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Computer Package Programs for Mathematical programming techniques to solve linear programming models

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Preface

Planning, industry, and business firms and their operations continue to increase in size and complexity. Accordingly, planners and managers must turn to the new tools and techniques to cope with the many critical decisions which must be made.

Mathematical programming is one of the newer scientific techniques to planning and managerial decision making. The ability to make long term plans and the cost reduction are often the objective of the mathematical programming objectives.

For this purpose, we present here a computer package programs for many mathematical programming techniques used for solving Lp Models.

A brief discription of the different techniques have been presented. The computer programs have been written in BASIC language and tested on the HP-9830A calculator. Throughout the programs, many remark statements were written to describe in details the model parameters and the comming steps of the algorithms procedures.

The flow charts are introduced to assist the user to code the computer programs for his model in any computer language he wishes.

The first section introduces the Original simplex algorithm that will be used for solving a Lp models in extended form (OSA/E). In these extended forms, the simplex table includes an identity matrix, the columns of the initial basic variables.

Section 2 presents the original simplex algorithm but in compact form since we note that the columns of unit vectors included in the previous algorithm offer no significant information other than to designate the initial basic-variables and it will be convenient to omit these columns and place instead the subscripts of the basic variables in only one column to the left of the simplex table (OSA/C).

The third section introduces the revised simplex algorithm(RSA) although it may appear to be unjustified since the OSA is much more simple from the theoritical point of view and numerically both algorithms appear to be identical but this algorithm is very useful specially in the economic activities.

Section 4 presents the dual simplex algorithm in extended form (DSA/E) to help the user to solve his model without using artificial variables. It is very useful for example in games and strategies used by players to find their optimal strategies and game values.

The dual simplex algorithm in compact form (DSA/C) has been introduced in section five.

By a combination of the OSA and the DSA, artifical variables may be avoided completely which reduces the size of the simplex table and makes a marked reduction in the number of iterations necessary to optimize the objective function of the model. For this purpose, the primal-dual algorithm in compact form (PDA/C) has been included in the last section.

Section: 1

The Original Simplex Algorithm for Solving Lp Models in Extended form (OSA/Extended):

Introduction:

In developing the simplex algorithm, G.Dantzig made use of the classical Gauss-Jordan elimination method which is familier to anyone solved a system of linear equations. The key idea is to take a multiple of one equation and add it to or subtract it form another equation in order to eliminate one of the unknowns from the second equation hoping to change the original system to an equivalent one easier to solve for the remaining unknowns.

The simplex algorithm is an efficient mehtod which is routinely used to solve the Lp models on today's computers.

Outline of the Simplex Algorithm:

Our Lp model has the following general standard form:

$$\text{Max}^{*}. \quad Z = \sum_{j=1}^{n} c_j x_j$$

S.to:

$$\sum_{j=1}^{n} a_{ij} x_j \leq b_i, \; i=1,2,\ldots,m$$

$$\& \; b_i > 0$$

$$\& \quad x_j > 0$$

ii, After converting this model to equality form, the initial simplex table takes the form:

eq.no:	Basic Variables	Coeffs of $x_1 \; x_2 \; \ldots \; x_n \; x_{n+1} \ldots \; x_{n+m}$	r.h.s.
o	z	$-c_1 \; -c_2 \ldots \; -c_n \; 0 \; 0 \ldots \; 0$	0
1	x_{n+1}	$a_{11} \; a_{12} \ldots a_{1n} \; 1 \; 0 \ldots \; 0$	b_1
2	x_{n+2}	$a_{21} \; a_{22} \ldots a_{2n} \; 0 \; 1 \ldots \; 0$	b_2
.	.	. .	.
.	.	. .	.
.	.	. .	.
m	x_{n+m}	$a_{mi} \; a_{m2} \ldots a_{mn} \; 0 \; 0 \ldots \; 1$	b_m

iii, now the simplex algorithm consists of the following 3 steps:

I- <u>The Initialization Step(Iteration no.o):</u>

It starts at a corner-point feasible solution (origin). This is equivalent to selecting the original variables (x_1, x_2, x_n) to be the initial non-basic variables (equal to zero), and the slack variables $(x_{n+1}, x_{n+2}, \ldots x_{n+m})$ to be the initial Basic variables (equal to r.h.s).

II. <u>The Stopping Rule:</u>

The algorithm stops when the current corner-point feasible solution is better than all its adjacent corner point feasible solutions. In this case the solution is optimal. The current basic feasible solution is optimal if and only if every coefficient in eq o is non-negative. If it is, stop; otherwise, go to the following iterative step to obtain a better basic feasible solution which involves changing one of the non-basic variable with one of the basic variables and vice-versa and then solving for the new solution.

III- The Iterative Step (Successive Iterations):

Move to a better adjacent corner-point feasible solution. This involves replacing one Non-basic variable (called the entering basic variable (EBV) by one of the old basic variables (called the leaving basic variable (LBV). This can be done through the following 3 parts.

1. Determine the EBV: Select the variable (automatically a Non-basic variable) with the greatest-ve coeff., in eq.0. Let it be X_{J1}; J1 refers to the column below this coeff. and called the Pivot-column.

2. Determine the LBV: The leaving basic variable X_{I1} can be determined such that:

$$\frac{b'_{I1}}{a'_{I1,J1}} = \min_{i} \left(\frac{b'_{i}}{a'_{i,J1}} \right) \quad a'_{i,J1}>0$$

I1 refers to the pivot-row no.,

$a'_{I1,J1}$ is called the pivot-element.

3. Determine the New basic feasible solution by constructing a new simplex table as follows:

i - New pivot-row = $\frac{\text{Old Pivot row}}{\text{Pivot element}}$

ii- Any other row = Old row - "Pivot-Column coeff" X new pivot-raw

; "pivot-column coeff" is the element in this row that is in the pivot-column.

(note that the pivot column except the pivot element became zero after this pivoting operation)

iii- The subscript of the leaving basic variable is replaced by the subscript of the Entering basic variable

Illustrative Example:

(This ex is due to F.S.Hillier (1))

Max. $= 3x_1 + 5x_2$

S.to: $x_1 \leq 4$

$2x_2 \leq 12$

$3x_1 + 2x_2 \leq 18$

&

$x_1, x_2 \geq 0$

The Equality form of this ex is:

max. Z

S.To:

$$Z - 3x_1 - 5x_2 = 0$$

$$x_1 + x_3 = 4$$

$$2x_2 + x_4 = 12$$

$$3x_1 + 2x_2 + x_5 = 18$$

&

$$x_j \geq 0 \quad \forall j = 1,2,3,4,5$$

The Initial Simplex Tableau is:

eq.no:	Basic Variables	Coeffs of					r.h.s.
		x_1	x_2	x_3	x_4	x_5	
0	Z	-3	-5	0	0	0	0
1	X_3	1	0	1	0	0	4
2	X_4	0	2	0	1	0	12
3	X_5	3	2	0	0	1	18

II. The Initialization Step (Iteration No. O)

From the above table, the Initial basic feasible solution is:

(0,0,4,12,18) & Z = 0

Now, go to the stopping rule to determine if this solution is optimal or not?

II. The Stopping Rule:

The ex has 2 -ve coeffs in eq o, -3 for x_1 & -5 for x_2, which means that this current solution is not optimal; so go to the iterative step.

III. The Iterative Step:

1. To determine the EBV; the largest -ve coeff in eq o is -5 for x_2, so x_2 is the EBV and J1, the pivot-column no., is equal 2.

2. To determine the LBV; it can be seen from the table that the LBV is x_4 which is associated with the minimum ratio indicated in the table, and there fore I1 is equal 2 also.

3. The new basic feasible solution can be determined by constructing a new simplex table as follows:

i,

$$\text{the new pivot-row} = \frac{\text{the old pivot-now}}{\text{the pivot-element}}$$

so, the table for this ex at this point has the appearance shown:

eq.no.	Basic Variable	Coeffs of x_1	x_2	x_3	x_4	x_5	r.h.s.
0	Z	-3	-5	0	0	0	0
1	X_3	1	0	1	0	0	4
2	X_4	0	2	0	1	0	12
3	X_5	3	2	0	0	1	18
0	Z						
1	X_3						
2	X_2	0	1	0	½	0	6 ←
3	X_5						

ii. To eliminate the new basic variable from the other equations, every row in the table, except the pivot-row, is changed for the new table by using the formula:-

new row = old row-"Pivot-Column coeff" X new pivot row,

for ex, row o becomes:

$$\begin{array}{lcccccc} & (-3 & -5 & 0 & 0 & 0 & 0\ ) \\ -(-5) & (\ 0 & 1 & 0 & \frac{1}{2} & 0 & 6\ ) \\ = & (-3 & 0 & 0 & \frac{5}{2} & 0 & 30) \end{array}$$

and so on for the remaining rows, i.e, the new simplex table becomes:

Iteration no:	eq.no.	Basic Variable	Coeffs of x_1	x_2	x_3	x_4	x_5	r.h.s.	
0	0	Z	-3	-5	0	0	0	0	
	1	X_3	1	0	1	0	0	4	
	2	X_4	0	2	0	1	0	12	
	3	X_5	3	2	0	0	1	18	
1	0	Z	(-3)	0	0	$\frac{5}{2}$	0	30	
	1	X_3	1	0	1	0	0	4	$\frac{4}{1}=4$
	2	X_2	0	1	0	$\frac{1}{2}$	0	6	
	3	X_5	3	0	0	1	1	6	$\frac{6}{3}=2$ ←

The solution at this iteration is:

(0,6,4,0,6) & Z = 30

Now, we have to go to the stopping rule to check whether this solution is optimal or not?

The stopping rule: Since the new eq.0 still has a -ve ceeff (-3 for x_1), this solution is not optimal and we return to the Iterative step to determine the next basic feasible solution.

Following the instructions of the Iterative step, we find that x_1 is the EBV and X_5 is the LBV. Then, the new simplex table for this iteration becomes:

Itteration	eq.no.	Basic Variables	Coeffs of x_1	x_2	x_3	x_4	x_5	r.h.s.
	0	Z	-3	-5	0	0	0	0
0	1	X_3	1	0	1	0	0	4
	2	X_4	0	2	0	1	0	12
	3	X_5	3	2	0	0	1	18
1	0	Z	-3	0	0	$\frac{5}{2}$	0	30
1	1	X_3	1	0	1	0	0	4
	2	X_2	0	1	0	$\frac{1}{2}$	0	6
	3	X_5	3	0	0	-1	1	6
	0	Z	0	0	0	$\frac{3}{2}$	1	36
2	1	X_3	0	0	1	$\frac{1}{3}$	$-\frac{1}{3}$	2
	2	X_2	0	1	0	$\frac{1}{2}$	0	6
	3	X_1	1	0	0	$-\frac{1}{3}$	$\frac{1}{3}$	2

The new basic feasible solution at this iteration is:

(2, 6, 2, 0, 0) & Z = 36

Going to the stopping rule, we find that the solution is optimal because none of the coefficients in eq o is -ve, so the algorithm is finished and the optimal solution for the problem is:

$$\left.\begin{array}{l} X_1 = 2 \\ X_2 = 6 \end{array}\right\} \text{the ordinary variables}$$

& $Z = 36$.