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Production Planning Models And Linear

Programming

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Contents

1- Introduction:

In the practical application of operations research plays the lenear programming methods the main roll in solwing many applied problems. This is hased on thier parctical simpel applications and also on thier simpel solution methods.

In economic field we can see that models can be formulated as linear programming problems due to liner structure and due to the non-negativity of the variables included in the Model.

The operations research process is considered from different byt intervelated perspectives.

From these per-pectives or components are: phasés, strategies and Factors. Each of these components consists of elements which are outlined. The relations between elements and between components are examined.

Even since the publication of the maximum principle the number of applications to economics problem has been steadly growing.

In this work we deal with the applicaltion of linear programming problem that can be formulated from production field.

Production fumctions form the basis of a preciese planning and control of costs.

In most accounting systems linear input output functions are supposed. Especially are presumed constant production coefficients and the possilsilty to allocate exactly at least variable costs to product units.

In this work it is analysed how for these presume correspond to modern production and cost theory. The influnce of multivariable and the ambiguous input-outp t relations in production processes and complex production structures to planning of costs is examissed.

The existence of several variables means that the differentiation according to production volume, normally used in cost paint of view. Therefore it is of on important to analysis the cost factor with it different types in order to achieve a more precise planning and control of costs.

In the following work we try to show how the input combination and different levels of costs affects productions planning.

At each section there is a mathematical linear programming model to allocates costs in it different types.

Also from the analysis of the characteristics of production conclusions are drawn for the planning of production processes taking in consideration the patterns of costs.

2- General describtion of production Models:

a- A linear programming problem is given as objective function under certain constraints, Model of linear programming are met in economic production planning. The traditional applied exampel is that economic Model of optimal production programm planning. In a production unit that produces X_j products, with m short term limited production factors V_i' (i=1,2, ... m), (j=1, ... n) The production is given due to leontief production function.

Now if a_{oj} represents the profit per unit of production and let a_{ij} represent the input of the factor i to produce the product j. The problem is then formulated as follows.

The total profit will be as follows

$$\sum_{j=1}^{n} a_{oj} \; x_j = \quad .... \quad \max \qquad (1)$$

Under the constraints

$$\sum_{j=1}^{n} a_{ij} \; x_j \lesssim a_{io} \quad \text{For all } (i=1,2, \ldots m) \; (2)$$

$$x_j \geqslant o$$

For this problem (1), (2), (3) there are many algorithms to get asolution of linear programming problems.

In applied field such problem is transformed to simple Integer programming, where the poducts x_j must be integer i.e the production sum x_j only integer is as (x_j=0 (mod1)) or in other formulation when the rest capicity must be integer.

The linear programming problem is transformed to

$$\sum_{j=1}^{n} a_{ij} \; x_j + \bar{x}_j = a_{,o} \quad (i=1,2, \ldots m) \quad (2')$$

by introducing the slack variabl $\bar{x}_i$ into (2) $\bar{x}_i$ represents the non-used capacity of the m factors V_i.

In case of integer value of the rest capacity there will be integer condition only for the variables $\bar{x}_j$

i.e $\bar{x}_j = 0$

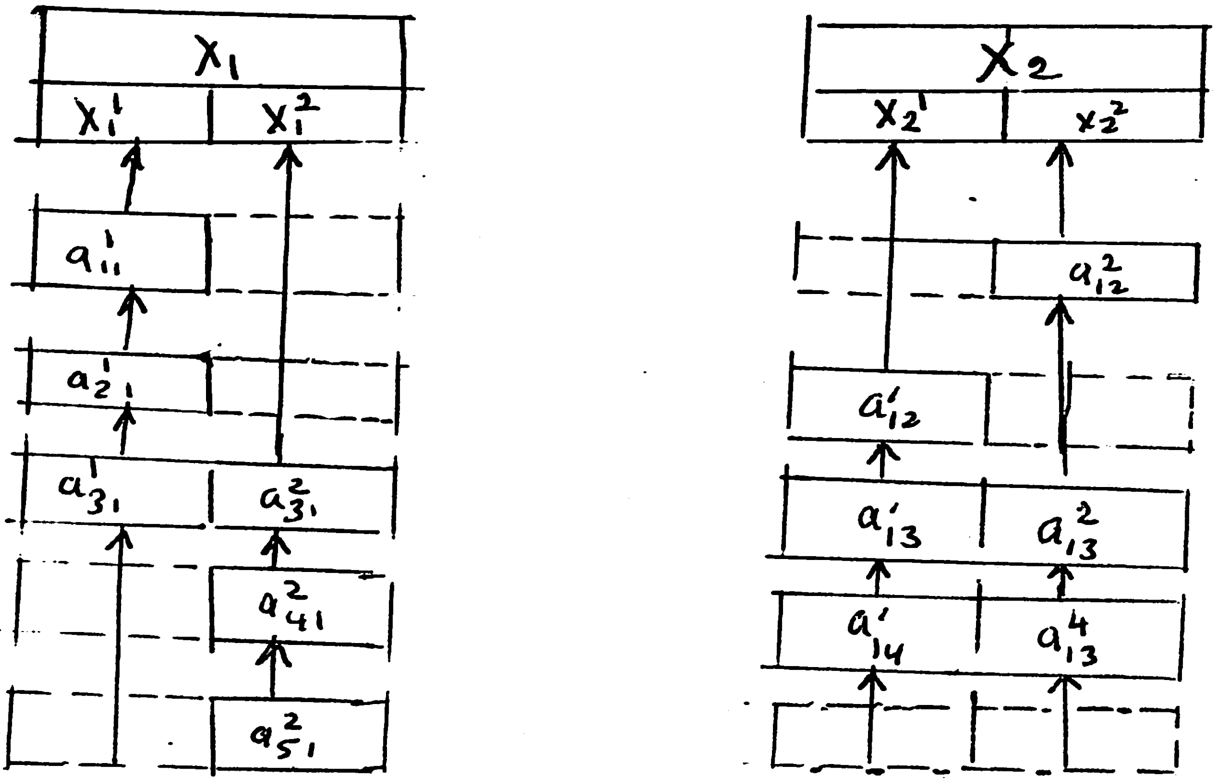
In the frist case (where the main variables are integers) The problem is called a real integer case. i.e for x_j with integer values of a_{ij} (i=1, ... m ; j=1,2, ... m) as also for x_i, but if for certain values of variables takes integer. Values then the problem is mixed - integer programming.

2-2 <u>Production Planning model with (all) or (ether-or) decesion</u>:

For production planning programm we mean heir that the production of one unit of product x_j (j=1,2, ... m) with the factors V_i, (i=1,2, ... m) and intensity a_{ij}.

Such Modeles of production have known and fixed production run. Such problems have itself solutions. The solution of these problems is given therough the answer of the question, how and with which combination of unites of products the production is produced.

A production unit is produced by different arts of mashines i.e for exampel any product as x_1 can be produced with V_1, V_2, V_3 or V_3, V_4, V_5 wile x_2 can be produced with V_2, V_3, V_4 or V_1, V_3, V_4, which means that for the product x_1, the factor product V_3 is neccessary while V_1 and V_2 can be replaced by V_4 and V_5 -; and for the product x_2, V_2 can be replaced by V_1



Fig(1)

Fig (1) shows that for x_1 there are two alternatives x_1^1 and x_2^2 auording to, the process of production, V_1, V_2, V_3 or $\underset{1}{V_3}$, V_4, V_5 also for x_2 either through V_2, V_3, V_4 which produce V_2 or V_1, V_3, V_4 which produce x_2^2. The production factors needs production coeffients a_{ij}^k (k=1,2)

These production coefficient have the following properties

$$a_{31}^1 = a_{31}^2 \quad ; \quad a_{32}^1 = a_{32}^2 \quad ; \quad a_{41}^1 = a_{42}^2 .$$

Which helps in solving the problem through seperation of variables.

Since the production means are known then the problem can be formulated as follows

$$\sum_{i=1}^{n} a_{oj} \; x_j = \eta$$

or

$$\sum_{j,k} a_{oj}^{k} \; x_j^{k} = \eta \qquad \begin{matrix}(j=1,2, \ldots n;. \\ k=1,2, \ldots p)\end{matrix} \qquad (8)$$

(number of avialable substituation)

Under the constraints

$$\sum_{j,k} a_{ij}^{k} \; x_j^{k} \leqslant a_{io} \qquad (9)$$

$$x_j^{k} \geqslant o \qquad (10)$$

For the number of product x_j

$$\sum_{k} x_j^{k} = x_j \qquad (11)$$

Problem (8), (9), (10), (11) is the some as problem (1), (2), (3) but the last problem show that, the production of the product x_j can be through the process x_j^1 as also through the process x_j, .. x_j^p produced.

i.e x_j can be produced through different process at the same time.

The other case of production is to produce x_j by either x_j^1 or x_j^2 or ... x_j^p i.e in Fig (1) either the production is x_1^1 or x_1^2 both are the same in the some planning period

The constraints in this case will be as follows

$$x_j^r \geqslant 0 \qquad (j=1,2, \dots n,\ k=1,\ \dots,\ r-1, r+1, \dots p)$$

$$x_j^k = 0$$

this means that one variable must equal to zero. but in the case of p=2 we can write (*)

$$x_j^1 . x_j^2 = 0$$

in case of $p > 2$ we get the following set of constrants.

$$x_j^1 \ x_j^2,\ x_j^1 \ x_j^3,\ \dots\ x_j^1 \ x_j^{p-1},\ x_j^1 \ x_j^p = 0$$

$$x_j^2 \ x_j^3,\ \dots\dots \qquad x_j^2 \ x_j^p = 0$$

$$x_j^{p-2} \ x_j^{p-1},\ x_j^{p-2} \ x_j^p = 0 \qquad (12)$$

$$x_j^{p-1} \ x_j^p = 0$$

If for exampel x_j^1 0 then form the frist row of constraint in (12) we see that all other alternatives processer equal to zero.

The same is in case of $x_j^r > 0$ this gives that all processes x_j^{k+r} equal to zero (from the $r^{\underline{th}}$ row)

The non-linear coustraints of (12) which have the value 0 or 1 are row transformed to a linear form.

The frist constraint in (12) is

$$x_1^1 \quad x_1^2 = 0 \qquad (12)$$

Now let us introduce as a new variable with the the values 0 or 1 then (12') can be written in the form

$$x_1^1 \leq M\,\delta \qquad (13a)$$

$$x_1^2 \leq M\,(1-\delta) \qquad (13b)$$

$$\delta \leq 1 \qquad (13c)$$

$$x_1^1,\; x_1^2,\; \delta \geq 0 \qquad (13d)$$

$$\delta = 0 \qquad (13e)\ \text{modl}$$

i.e greater than or equal to zero (13d)

smaller than or equal to 1 (13c)

and integer (13e), it can take the value 0 or 1 only

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* See: Dantzig, G.B: on the significeuce of solving linear programing problem with sowe integer variables Econometrica 1960.

If δ equal to 1, that means x_1^1 equal to or smaller than M and x_1^2 must be smaller than or equal to zero (13b) and greater than or equal to zero (13d), which mean that it still for x_1^2 the value zero only.

In case of $\delta = 0$ then x_1^2 smaller or equal to M and x_1^1 can have the value zero.

The number M is a constant and it must be at least of great value, in order that x_1^1, x_1^2 not to be strong bounded.

For each of relation (12) can also have system of the following bounds as (13a) _____ (13e)

$$x_j^1 - M\delta_1 \leq 0$$

$$x_j^2 + M\delta_1 \leq M, \; x_j^3 + M\delta_1 \leq M; \; \ldots, \; x_j^{p-1} + M\delta_1 \leq M$$

$$; \; x_j^p + M\delta_1 \leq M$$

$$x_j^2 + M\delta_1 \leq 0$$

$$x_j^3 + M\delta_2 \leq M; \; \ldots. \; x_j^{p-1} + M\delta_2 \leq M, \; x_j^p + M\delta_2 \leq M$$

$$x_j^{p-2} - M\delta_{p-2} \leq 0 \qquad (14)$$

$$x_j^{p-1} + M\delta_{p-2} \leq M,$$

$$x_j^{p-1} - M\delta_{p-1} \leq 0$$

$$x_j^p + M\delta_{p-1} \leq M$$

with

$$1 \geq \delta_k \geq 0 \qquad (15)$$

$$\delta = 0 \qquad (\text{Mod } 1) \qquad (16)$$

the relations (14) are written as

$$x_j^k - M\delta_k = 0 \qquad (k=1,2, \ldots, p-1)$$

$$(14')$$

$$x_j^{k+1} + M\delta_k \leq M \qquad (j=1,2, \ldots, P)$$

If we let for exampel in (14) x_j^2 greater than zero, then δ_2 must equal to 1, from the next row we find that

$$x_j^3; \ldots, x_j^P = 0$$

and from the relation of the row we see that for x_j^2 greater than zero, S_1 must be equal to zero

$$\text{jf } \delta_1 = 0 \quad \text{then} \quad x_j^1 = 0$$

The system (14) i.e (14') shows that for every j there is only one variable equal to zero.

This means that there are different ways of producing any product. So for exampel if we take 4 ways of production say x_1^1, x_1^2, x_1^3, x_1^4 for the production of x_1 and only two of this production ways are of maximum realiziation then we can get the following set of relations.

$$x_1^1 \; x_1^2 \; x_1^3 = 0 \qquad (17a)$$

$$x_1^1 \; x_1^3 \; x_1^4 = 0 \qquad (17b)$$

$$x_1^1 \; x_1^2 \; x_1^4 = 0 \qquad (17c)$$

$$x_1^2 \; x_1^3 \; x_1^4 = 0 \qquad (17d)$$

If two values of the variable x_1^i $(i=1,2,3,4)$ are equal to zero, means directly from (17a ... 17) that the values of the two other must equal to zero.

(17a) can be written as a linear relation as

$$x_1^1 \leq M\,(\delta_1 - \delta_2)$$

$$x_1^2 \leq M\,(1 - \delta_1)$$

$$x_1^3 \leq M\,(1 - \delta_2) \qquad (17a)$$

with

$$0 \leq \delta_1 \;;\; \delta_2 \leq 1 \qquad (18)$$

$$\delta_1, \delta_2 \equiv 0 \quad (\text{mod } 1) \qquad (19)$$

If we use (17a) in (18) and (19) we get

δ_1	δ_2	x_1^1	x_1^2	x_1^3
0	0	0	$0, > 0$	$0, > 0$
0	1	$0, > 0$	$0, > 0$	0
1	0	$0, > 0$	0	$0, > 0$
1	1	$0, > 0$	0	0