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THE TRIM - LOSSREDUCTIONPROBLEM

IN

INDUSTRY

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CHAPTER - I

I - 1 Introduction

The process of finding a best way is called "optimisation". Most industrial projects involve the optimization of a system, such as minimizing production costs, or minimizing the cost of achieving certain technical properties for some engineering entity or operation. On the other hand, optimization may be maximizing profits or capacity of a flow of goods or informations through a network, ...etc.

One of the most applicable industrial projects, and of primary significance to a variety of industries is the so-named "Trim Loss" or "Cutting stock" problem. This problem is concerned with cutting rolls of paper, textiles, metallic foils, cellophane, or other materials into a desired number of subparts such that the amount of "waste" is minimized.*

Nowadays, large general purpose computers used to solve problems of many factories , businesses , universities, ...etc.

In our research, after formulating the problem, we will expose the different approaches and techniques to solve it. Finally, a description for a personal computer program designed and written in order to put the problem in an executable way. The research could be applied in paper mill industries as well as in metallic and textile socities of small or large scales.

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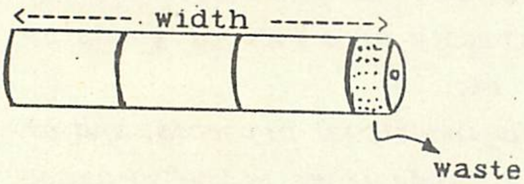
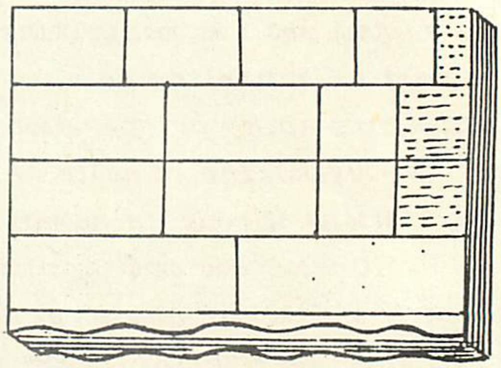
* Waste is defined in our problem, as any left over portions of a jumbo reel which cannot be used to meet demand.

I - 2 Technique of cutting process

In textile, metallic, wooden and paper mill industries, the raw material take the form of rolls or jumbo reels or slices of certain width, (fig. 1).





(fig. 1)

The company receives orders to cut, some of these reels or slices, to a set of sub reels of various width, suitable for the manufacturing necessities or the customer's demands. A number of cutting machines are at the disposal of the company, and their knives can be set for any combination of widths in which the total combined does not exceedthe overall roll width.

In fitting the list of orders to the available rolls and machines, it is generally found that trimming losses are unavoidable ; this wasted material represents a total loss, which may be somewhat alleviated by selling it as scrap. The firm wishes to determine how to meet these orders so as to minimize total waste.

As a start, the firm has to determine the different patterns which can be used in slitting a jumbo reel, and note the attendant loss. It will only consider patterns which yield waste less than the smallest required width.

I - 3 Formulation of the problem

Before formulating the problem, suppose the following example .

Example :

A paper mill produces paper in reels of standard width 60 inches, the required minimum number of rolls or slices for each width ,the have been received as follows :

Width in inches	28	20	15
Length ordered	30	60	48

To begin with, we determine the following possible partitions of 60 inches in to the required width..

Width \ Combination	1	2	3	4	5	6	7	Minimum no of rolls..
28	2	1	1	0	0	0	0	30
20	0	1	0	3	2	1	0	60
15	0	0	2	0	1	2	4	48
T R I M	4	12	2	0	5	10	0	

The requirement for a length of 30 and width 28 will be satisfied if :

$$2 X1 + X2 + X3 \geq 30$$

where Xi, denote the length of standard reel that will be processed according to combination i .

Also for the other lengths and widths :

$$X2 + 3 X4 + 2 X5 + X6 \geq 60$$

$$2 X3 + X5 2 X6 + 4 X7 \geq 48$$

The total trim loss to be minimized is ,

$$4 X1 + 12 X2 + 2 X3 + 5 X5 + 10 X6 + 28 X8 + 20 X9 + 15 X10 \quad (1)$$

where X8 , X9 and X10 are slack variables .

However, it is simpler to look at the problem from another point of view. The lengths of the reels for the various combinations are X1, X2, ..., X7 and therefore the total area actually cut will be 60 * (X1 +X2 + ... +X7).

The total area of the paper ordered is :

$$28 * 30 + 20 * 60 + 15 * 48 .$$

The difference between the two is the trim, and the latter will be minimized if :

$$X1 + X2 + ... + X7 \text{ is minimized.}$$

Equation (1) with the above constraints represent a linear programming problem which could be easily solved to obtain :

$$X1 = X4 = X7 = 1 \text{ and}$$

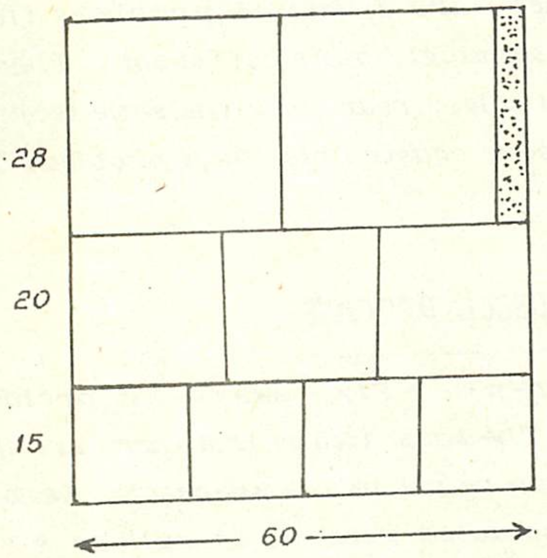
$$X2 = X3 = X5 = X6 = 0.$$

This means, that the combinations which will give minimum trim loss are :

Combination number 1 :	2 x 28
Combination number 4 :	3 x 20
Combination number 7 :	4 x 15

Since the ordered length of width 28 inches is 30, then we should have to cut 30 / 2 = 15 large rolls. Similarly, we should have to cut 60 / 3 = 20 large rolls to satisfy the ordered length with combination number 4, and to cut 48 / 4 = 12 large rolls for the last ordered length, with a total loss trim equal to 60 .

(fig.2)



It could be happened that, there are more than one optimal solution (multiple solutions). For example :

2 * 28 & 3 * 20 & 4 * 15 gives the same total loss trim.

So, the problem could be stated as :

Given quantities of goods (rolls or slices) of different shapes to be cut from a material that comes in various sizes, a number of possible efficient patterns are considered. How much of each pattern should be cut ? The number of each pattern cut represent the decision variables. The constraints are given by the required quantities and by the amount of material of each size available. The objective may be to minimize the cost of the material used or the amount of waste produced.

I - 4 Some coherent problems

Although, our problem "the trim loss " was tackled in some other references under the name of " Cutting stock problem*" but, there are a host of problems that fit this structure, some are seemingly quite different . These problems could be treated and solved nearly by the same technique, after adding or deleting some constraints .Such problems like the so-called :

I - 4 - a: The knapsack problem

It is also known as " Fly - away - kit problem**", or / and loading problem. The basic idea is that there are N different types of items that can be put in to a knapsack. Each item has a certain weight associated with it, as well as a value. The problem is to determine how many units of each item to place in the knapsack in order to maximize the total value. The problem could be formulated as follow :

(*) " Principles of operations research for management" Budnick, Mojena, Vollmann - 1977 IRWIN.

(**) " Introduction to operations research techniques" H.G.Dacllenbach & J.A.George & D.C.Mc Nickle-1983 ALLYN&BACON

$$\text{Maximize } Z = \sum_{i=1}^{N} X_i \cdot V_i .$$

subject to :

$$\sum_{i=1}^{N} X_i \cdot w_i \leqslant W .$$

$$X_i = \begin{cases} 0 \\ 1 \end{cases} \quad \text{for } i = 1,2,\dots, N ;$$

where V_i , w_i are the value and weight of item i .

I - 4 - b: The allocation problem :

This type of problems is interest in covering an area of vital concern - as possible as we can by a limited personnel (fire fighters, police, ambulance drivers, ... etc.), supported by a suitable equipment (fire trucks, patrol cars, ambulances or rescue trucks, ... etc.).

The problem is to deploy these resources in order to achieve as close as possible the objectives of the emergency response function. The problem could be formulated as follow :
For any vertex i, only the set N_i of vertices within T (maximum response time) of i can provide an acceptable emergency service to i;

$N_i = \{ j; d_{ji} \leqslant T , i \subset X$ and j is a possible node for a facility location $\}$.

The decision variables x_j are defined as :

$$x_j = \begin{cases} 0 & \text{if no facility is established at vertex } j. \\ 1 & \text{if a facility is established at vertex } j \text{ for all possible facility location vertices } j. \end{cases}$$

Also :

$$\sum_{j \in N_i} x_j \geq 1 \text{ for } i = 1,2, \ldots ,n.$$

(n : number of vertices which have to be served);

and the objective Z is to minimize the total number of facility locations used, i.e :

$$\min : Z = \sum_{j=1}^{m} x_j$$

(m : the total number of possible facility locations).

Chapter - II

Some obstacles in solving the trim loss problem :

To put the problem in a practical use and application, we have to overcome two major problems :

II - 1 : Fractional solutions

It is clear from the formulation of the problem that, the trim problem is one in which the decision variables xi must be integers . In applying linear programming techniques, we cannot guarantee an integer optimal solution. Also, the approach of rounding the linear programming solution to the nearest integer solution is not a good strategy. For example, the following linear programming problem :

$$\text{Max } 2x_1 + 3x_2$$

such that

$$130x_1 + 182x_2 \leq 910,$$
$$4x_1 + 40x_2 \leq 140,$$
$$x_1 \leq 4$$
$$\text{and } x_1, x_2 \geq 0;$$

has an optimal solution at $x_1 = 2.44$ & $x_2 = 3.26$.

If this solution is rounded to :

$$x_1 = 2 \text{ and } x_2 = 3,$$

the objective function will have the value of 13 ; whereas the optimal integer solution is at : $x_1 = 4$ and $x_2 = 2$, with an objective function equal to 14 . For this reason, it is essential to put the linear integer programming techniques in to consideration, to solve this kind of problems .

Unfortunately, there is no single method, such as the simplex method - in linear programming problems - that has been accepted as the "only" method for solving all types of integer linear programming models . However, all of the known methods in integer linear programming are primarily based on one of the following four approaches :

i) Some type of enumeration. ii) Cutting plane .

iii) Bender's decomposition. vi) Group theory .

We will apply in this research two computer programs for the first two approaches, if the linear programming technique fails in giving us the desired integer solution .

II - 2 : Generating the cutting patterns

Each setting of the cutting blades yields a set of smaller rolls, and the problem then becomes the determination of how the blades should be positioned and, for a given setting of the blades, how many large rolls should be cut. Thus, the first thing to be done is to determine which setting of the blades yields rolls that can be used. Each distinct setting is a variable of the problem and the value of each variable represents how many rolls of standard width should be cut at the corresponding setting of the blades.

Returning to the table of the different combinations in (I - 3). After arranging the required widths in descending order, we notice that, for every combination there is a cutting - pattern vector :

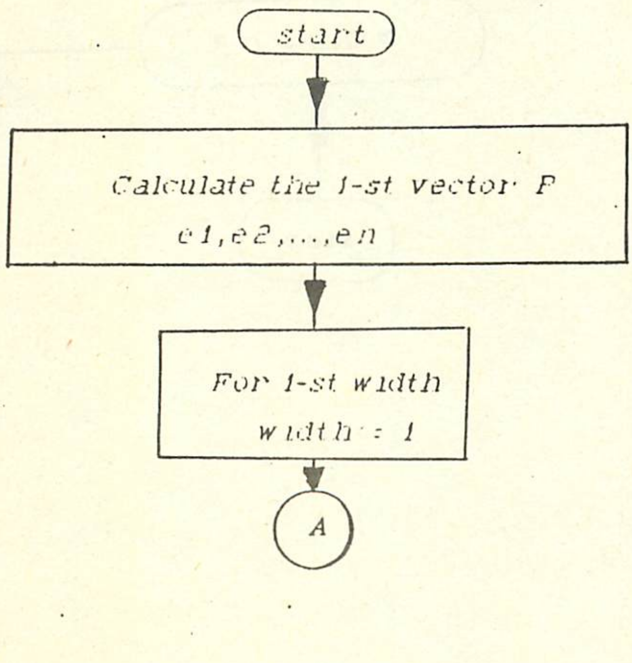
$$P = \begin{bmatrix} e1 \\ e2 \\ e3 \end{bmatrix} \text{ satisfies the condition :}$$

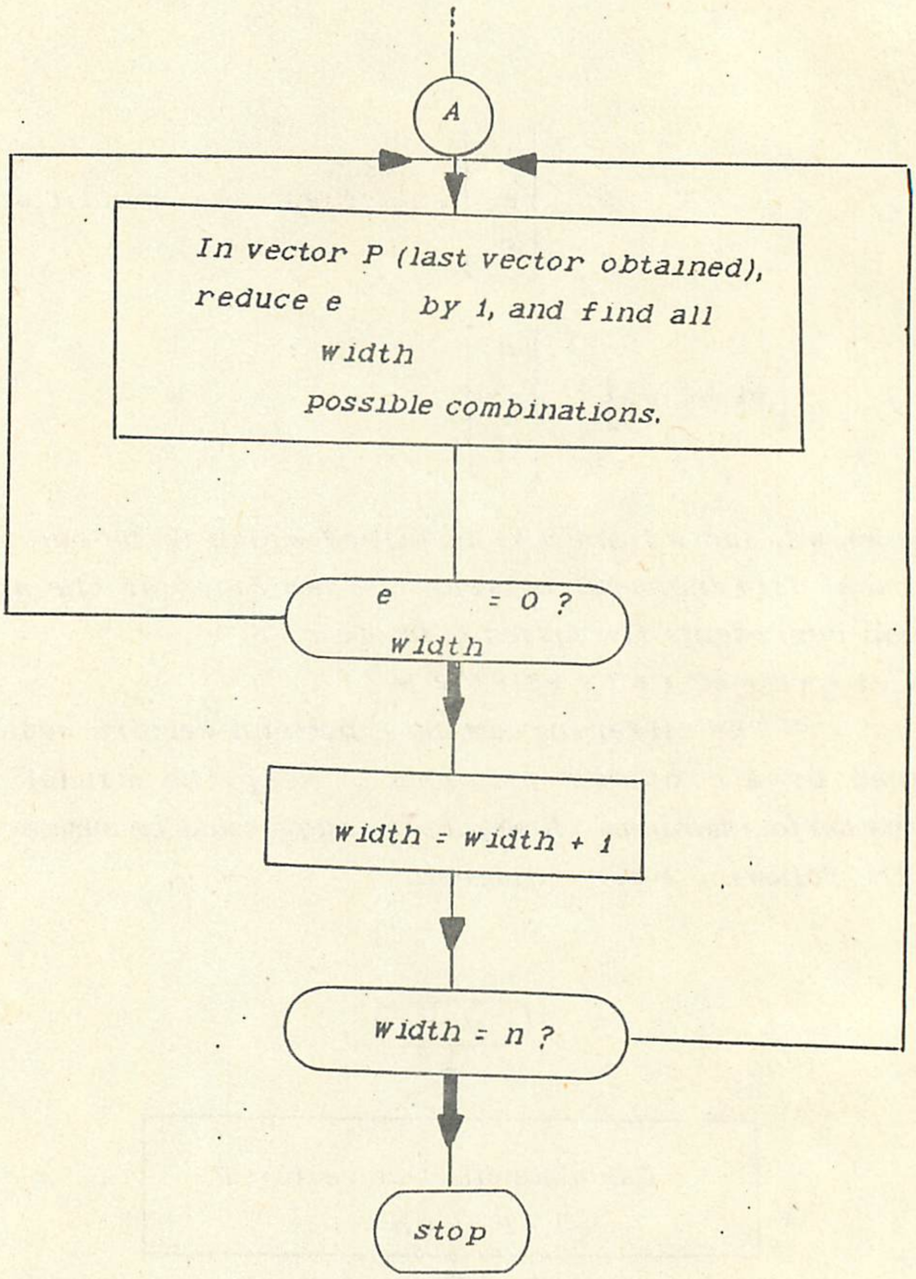
$$\begin{bmatrix} w1 & w2 & w3 \end{bmatrix} \begin{bmatrix} e1 \\ e2 \\ e3 \end{bmatrix} \leq W$$

where w1, w2, and w3 are the required width to be cut. The numerical difference C between the two sides of the above equation represents the lossed trim, i.e ;

$$w1 * e1 + w2 * e2 + w3 * e3 + C = W$$

The different cutting - pattern vectors could be obtained by a computer program to keep the manual data preparation to a minimum. However, the steps could be summarized as in the following block - diagram.




A
In vector P (last vector obtained),
reduce e
width
by 1, and find all
possible combinations.
e
width
= 0 ?
width = width + 1
width = n ?
stop