{k=exp} S_{ikt} \leq \min_{t=1,\ldots T} (-S_{ikt})$$

Here it is clear that the maximization of sector objective function will leads to the maximization of the objective function on the national scale.

<u>The dual sector model:</u>

The dual problem of the above formulation of the sector model is as follow and the set of constraints

$$f_{ikt}\,(x_{it} - \nu_{it}) + \sum_{\substack{j=1 \\ j=i}}^{n} g_{ijkt}\, \lambda_{ijt} + h_{ikt}\, W_{it}$$

$$+ \sum_{i=spec} a^{o}_{ilkt}\, \sigma_{ie} \geq S_{ikt}$$

where

where

k = production, export, import, 0

$t = 1,\ldots,T$

$$f_{ikt}(x_{it} - \lambda_{it}) + \sum_{\substack{j=1 \\ j\neq i}}^{n} g_{ijkt} \quad {}_{ijt} + h_{ikt} W_{it}$$

$$+ \sum_{l=spe} a^{0}_{iekt} \quad \sigma_{i\ell} \geq S_{ik}$$

k = investment only, $t = 1,\ldots T$.

and

$\nu_{it} \geq 0$ ($\lambda_{ijt} \geq 0$ ($W_{it} \geq 0$

, $x_{it} \geq 0$, $\sigma_{i\ell} \geq 0$

$j = 1,\ldots n$, $j \neq i$, $t = 1,\ldots T$

ℓ = spec.

The dual objective function is

$$\sum_{t=1}^{T} (V_{it}\,\gamma_{it} - \nu_{it}\, v_{it}) + \sum_{\substack{j=1 \\ j\neq i}}^{n} Z_{ijt} \quad \lambda_{ik} + W_{it}\, W_{it})$$

$$+ \sum_{l=spec} b^{0}_{i\ell}\, \sigma_{i\ell} \; \ldots\ldots \; \min \, !$$

In the above dual sector model ν_{it}, is the shadow price for the supply, task v_{it} , λ_{ijt} is the shadow price for material quota Z_{ijt} and also w_{it} is that of manpower quota. Let γ_{it} to gives the shadow price of the upper boundary V_{it} of the i^{th} supply task and $\sigma_{i\ell}$ that of the boundary $b^{o}_{i\ell}$ in the ℓ^{th} special costrant.

Economic Interpretion And Calculation Process:

The aim of this study is to give an idea of the two-level planning models and how it can be used for planning process. The programms of sectors are not our subject here and it can be left for a detailed study. Were will be an illustration of the main relations of the two-level planning model and the iteration process to get the optimal solution.

The calculation process consist of a number of phases, these numbers are not mor than four phases, each phase consist of a number of steps. The phase mainly concentrate on a given objective function. The calculations are done through an iterative procedure which depend on upper and lower optima. In our problem the value of the objective function will certainly fall between the upper and lower optima in any step of iteration.

Now let U^*_{n-1} , L^*_{n-1} be the uper and lower optima in the $|n-1|$ iteration ant let δ be any small incremental value. Now-the four phases are as follows.

Phase I.

This phase is a centeral level phase (i.e. the calculation in done in the center of plan.) and as a test for calculations is that.