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AN INTEGRATED FRAME - WORK FOR EXPERIMENTAL INVESTIGATION BY SIMULATION MODELS

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AN INTEGRATED FRAME-WORK FOR EXPERIMENTAL INVESTIGATION BY SIMULATION MODELS

By

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This paper is addressed mainly to researcher who wishs to conduct simulation experiments on models representing management and economic systems. Our main ambition is to show him how can controled experiments be achieved through discrete event simulation and to make him aware of the important statistical aspects of this technique.

Once a particular model is build and its computer program is prepared, the main task will be to manipulate computer runs in a way to get the desired information about the behavior of the simulated system. In the present research, we develop an integrated, framework for investigating simulation models and analyze the following three main strategic and tactic problems;

i - How each of test runs is to be executed and how to estimate simulation run length?

ii - How to design an experiment in order to explore the underlying mechanism governing the behavior of the simulated system?

iii - How to select an experimental plan in order to find the optimum operating conditions of the simulated system.

1. Introduction

The use of computer simulation technique to conduct artificial experiments on numerical models of complex systems, is an increasingly important tool in many desciplines today. Computer simulation offers many features that make it an attractive experimental method for studying management and economic systems.

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Some examples of such features are, the ability to test and evaluate new systems in advance, the ability to identify and control the source of variation in the experiment, etc. [16,29].

These advantages have encouraged operations researchers and statisticiens to improve its practice through the use of different statistical techniques to design and analyze simulation experiments [16,18,21, 28,29,30,34]. The results of these studies demonstrate the need to consider two problems. First, the special circumstances of simulation that lead to misinterpretation of results and then misunderstanding the simulated systems. Second; the difficulty to achieve the assumptions of the statistical theory, as independence and homogeneity of variances. So either we manipulate simulation runs to match these assumptions, or we hope that the selected techniques are not affected by their violation.

The purpose of this paper is to develop an integrated framework for investigating, management systems by simulation technique and to find satisfactory solutions to problems that we might face when experimenting simulation models.

We suppose that simulation experiment is conducted in order to achieve two objectives; i) Investigating the relatiohship of simular response to input specifications in order to determine the underlying mechanism governing the simulated process; ii) Finding the levels of input specifications at which simular response is optimized.

As any statistical investigations, we begin by selecting a sampling plan which specify, how each of test runs is to be executed, and how to determine simulation run length. The second phase, is to design an experiment that will yield the desired information. Finally, a data analysis technique is to be choosen in order to reach some conclusion about the simulated system.

In section 2, the mathematical base of simulation experiment is presented, the simular response function is defined, and different experimental designs are formulated. A detailed discussion of the steps needed for investigating simulation models, apears in the remaining sections.

2. The Mathematical Model

In many simulation models the process of interest appears as a stochastic process[(1)], $\{Y(t), -\infty \leq t \leq \infty\}$. Considering discrete event digital simulations, we assume that during an interval Δt the process shows

(1) We will consider only the stochastic simulation models as most management or economic systems inevitably appear random to some degree in nature.

little, if any, change so that observing $Y(t)$ at periodic interval Δt result in no loss of information. For convenience, let Δt be unity, then

$$(1) \qquad Y_t \equiv Y(t)$$

so that the sequence $\{Y_t;\ t=0,1,2,\ldots,\infty\}$ corresponds to the Process $\{Y(t)\}$ at all integer values of the index t.[1]

In order to study several processes of interest, generated by different environmental conditions or input specifications, we would like to aquire a quantitative characterization of each of them. The mean of the process serves generally as the mathematical discriptor. Let $\{Y_t;\ t\varepsilon n\}$ be a time series of length n observed during the simulation run, the mean of the process "u" can be estimated by:

$$(2) \qquad \bar{Y} = n^{-1} \sum_{t=1}^{n} Y_t$$

where $\bar{Y}$ is called "simular response".

Since the stochastic features are spawned in the simulation model by incorporating the random number seed as an integral part of input specifications, the response $\bar{Y}$ becomes a random variable, because it is a transformation not only of the environmental conditions "$x_1, x_2, \ldots, x_p$", but also of the randomly selected seed "r".

(1) The index may be the time, for example Y_t may define the number of jobs in a production system. It may simply denote order; for example Y_t may represent the waiting time for the t^{th} job to receive service.

This relation is defined as:

$$(3) \quad \overline{Y} = \phi(x_1, x_2, \ldots, x_p; r) = \phi(\vec{x}, r).$$

Then for each permissible specification of environmental conditions $\vec{x}$, the set of all possible responses, (which arise from the selection of different random number seeds), might form a probability density function for simular response $\overline{Y}$.(1)

Consequently, the aim of the experimenter will be to estimate the moments of this distribution. Specifically, expected simular response "u" and variance of simular response var($\overline{Y}$), can help him in explaining the particular nature of the simular density function.

Then, regardless of the experimental objectives, we should define a procedure for estimating the mean and the variance of simular response; i.e to select a sampling plan. Once a method for their estimation is selected, we can proceed to the study of $\overline{Y}$ as a function of the p environmental conditions.

The environmental conditions or experimental factors are categorized as qualitatives and quantitatives.(2) Although the random number seed "r" consists of real numbers, it could not be classified as

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(1) A detailed discussion of this point can be found in Mihram [34] pp 261-267.

(2) Examples of qualitative factors are policy specification, or discrete environmental conditions. Quantitative factors are examplified by input parameters that can usually be thought as continious variates.

quantitative factor because $\overline{Y}$ will probably not be continious function of it. The random number seed is then unique among quantitative factors, and relation (3) can be written:

$$(4) \qquad \overline{Y} = \phi(x_1, x_2, \ldots, x_p) + \varepsilon(r)$$

where $\varepsilon(r)$ is a random effect dependent upon the random number seed r. Further, if we assume that $\varepsilon(r)$ is independent of the factors $(x_1, x_2, \ldots, x_p)$ and that $E\{\varepsilon(r)\}=0$, the expected simular response can be defined:

$$(5) \qquad E(\overline{Y}) = \phi(x_1, x_2, \ldots, x_p) = \phi(\vec{x}).$$

It is the nature of the unknown function $\phi(\vec{x})$, termed <u>simular response function</u>, that we try to investigate by simulation experiment.

In practical simulation situations, any attempt to develop the exact form of $\phi(\vec{x})$ could not be justified from economical point of view. In addition, for many experimental purposes, it is unnecessary to consider the form of the true function, a flexiable graduating function, for example a polynomial, will often be satisfactory to express the relationship between $E(\overline{Y})$ and the "p" factors. Further more, many experimental strategies proceed by dividing the whole operability region of factors space, into a number of smaller regions of immediate interest. Withen these regions of interest, the experimenter may feel it is reasonable to represent the response function by a known functional form, although he may know that such representation would be quite inadequate over the whole operability region.

As a result of the previous discussion, the simular response function may be approximated by

$$(6) \quad E(\bar{Y}) \simeq f(x_1, x_2, \ldots, x_p; \theta_1, \theta_2, \ldots, \theta_\ell) = f(\vec{x}, \vec{\theta})$$

where f is a known functional form indexed by some unknown vector $\vec{\theta}$.
The way by which we investigate the function $f(\vec{x}, \vec{\theta})$, in order to yield information about simulated system, depends on the experimental objectives. Accordingly we destinguish between two types of experiments, exploratory and optimization.

2.1. Exploratory Experiments

If the experimenter wish to study the relative importance of the factors $\vec{x}$ as they affect the expected simular response, he may select one of the following experimental designs.(1)

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(1) In most designs, the constraint of experimental budget is considered either by fixing the number of experimental points or by selecting the plan that reduce this number as possible.

i) Screening designs

At the begining of investigation, specially with complicated simulation models, the experimenter may face the problem of so many factors. It may happen that not all the p factors are important but only a few, say p' factors. Therefore we may screen for these factors.

ii) Designs for Estimating Parameters

When experimenter has a prior knowledge about the simulated system due to theoritical backgrund or from previous investigations. He may assume that a particular functional form $f(\vec{x},\vec{\theta})$ is a good approximation to the true response function $\phi(\vec{x})$ in such a way that bias due to inadequacy of $f(\vec{x},\vec{\theta})$ to represent $\phi(\vec{x})$ can be neglected. In such case, his goal will be to select an experimental plan to estimate the unknown parameters $\vec{\theta}$ so that the variances of the estimators are minimized.

iii) Designs for Exploring Response Surface.(1)

When knowledge about simulated system is limited, the object is to approximate, withen a given region of the factors space, the function $\phi(\vec{x})$ by some graduating function $f(\vec{x},\vec{\theta})$ which most closely represent the true simular response function. The criteria of closeness is measured by the variance error caused by sampling

(1) These designs treat only the case of quantitative factors.

variation and bias error resulting from inadequacy of $f(\vec{x},\vec{\theta})$ to exactly represent $\phi(\vec{x})$.

2.2. Optimization Experiments

The purpose of this type of experiments is to find the combination of factor levels at which the simular response function $\phi(\vec{x})$ is optimized. Researchers of management science face frequently this experiment, The maximization of profit or the minimization of cost is a common objective in management studies.
To conclude, any attempt to develop an experimental method for investigating management systems by simulation, necessitate the choice of a sampling plan which defines an efficient procedure for estimating the variance of simular response. The estimated variance measures the accuracy of results and then can be used to determine the appropriate run length. Having accomplished this task, an experimental strategy may be defined for investigating the inter-dependence between the simular response and the experimental factors.

The rest of this paper will be devoted to the detailed discussion of the previous statistical aspects of simulation experiment.

3. The Stochastic Sequence generated by Simulation

At the begining of investigation, the study of the stochastic sequence, $\{Y_t, t=1,2,...\}$, generated by simulation,is important for the understanding of the process under study, and the reduction of the experimental effort needed in the next steps. The following three characteristics may provide the required information.

Stationarity. A sequence is said to be strictly stationary if every series, $\{Y_s, Y_{s+1}, ..., Y_{s+n}\}$, for $s=1,2,...,\infty$; will have the same probability density function. A wide sense stationary sequence, which is less restrective, will have the mean:

$$(7) \qquad E(Y_t) = \mu < \infty$$

and the autocovariance function

$$(8) \qquad R_s = E\left[(Y_t-\mu)(Y_{t+s}-\mu)\right] \quad , \quad s=0,1,2,...$$

The importance that the sequence, generated by simulation, be a stationary one is explained by the fact that its autocovariance function R_s depends only on one variable. Moreover, the spectral density function can be represented as the fourier transformation of the autocorrelation function [16]. These two facts are of immense assistance to facilitating the analysis of the sequence.

The existence of a trend in the generated squence will cause non-stationarity. In case of simulation, we can eliminate such trend, either by using an elimination technique [34], or simply by the clever choice of simular response. If we cannot avoid non-stationarity, replicating simulation runs will be recommended in order to generate uncorrelated observations and then to avoid the problems associated with the estimation of R_s.

Autocovariance Function. This function gives the experimenter an initial guess about the independence between events and then the degree of congestion of the simulated system. Since high congested systems need longer run lengths to liberate results from the imposed initial conditions, the choice of a starting policy(1), and the determination of sample size benefit from knowledge about autocovariance function. This function is also used in estimating the precision of simular response $\overline{Y}$ in case of autocorrelated observations.

Spectral density function. This function represents another measure of dependence between observations in the stochastic sequence. It is defined as:

$$(9) \qquad A_\omega = \pi^{-1} R_0^{-1} \sum_{\tau=-\infty}^{\infty} R_s \cos \omega s; \quad \omega \varepsilon \{0,\pi\}$$

(1) See section 5.1.

The estimate of A_w reveals the prominent periodicities in the generated time series. In simulation experiment the periodic components may apear as a consequence of building in the experiment rules that contribute an element of regularity recurring behavior to the sequence of interest. The existence of periodicity is un desired because it adds unnecessary variation to the sequence and create statistical problem when estimating R_s [16]. The formulas for estimating R_s, A_w can be found in references [16,34]. For their theoritical development see [22,36].

4. Termination rules in simulation

When conducting simulation experiments on models representing real systems, two situations can be faced:

i) Simulation run can be prolonged indifinitely. In that case we can increase sample size either by continuing the run or by replicating it. In either cases a stopping rule is needed to end simulation experiment. This situation is designated "non-terminating systems". Many simulation models behave as non terminating systems, for example, Jobshop, inventory or queueing models.

ii)- Simulation run ends with the occurance of a particular event. In that case the only way to increase sample size is replicating simulation runs. This situation is designated "terminating systems". It can take one of the following forms [30]: