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Predicting the National Freight Transport Demand An Application of Multivariate Autoregressive Moving Average Time Series Analysis

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I. Introduction

The development of many sectors of the economy is often hampered by the insufficiency of the exisitng facilities of physical distribution. Ridding of such a problem is expected to yield a returns to different sectors far more than the cost involved in the additional facilities. To estimate future needs for each mode of freight transport (waterways, railroad, and motor carriers), it is logical to start by predicting total needs of freight transport, then break down the total to find the size of demand on each mode.

This study concerns itself with the first part, i.e. with developing a proper model which could be used to predict total annual needs of freight transport services.

It is doubtful that analysis of time series of past commodity shipments would by itself suffice to predict such future needs. It does not account for external factors which affect the variable being forecast. In a previous study in which the author took part,[1] it had been suggested that Gross National Product at constant prices would be about the most relevant single variable affecting freight transport needs since most domestic product is physically distributed within the economy.

However, direct application of regression analysis to estimate the relationship between the two series (Demand for freight transport as the dependent variable Y, and GDP at constant prices as the independent variable X) would present some dificulties.

On the one hand, the disturbance terms of each series would not be white noise (random) as required by the model. Rather, they would most likely be serially related.

On the other hand, the trend in both series would tend to dominate the regression thus obsocuring the true regression relation making its identification and estimation rather difficult.

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1) This was in an unpublished study by the Institute of National Planning, Cairo, 1973.

Often in practical studies, these problems are ignored and regression analysis is applied to time series as are. The results would be unbiased estimates of the regression parameters but their estimated variances would be biased. This in turn leads to unreiable tests and inacurate interval estimates.

Therefore, for the correct specification of the regression model which will be used for prediction purposes, it is important to rid each series of any of these these difficulties whenever present before applying regression analysis to them. The suggested procedure, known as the Box-Jenkins Approach, will be applied in the following sequence:

a) Checking for the existence of a trend in each series, in which case data should be detrended first.

b) Checking whether disturbances of each series is white noise, if not, transform them into white noise through the specification and application of the correct ARMA model. This is knows as the prewhitening stage.

c) Applying regression analysis to the prewhitened series of X and Y. Modify until reaching the correct MARMA model.

d) Checking the model for adequacy. This stage is known as diagnostic checking stage.

e) Using the estimated MARMA model for prediction.

II. Detrending and Prewhitening

Prewhitening is applied to both the independent variable X and the dependent variable Y. The purpose is to eliminate the trend within each series leaving what could be called white noise, therefore allowing the application of regression analysis to estimate the regression of the prewhitened y series on the prewhitened X series.

Starting with the X series, the general class of autoregressive moving average (ARMA) model expressing the way an X value is related to its own past values and to current and past error terms is:

$$x_t = \phi_1 x_{t-1} + \phi_2 x_{t-2} + \ldots + \phi_p x_{t-p}$$

$$+ u_t - \theta_1 u_{t-1} - \theta_2 u_{t-2} \ldots\ldots - \theta_q u_{t-q} \qquad (1)$$

Equation (1) should be applied to stationary time series to find out the degree of the ARMA model, i.e. to determine p and q.

If the data are not stationary, they should be made so by first removing the trend in them.

Therefore, the very first step is to find out whether the X series is stationary. Since stationarity exists when the data are horizontal or flutuate around a constant mean, autocorrelations are used to detect the presence of stationarity. Autocorrelation for time lag (k) is equal to

$$r_k = \frac{\sum_{t=1}^{n-k} (X_t-\bar{X})\ (X_{t+k}-\bar{X})}{\sum_{t=1}^{n} (x_t-x)^2}$$

If autocorrelation drops rapidly to zero, the data are stationary and the ARMA model could be applied to them. Otherwise, if autocorrelations drop slowly to zero and several of them are significantly different from zero, it would be a sign of an existing trend within the series, i.e. the data would be nonstationary. Table 1 shows the autocorrelations for the X series for various time lags.[1)]

Table 1
Autocorrelations of Original X Series

Time lag	Autocorrelation
1	0.865
2	0.748
3	0.641
4	0.537
5	0.447
6	0.358
7	0.283
8	0.211
9	0.137
10	0.063

1) The Data used are:
X = GDP at 1954 prices million £ for 1953-1980/1981
Y = Total volume of Freight transport in million ton/kelometer for 1953-1980/1981.

Since all autos up to the fifth time lag are significant[1], and the decline in them is rather slow, there exists a trend in the X series which will be removed by differencing.

Taking first differences instead of the original X data. The autos for various time lags are as shown in table 2.

Table 2
Autocorrelations of Differenced X Series

Time lag	Autocorrelation
1	.39
2	.305
3	-.136
4	.037
5	.028
6	.318
7	.045
8	.106
9	-.104
10	.054

The values of the table indicates that the new series of first differences has much lower autos which decline exponentially.

To determine the proper order of the ARMA model we examine the autos and partial autos for various time lags. Table 3 shows the partial

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1) Standard error of the autocorrelation coefficient is equal to

$$1/\sqrt{n-k}$$

autos for the differenced series.

Table 3

Partial Autocorrelations of Differenced X Series

Time lag	Partial Autocorrelation
1	.39
2	.227
3	-.354
4	.30
5	.256
6	.54
7	-.258
8	.40
9	.336

Notice that the first auto, the first and sixth partials are significantly different from zero. This suggests an AR of up to (6), and a MA of (1). However, we will start with the simplest model, an ARMA (1,1) and proceed with the remaining steps to find out whether such a model is adequate or should be modified.

An ARMA (1,1) model is in the form:

$$(X_t - X_{t-1}) = \phi_1 (X_{t-1} - X_{t-2}) + u_t - \theta_1 u_{t-1} \qquad (2)$$

where u_t is white noise such that

$E(u_t) = 0$, $E\, u_t^2 = \sigma^2$, and $E\, u_t\, u_{t-1} = 0$

Initial estimates of ϕ_1 and θ_1 should be obtained. These are obtained using the autocorrelation coefficients.

Let us first use $x_t = X_t - X_{t-1}$

and $x_{t-1} = X_{t-1} - X_{t-2}$

Therefore, eq (2) becomes

$$x_t = \phi_1 x_{t-1} + u_t - \theta_1 u_{t-1} \tag{3}$$

The variances and covariances of the mixed ARMA process would then be

$$\gamma_o = E(x_t^2) = E(\phi_1 x_{t-1} + u_t - \theta_1 u_{t-1})^2 \tag{4}$$

$$= \phi_1^2 \gamma_o - 2\phi_1\theta_1 E(x_{t-1} u_{t-1}) + \sigma^2 + \theta_1^2 \sigma^2 \tag{5}$$

$$= \phi_1^2 \gamma_o - 2\phi_1 \theta_1 \sigma^2 + \sigma^2 + \theta_1^2 \sigma^2 \tag{6}$$

since $E(x_{t-1} u_{t-1}) = \sigma^2$

Therefore,

$$\gamma_o (1 - \phi_1^2) = \sigma^2 (1 + \theta_1^2 - 2\phi_1 \theta_1) \tag{7}$$

and, the variance γ_o is

$$\gamma_o = \frac{(1 + \theta_1^2 2\phi_1 \theta_1)}{1 - \phi_1^2} \sigma^2 \tag{8}$$

Likewise, the covariance γ_1 is

$$\gamma_1 = E\,(x_{t-1}\,x_t) = E\left[x_{t-1}\,(\phi_1\,x_{t-1} + u_t - \theta_1\,u_{t-1}\right]$$

$$= \phi_1\,\gamma_0 - \theta_1\sigma^2 \qquad (9)$$

$$= \phi_1 \cdot \frac{1 + \theta_1^2 - 2\phi_1\,\theta_1}{1-\phi_1^2}\,\sigma^2 - \theta_1\sigma^2 \qquad (10)$$

$$= \frac{\phi_1 + \phi_1\,\theta_1^2 - 2\phi_1^2\,\theta_1 - (1-\phi_1^2)\,\theta_1}{1-\phi_1^2}\,\sigma^2 \qquad (11)$$

$$= \frac{\phi_1 + \phi_1\,\theta_1^2 - 2\,\phi_1^2\,\theta_1 - \theta_1 + \phi_1^2\,\theta_1}{1-\phi_1^2}\,\sigma^2 \qquad (12)$$

$$= \frac{\phi_1 + \phi_1\,\theta_1^2 - \phi_1^2\,\theta_1 - \theta_1}{1-\phi_1^2}\,\sigma^2 \qquad (13)$$

$$= \frac{(\phi_1 - \theta_1) - \phi_1\,\theta_1\,(\phi_1 - \theta_1)}{1-\phi_1^2}\,\sigma^2 \qquad (14)$$

Thus,

$$\gamma_1 = \frac{(\phi_1 - \theta_1)\,(1 - \phi_1\,\theta_1)}{1-\phi_1^2}\,\sigma^2 \qquad (15)$$

ƙewise γ_2 is

$$\gamma_2 = E\,(x_{t-2}\, x_t)$$

$$= E\left[x_{t-2}\ (\phi_1\, x_{t-1} + u_t - \theta_1\, u_{t-1})\right] \quad (16)$$

$$= E\left[x_{t-2}\ (\phi_1\ (\phi_1\, x_{t-2} + u_{t-1} - \theta_1\, u_{t-2}) + u_t - \theta_1 u_{t-1}\right] \quad (17)$$

$$= \phi_1^2\, \gamma_\circ - \theta_1 \phi_1 \sigma^2 \quad (18)$$

Thus,

$$\gamma_2 = \phi_1\ (\phi_1 \gamma_\circ - \theta_1 \sigma^2) = \phi_1 \gamma_1 \quad (19)$$

in the same fashion

$$\gamma_k = \phi_1\, \gamma_{k-1} \qquad \text{for} \quad k \geqslant 2 \quad (20)$$

where k is the number of time lags.

Thus, the autocorrelation functions would be

$$r_1 = \frac{\gamma_1}{\gamma_\circ} = \frac{(\phi_1 - \theta_1)\ (1 - \phi_1\, \theta_1)}{1 - \theta_1^2 - 2\phi_1\, \theta_1} \quad (21)$$

and

$$r_2 = \frac{\gamma_2}{\gamma_\circ} = \frac{\phi_1\, \gamma_1}{\gamma_\circ} = \phi_1\, r_1 \quad (22)$$

Therefore, $$\phi_1 = \frac{r_2}{r_1} \quad (23)$$

Next, the autocorrelation estimates for various time lags are used to obtain initial estimates for ϕ_1 and θ_1.

Starting with ϕ_1 ,

$$\phi_1 = \frac{r_2}{r_1} = \frac{.305}{.39} = .781$$

Substituting this value of ϕ_1 in eq. (21) we can solve the resulting function, which is nonlinear, to get an initial estimate for θ_1

$$.39 = \frac{(.781 - \theta_1)(1 - .781\,\theta_1)}{1 - \theta_1^2 - 2 \times .781 \quad 1}$$

or

$$-.39\,\theta_1^2 + \theta_1 - .39 = 0$$

Thus,

$$\theta_1 = \frac{-1 \pm \sqrt{1 - 4(.39)^2}}{-2 \times .39} \qquad (24)$$

$$\theta_1 = 0.479 \quad \text{or} \quad \theta_1 = 2.085$$

The first value θ_1 = °479 meets the constraint for stationarity (otherwise, with the other value of θ_1 the series would be explosive). Therefore, the initial values of the parameters are

$$\phi_1 = .781 \qquad \theta_1 = .479$$

Final parameter estimates are obtained using Marquardt's method for solving nonlinear equations. This method is often preferred for its practical advantages over the other methods.[1)]

Using the initial estimates of ϕ_1 and θ_1 on the differenced data x_t we get its corresponding estimate

$$\hat{x}_t = .781 \; x_{t-1} - .479 \; e_{t-1}$$

where e_t is the observed residual value for time t. Thus the mean square error MSE is

$$MSE = \frac{\sum_{t=3}^{28} e_t^2}{26} = \frac{368622}{26} = 14178 \qquad (25)$$

where $e_t = x_t - \hat{x}_t$.

The calculations using initial parameter values are shown in table 1 in the appendix.

Next, to determine the direction of changing the parameter estimates, we introduce a small change, say 1% of original value, once added and once subtracted to the initial value of ϕ_1 while holding θ_1

1) Other methods for solving nonlinear equations are; linearization of the above nonlinear function around the initial values of the parameters using Taylor series expansion (see 7, p. 482), and steepest descent method (see 2, p. 267).

constant. The predicted values of x_t using these values, called f_{1t} are calculated. The difference between f_{1t} and $\hat{x}_t$ is also found. The resulting values of MSE will determine the appropriate direction of change in ϕ_1.

Interchangeably, ϕ_1 is held constant at its initial value and θ_1 is changed by a small increment in each direction. The predicted values of x_t using these values, called f_{2t} are also calculated. The difference between f_{2t} and $\hat{x}_t$ could also be found. Again, the values of MSE will determine the appropriate direction of change in the value of $_1$. Table 4 shows alternative values of ϕ_1 and θ_1 and their corresponding values of MSE.

Table 4
MSE for alternative Estimates
ϕ_1 and θ_1

Parameter Estimates	MSE
$\phi_1 = .781$, $\theta_1 = .479$	14178
$\phi_1 = .789$, $\theta_1 = .479$	13915
$\phi_1 = .773$, $\theta_1 = .479$	14450
$\phi_1 = .781$, $\theta_1 = .484$	14262
$\phi_1 = .781$, $\theta_1 = .474$	14096

Notice that the value of MSE decreases whenever ϕ_1 is increased or θ_1 is decreased and vice versa. Thus, the search for final parameter