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Multi - Farametric Linear Programming

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Introduction:

It is expected that the programming techniques that will be useful to solve different problems in different fields will differe from the classical techriques. Since the increased computer capability will enable to develop techniques which recognize the particular charaeteristics of the problem being studied. Instead of using some standard techniques which make assunptions about the problem that can not be met. Also A typical manufacturing orgainization, the various operating departments are for most part goal oriented. Since each department has specific tangible or intangible objectives that are to be optimized conflict may often result. Such problems can be formulated as linear parametric programming. In general various other multiparametric linear programmes may be possible. Linear programms wilh multipile parameter either in abjective function or in constraints or both are obviousely not limited to manufacturing Organiziations. Therefore similar motivation could have been given from other areas. It has the advantage of representing many real prablems. In anticipation of such approach, it would be quite advantageous to have solutions techniques for such problems. The purpase of our work is to develop auseful technique for solving the multiple parametric linear programming.

1- Multi-parametric linear programming

1-1 Mathematical formulation of linear programming problem:

A linear programming problem can be written as fallows:

Find the vector X that (maximize or minimize) the linear objective function

$$\underline{Z}' = \underline{C}'\ \underline{X} \qquad (1)$$

under the constraints

$$\underline{A}\ \underline{X} \quad \underline{b} \qquad (2)$$

and

$$\underline{X} \quad 0$$

where

$$\underline{A} = A\ (m,n)$$

$$\underline{b} = b\ (m,1)$$

$$\underline{C} = C\ (m,1)$$

Equation (1) Form the optimality criterion and define the solution of the problem. i.e the oprimal result by the given constraints. System (2) gives the technical, economical and special constraints which bound the salution.

The solution is always seen as a relative solution. The solution of economic problems by such metheds is a relative solution, since it can not represent all economic factor in linear programming problem i.e it represents only one side of that problem. Fer this reason the optimal solution is relative. A better result, then must be given this result, is obtoined from the parametris programming. The parametric optimiziation programming takes in consideration the main factors of the problem in formulating the model. Thess main factors are to be taken as parameters and defin the objective function with any change in the main factors. Form this we develop a useful techmique for solving more parametric programming problem.

Another advantage of such parametric programming problem is that programme with more optimal criteria can be represented.

Mathematical for mulation of a parametric programming problem is to be done under the fellowing types.

1- Parametric objective functons coefficient

2- parametric boundes (constrints)

3- parametric costs coefficients.

1-2 In the follwing part If will deal with more-parameteric linear programming there will be four types of problems.

1- For the 1st type of those problems will be the optimurning probem as follows

$$Z_1 = \underline{U}' \ \underline{C}' \ \underline{X} \quad \text{Maxumum} \quad \underline{U}E.U \qquad (1)$$

under the constraints

$$\underline{A} \ \underline{X} \leqslant b$$
$$\underline{X} \geqslant 0$$

with

$$\underline{U}' = (U_1, U_2, \ldots U_a)$$

and

$$\underline{C}' = \begin{bmatrix} \underline{C}'_1 \\ \underline{C}_2 \\ \vdots \\ \vdots \\ \vdots \\ \underline{C}'_d \end{bmatrix}$$

The $\underline{C}_1$, $\underline{C}_2$, ..., $\underline{C}_d$ must be at frinstfree wneights of the paramelers U_1, U_2, ..., U_d verfied

the aim is to see

1- For which paraweter branch is an extrem point an optimal solution

- The properties of the parameter branch
- The properties of Z in this branch
- Which solution can be on optrmal solution
  the parmeters in this sense are free selected

  ($U = E^a$) or limited ($U < E^d$).

2- the <u>second problem</u> is

$$Z = \underline{C}' \underline{X} \qquad \text{maxumum} \qquad (4)$$

and the constraints

$$\underline{A} \quad \underline{X} \leq \underline{B} \ \underline{V} \qquad \underline{V} \ E \quad V \qquad (5)$$
$$\underline{X} \geq \underline{0}$$

Where the vector $\underline{b}$ is not given and

$$\underline{b} = \underline{b}1 \ V_1 + \underline{b}_2 \ V_2 + \ldots + \underline{b}_g \quad V_g$$

with the parameters

$$V_1, V_2, \ldots\ldots, V_g$$

then we have

$$\underline{b} = \underline{B} \ \underline{V}$$

wher

$$\underline{B} = (\underline{b}_1, \underline{b}_2, \ldots . \ b_g)$$

3- Problem (1) and (2) can be combined together give problem (3) which can be formulated as

$$Z_3 = \underline{U}' \ \underline{C}' \ \underline{X} \qquad \text{maximum} \qquad (7)$$

and

and

(U E U)

under the constraints

$$\underline{A}\ \underline{X} \leq \underline{B}\ X \qquad (V\ E\ V) \qquad (8)$$

$$X \geqslant 0 \qquad (9)$$

4) <u>The fourth problam</u>

If the Matrix A is not before given, and there is two kinds of Matrices $\underline{A}^{1}$ and $\underline{A}^{-2}$ to be calculated then the problem is as

$$Z_4 = \underline{C}'\underline{X} \qquad \max \qquad (10)$$

Under the conditions

$$(\underline{A}^1 W_1 + \underline{A}^2 W_2)\ \underline{X} \leqslant \underline{b},$$

$$(\underline{W} = \begin{Bmatrix} W_1 \\ W_2 \end{Bmatrix} \quad E \quad W) \qquad (11)$$

$$\underline{X} \geqslant \underline{0}$$

For problem (1) the mor - parameteric Programming problem is

$$\underline{Z}_1 = \underline{M}'\underline{C}'\,\underline{X} \quad \text{maxumin} \; (\underline{U} \in U)$$

$$\underline{A}\,\underline{X} \leqslant b$$

$$\underline{X} \geqslant 0$$

the dual of this problem will be

$$Z'_1 = \underline{b}'\,\underline{y} \qquad \text{minimum}$$

$$\underline{A}'\,\underline{y} \geqslant \underline{C}\,\underline{M} \qquad (\underline{U} \in U)$$

$$y \geqslant 0$$

where

$$\underline{C}' = \underline{C}'\,(1,n)$$

$$\underline{U}'\,\underline{C}' = (\underline{U}'\underline{C}')\quad(1,n)$$

as an intial table for the problem

A- <u>the intial table for the problem is</u>

	$\underline{X}'$	
$\underline{Y}$	$\underline{A}$	$\underline{b}$
	$\underline{C}'$	$\underline{O}$ (d,1)

the $1^{\underline{st}}$ table is

	$\underline{X}_1{}'$	$\underline{X}_2{}'$	
Y_1 Y_2	$\underline{A}_{11}$ $\underline{A}_{21}$	$\underline{A}_{12}$ A_{22}	$\underline{b}_1$ $\underline{b}_2$
$\underline{U}'$	$\underline{C}_1'$	$\underline{C}_2'$	$\underline{0}$

(13)

with which $\underline{A}_{11} = \underline{A}_{11}$ (r,r) and

$$\underline{C}^1 = \begin{pmatrix} \underline{C}_1' \\ \underline{C}_2' \\ \\ \underline{C}_d' \end{pmatrix} = \begin{pmatrix} C_{11} & C_{12} & \cdots\cdots & C_{1n} \\ C_{21} & C_{22} & \cdots\cdots & C_{2n} \\ \\ C_{d1} & \cdots\cdots\cdots\cdots & & C_{dn} \end{pmatrix} = C(d,n)$$

it is also

$$\underline{C}_1' = \begin{pmatrix} C_{1,J_1} & C_{1,J_2} & \cdots\cdots\cdots C_{1,J_r} \\ \\ C_{a,J_1} & C_{d,J_2} & \cdots\cdots\cdots C_{d,J_r} \end{pmatrix} = \underline{C}_1 \ (d,r)$$

and

$$\underline{C}_2' = \begin{pmatrix} C_{1,J_{r+1}} & C_{1,J_{r+2}} & \cdots\cdots\cdots\cdots & C_{1,J_n} \\ \\ C_{d,J_{r+1}} & C_{d,J_{r+2}} & \cdots\cdots\cdots\cdots & C_{d,J_n} \end{pmatrix} = \underline{C}_2' \ (d,n-r)$$

From table (13) through the privot element $\underline{A}^{*}_{11}$ we get the following table

	$\underline{Y}'_1$	$\underline{X}'$	
$\underline{X}_1$	$\underline{A}^{-1}_{11}$	$\underline{A}^{-1}_{11}\ A_{12}$	$A^{-1}_{11}\ \underline{b}_1$
$\underline{Y}_2$	$-A_{21}A^{-1}_{11}$	$A_{22}-\underline{A}_{21}\underline{A}^{-1}_{11}\underline{A}_{12}$	$\underline{b}_2-\underline{A}_{21}A^{-1}_{11}A_{12}\ b_1$
$\underline{U}'$	$-\underline{C}'_1A^{-1}_{11}$	$\underline{C}'_2-\underline{C}'_1A^{-1}_{11}A_{12}$	$-\underline{C}'_1A^{-1}_{11}\ \underline{b}_1$

From table (14) is the optimal Solution of the primal problem (1) as

$$\underline{X}^{+} = \begin{pmatrix} \underline{A}^{-1}_{11} \quad b_1 \\ \underline{0} \end{pmatrix} \tag{15}$$

the Solution of the dual problem is

$$\underline{Y}^{+} = (\underline{U}'\underline{C}'\underline{A}^{-1}_{11},\ \underline{0}') \tag{16}$$

with the following constraints

$$\underline{A}^{-1}_{11}\ \underline{b}_1 \geq \underline{0} \tag{17}$$

$$\underline{b}_2 - \underline{A}_{21}\ \underline{A}^{-1}_{11}\ \underline{b}_1 \geq \underline{0} \tag{18}$$

$$-\ \underline{U}'C'_2 - \underline{A}^{-1}_{11} \leq \underline{0} \tag{19}$$

$$\underline{U}'\ C'_2 - \underline{C}'_1\ \underline{A}^{-1}_{11}\ \underline{A}_{12} \leq \underline{0} \tag{20}$$

the inequalties (19) and (20) are

$$\underline{A}_{11}^{-1} \ \underline{C}_1 \ \ \underline{U}_1 \geqslant \underline{0} \text{ and}$$

$$(- \underline{C}_2 + \underline{A}_{12} \ \underline{A}_{11}^{-1} \ \underline{C}_1) \ \underline{U} \geqslant \underline{0}$$

the optimal value of the abjectine

function of problem (1) is

$$z^{+} = \underline{U} \ \underline{C}_1' \ \ \underline{A}_{11}^{-1} \ \ \underline{b}_1 .$$

Initial Table for solving problem (2):

The problem is given as

$$Z_2 = C^1 \; X \qquad \text{maximum !} \quad (21)$$

under the constraints

$$\underline{A}\;\underline{X} \leq \underline{B}\;\underline{V} \qquad V \;\; E \;\; V, ; \; B = B(m, g)$$

$$\underline{V} = \underline{V}\;(g, 1)$$

$$\underline{X} > \underline{0}$$

the dual of this problem is as follows

$$Z_2 = \underline{V}' \cdot \underline{B}' \cdot \underline{Y} \qquad \text{minimum !} \quad (22)$$

under the constraints

$$\underline{A}'\;\underline{Y} \geq \underline{C}$$

$$\underline{Y}' \geq 0$$

as in problem (1) the intital table is as follows:

	X_1	X_2	V
Y1	A11	A12	B1
Y2	A21	A22	B2
	C1′	C2′	0 (1, g)

with the Matrix A_{11} as a pivot element then table 23 will be in the following form.

	Y1	X2	
X1	$\underline{A}_{11}^{-1}$	$\underline{A11}\ \underline{A12}$	$\underline{A}_{11}^{-1}\ \underline{B1}$
Y2	$-\underline{A21}\ \underline{A}_{11}^{-1}$	$\underline{A22} - \underline{A21}\ \underline{A}_{11}^{-1}\ \underline{A12}$	$\underline{B2} - \underline{A21}\ \underline{A}_{11}^{-1}\ \underline{B1}$
	$-\underline{C1}\ \underline{A}_{11}^{-1}$	$\underline{C2} - \underline{C1}\ \underline{A}_{11}^{-1}\ \underline{A12}$	$-\underline{C1}\ \underline{A}_{11}^{-1}\ \underline{B1}$

the optimal solution of primal problem 21 is

$$\underline{X}^{+} = \begin{pmatrix} \underline{A}_{11}^{-1}\ \underline{B1}\ \underline{V} \\ \underline{0} \end{pmatrix}$$

and the optimal solution of the dual

$$\underline{y}^{+} = (\underline{C1}'\ \underline{A}_{11}^{-1}\ ,\ \underline{0})$$

the objective functions of both problem will be

$$Z^{+} = \underline{C1}'\ \underline{A}_{11}^{-1}\ \underline{B1}\ \underline{V1}$$

for the optimal solution of 24 it is necessary that

$$\underline{A}_{11}^{-1}\ \underline{B1}\ \underline{V} \geq 0 \qquad (26)$$

$$(\underline{B2} - \underline{A21}\ \underline{A}_{11}^{-1}\ \underline{B1})\ \underline{V} \geq 0 \qquad (27)$$