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A Developed Simplex Algorithm For Solving Transportation Problem

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Introduction.

As an economic interpretation of the linear programming model encompassing a wide variety of applications is the following. The modeled system includes several activities which shares limited resources. The objective of the model is to determines the level of each activity that optimizes the output of all activities without violating the limits stipulated on the resources.

The classical "transportation" and "assignment problems have the fortunate property that the linear programming solution or more precisely, every extrem point solution automatically assigns integer values to the variables. In this paper there is a representation of a transportation problem with initial and variable costs. The solution of such problem depends originaly on the distribution method which helps to get an initial feasible solution.

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1. Transportation Problem With intial cost.

1.1. The Nature of the Problem.

There is a special class of linear programming models that are of particular interest because of their structures as well as applications. The transportation model deals with the transportation of a commodity from m sources to n destinations. The supply available at source i is a_i units and the demand required at destination j is b_j units. X_{ij} will represents the level of activity (amount transported) from source i to destination j. The case to be studied heir will take incondieriation the fixed cost and also the interval fixed cost which is a new extension in dealing with the transportation problems. To do this let us assume the pollowing.

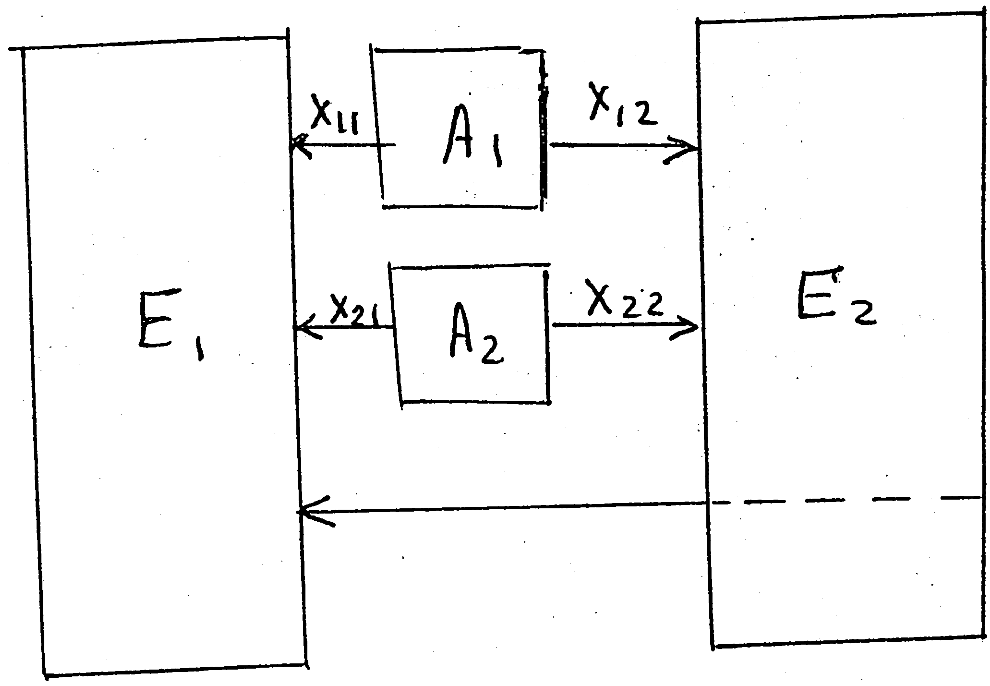
Given A_i sources (i=1,2,3) and E_j destinations (j=1,2) and it is asked about the minimum transport cost from A_i to E_j. Let T_1 and T_1^2 represents the arrangement of the transport standsill of the given places. Also it is assumed that we have two types of costs

a. Fixed costs K_i

b. Interval Fixed costs K_{ij}

These two costs are needed also for the transportation of a unit. The transportation problem is now considered as a mixed transportation problem, given the transports mittel(transit transport mittel) as t_i, which means that there is for the amount transported from A_i to E_j (X_{ij}) X_{11}, X_{12}, X_{21} and X_{22} transport mittel to be fullfild. This means that the transport-mittels allows only on E_j to be supplied, IF we want to supply the other E_j, then there must be new transport mittel.

This condition is also applied for A_3. IF for example we have a transport mittel for supplying from A_3, so the transport mittel is divided between E_1 and E_2 as seen in Fig(1). This condition is not applied for A_3, i.e. new transport mittel must be used.



Fig(1)

Beside the fixed costs and the interval fixed cost there exist also the transport cost for transporting a unit from A_i to E_j which are given by the following table

C_{ij}	E_1	E_2
A_1	100	120
A_2	100	100
A_3	160	110

Also the fixed cost for transporting from A_1, A_2 and A_3 will be

$$K_1 = 1500 \quad , \quad K_2 = 1680 \, , \qquad K_3 = 1200$$

The order of transports mittel for the standing A_1, A_2 and A_3 will be given by

$$T_1^1, T_1^2, \ldots\ldots = 2u$$

$$T_2^1, T_2^2, \ldots\ldots = 3u$$

$$T_3^1, T_3^2, \ldots\ldots = 2u$$

For a seperate transports mittel for the given sources A_1 and A_3 the fixed cost will be of maximum value of 1000 and for the source A_2 will be of a value of 1320. This fixed cost of the transports mittel will be the total transport plan for the intervals

$$0,2,4 \quad x_{11}, x_{12}, x_{11}, x_{32} \quad 2,4,6$$

$$k_{11} = k_{12} = k_{31} = k_{32} = 1000$$

but for the source A_2 we assume that

$$k_{2j} = 1320 \qquad \text{for an interval of } 3u$$

Let now a_i represents the capacity of A_i and e_j be the needed sum from E_j then we have

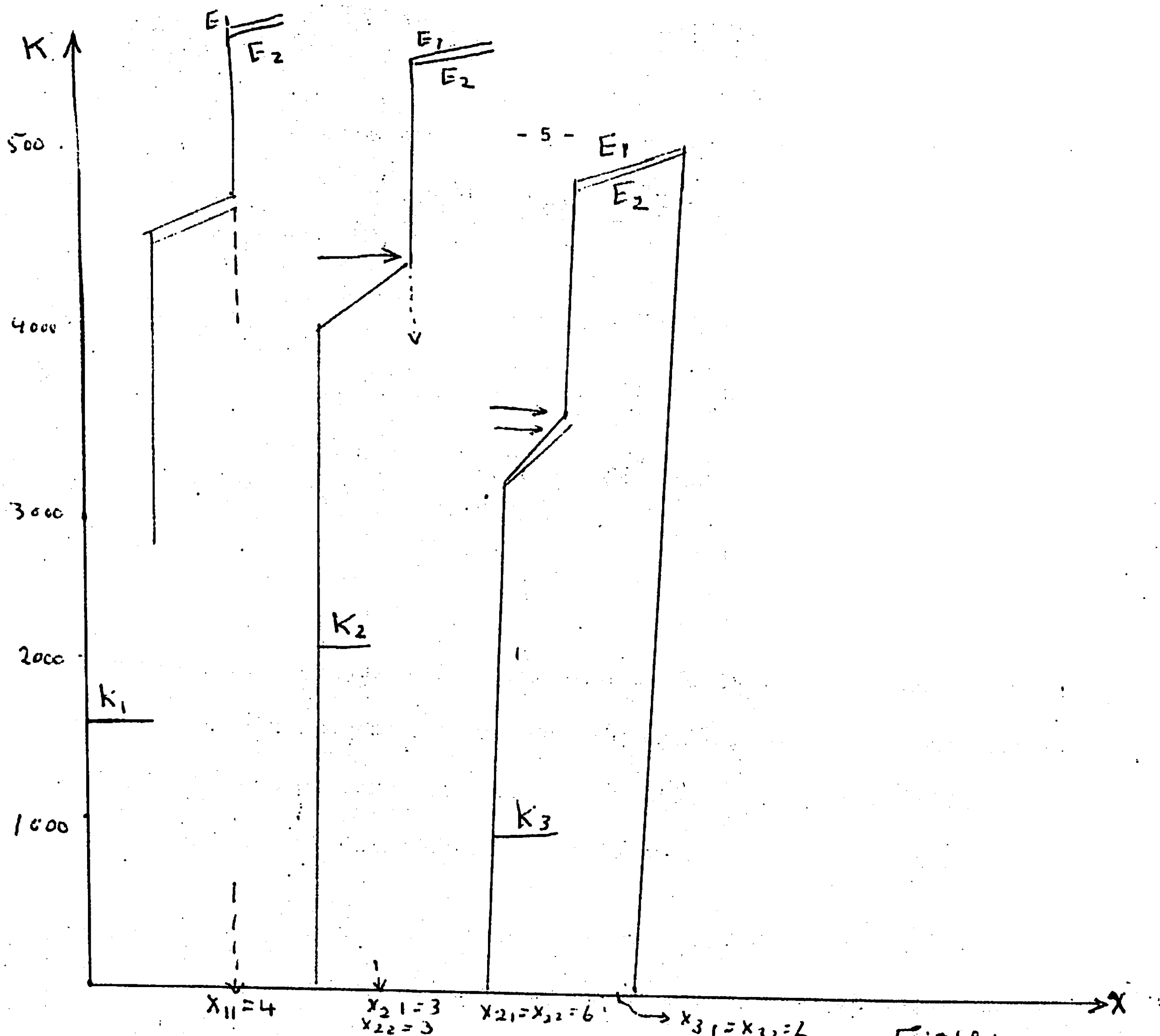
$$a_1 = 5 \qquad e_1 = 5$$

$$a_2 = 6 \qquad e_2 = 5$$

$$a_3 = 4$$

Since the total sum of a_i greater than that of e_j, then we assume that there is a thrid destination E_3 without transport costs ($C_{i3} = 0$).

The following fig (2) gives an illustration of the cost run for a seperate supply of one source.



Fig(2)

3. <u>The Formulation of the Objective Function.</u>

The transport cost function includes the total variable transport costs

$$K_v = \sum_i \sum_j c_{ij} \quad x_{ij} \qquad (1)$$

The transport cost theory with the matrix C_{ij} as the costs of transportation.

For the total fixed costs for supplier resources we have

$$\sum K_i = K_1 v_1 + K_2 v_2 + K_3 v_3 \tag{2}$$

with

$$x_{11} + x_{12} \leqslant a_1 v_1 \tag{3a}$$

$$x_{21} + x_{22} \leqslant a_2 v_2 \tag{3b}$$

$$x_{31} + x_{32} \leqslant a_3 v_3 \tag{3c}$$

and

$$v_1,\ v_2,\ v_3 = \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \tag{3d}$$

The fixed cost K_1 is real only, when there is a supply from a given source. Also we can see for example that when A_1 supplies either A_1 $(x_{11} \geqslant o)$ or/and E_2 $(x_{12} \geqslant 0)$, then v must be equal to one this is from (3d), which gives K_1 as the total cost and relation (2) is fullfiled.

Relation (3b) to (3c) represents the fixed costs K_2 and K_3.

For the interval fixed costs in the position A_1 we have

$$K^1_{jt} = k_{11} W^1_1 + k_{12} W^2_1 \qquad (4)$$

$$x_{11} \leqslant 2w^1_1 \qquad (4a)$$

$$x_{12} \leqslant 2w^2_1 \qquad (46)$$

$$w^1_1, w^2_1 = 0 \qquad (4c)$$

$$w^1_1 , w^2_1 \geqslant 0 \qquad (4d)$$

The interval fixed cost k_{11} given by relation (4) is for example availble, when $0 \leqslant x_{11} \leqslant 2$, but if $2 \leqslant x_{11} \leqslant 4$, then the value of w^1_1 must be at least equal to 2, this can be deduced from relations (4a) with (4c) and (4d).

Similarly for k_{12} we get a set of relations coresponding to A_1 and A_3 as follows.

$$K^2_{jt} + K^3_{jt} = k_{21} w^1_2 + k_{22} w^2_2 + k_3 w_3 \qquad (5)$$

with

$$x_{21} \leqslant 3w^1_2 \qquad (5a)$$

$$x_{22} \leqslant 3w^2_2 \qquad (5b)$$

$$x_{31} + x_{32} \leqslant 2w_3 \qquad (5c)$$

$$w^1_2, w^2_2, w_3 = 0 \qquad (5d)$$

$$w^1_2, w^2_2, w_3 \geqslant 0 \qquad (5e)$$

Relations (5) and (5c) shows that the source A_3 supply the destinations E_1 and E_2 only in one direction, which means that only one transport mittel is used. In this case we see that the interval transport costs are divided between x_{31} and x_{32}.

Also relations (4) to (5c) shows that there is no interval fixed costs for the supply of destination E_3.

The final objective function which is the total cost function is formulated as follows

$$K = \sum_i \sum_j a_{ij} x_{uj} + \sum_i K_i v_i + \sum_{j=1}^{2} k_{ij} w_1^j + \sum_{j=1}^{2} k_{2j} w_2^j + k_3 w_3 \qquad (6)$$

In the above cost function (6) the set of constraints (3a) to (3e); (4a) to (4d) and (5a) to (5e) are to be taken into consideration.

1.3 The Constraints

The objective function (6) is formulated without any condition on the variables x_{ij} (the non-negativity condition on the variables). The capacity conditions are included in the constraints (3a), (3b) and (3c).

Beside the non-negativity of the variables, we most also represents the supply demand conditions between A_i and E_j which gives

$$\sum_i x_{ij} \geq e_j \qquad (7)$$

If we introduce the variable V_i, which means that if the supply of A_i exist then the value of V_i is equal to 1, and the value of x_{ij} is not greater than a_i.

Now we are in a position to get the mathematical formulation of the problem as follow.

Given an objective function

$$\begin{aligned} K = {} & 100x_{11} + 120x_{12} + 100x_{21} + 100x_{22} + 160x_{31} \\ & + 110x_{32} + 1500V_1 + 1660V_2 + 1200V_3 \\ & + 1000w_1^1 + 1000w_1^2 + 1320w_2^1 + 1320w_2^2 \\ & + 1000w_3 \ldots\ldots\ldots\ldots \end{aligned} \qquad (8)$$

is to be maximized under the constraints.

$$x_{11} + x_{12} - 5v_1 \leq 0 \qquad (9)$$

$$x_{21} + x_{22} - 6v_2 \leq 0 \qquad (10)$$

$$x_{31} + x_{32} - 4v_3 \leq 0 \qquad (11)$$

$$x_{11} - 2w_1^1 \leq 0 \tag{12}$$

$$x_{12} - 2w_1^2 \leq 0 \tag{13}$$

$$x_{21} - 3w_2^1 \leq 0 \tag{14}$$

$$x_{22} - 3w_2^2 \leq 0 \tag{15}$$

$$x_{31} + x_{32} - 2w_2 \leq 0 \tag{16}$$

$$x_{11} + x_{21} + x_{31} \geq 5 \tag{17}$$

$$x_{12} + x_{22} + x_{32} \geq 5 \tag{18}$$

$$v_1,\ v_2,\ v_3 \leq 1 \tag{19}$$

$$v_1,\ v_2,\ v_3,\ w_1^1,\ w_1^2,\ w_3 = 0 \tag{20}$$

$$\begin{matrix} x_{ij} \geq 0 \\ v_i \geq 0 \\ w_i^j \geq 0 \end{matrix} \qquad \begin{pmatrix} i=1,2,3 \\ j=1,2 \end{pmatrix} \tag{21}$$

If we use the simplex method to get a solution for the above problem, this means that we are dealing with 14 basic variables and 13 slack variables due to the constraints from (9) to (19). But in case of using the M-Method there must be two additional salack variables. For the two constraints (17) and (18) i.e. our problem consists of 29 variables.