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THE DEMAND - INCOME FUNCTION - A STUDY IN AGGREGATION

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CONTENTS

I. Models and Structures:

Econometric analysis is concerned with the empirical investigation of quantitative relationships between economic variables as envisaged by economic theory. The analysis is related to the concept of a model, and its outcome defines a certain structure. By a model we mean a given hypothetical set of (one or more) relationships which relate economic variables to each other in a well-defined manner, each of which aiming at explaining a certain economic phenomenon. This might lead to the introduction of non-economic magnitudes which are either variable or relatively constant, hence the term "parameters". In theoretical analysis one can do without an explicit formulation of the exact functional relationships, provided sufficient information is available for defining relevant properties (e.g., first derviatives). For econometric investigation this is not sufficient.

Three conditions have to be met before a theoretical model is to be considered as workable from an econometric point of view:

1. The explicit formulation of relationships; defining their form, and the parameters to be included.

2. The definition of variables in a manner which makes them capable of actual statistical observation.

3. The specification of the distributions of stochastic variables which have to be introduced in order to account for discrepancies between ideal and observed concepts (errors of observation), and between theoretical and actual economic relationships (errors of equations).

By a structure we mean a given point of parameter values which the model assumes in a particular case. Thus a model is the complete set of structures which satisfy its full specifications. This implies that the relevant parameters are not only those of the functional economic relationships; they include also the parameters of the distributions of the stochastic element. If we confine ourselves to the former group of parameters, we would be dealing with a purely economic structure, related to a given economic model. But if we include the complete set of parameters, we would be dealing with the full econometric structure corresponding to a given econometric model.(1)

(1) In many cases in the literature an econometric model is meant to satisfy the first two conditions only. This has led to a confusion of the term "econometric" itself.

II. Measurement of Economic Variables:

Whether for purposes of theoretical analysis or of econometric investigation, it is of fundamental importance to define precisely the dimensions of the economic variables to be included. This requires the specification of the exact types of units of each of the following aspects:

1. The economic agents: individuals, groups of individuals and organisms.

2. The economic entities: commodities and groups of commodities, defined according to their nature and uses.

3. The time unit of measurement. In this connection we have to differentiate the timeless stocks and ratios (e.g., prices) from flows. Even when we choose to consider flows as continuous functions of time, their expression should be in terms of given time rates per discrete time units.

It can be easily noticed that in each case one could differentiate elementary units of measurement, below which any further subdivision is either impossible or operationally meaningless. Any other units should be considered as composite units which can be defined as given functions of elementary units.

Thus the elementary unit of economic agents can be considered as the smallest unit capable of formulating and implementing decisions with respect to a given economic phenomenon. For example the individual consumer is such an elementary unit, though it might be a group of natural persons forming a household. Similarly for an individual firm. If we apply this principle we can say that Government itself is an elementary agent with respect to certain types of action, although it is constituted of a number of separate administrations.

The elementary unit of economic entities is directly related to the concept of commodity. It is known that the homogeneity condition should be strictly satisfied, meaning by that the perfect substitutability with respect to a given type of uses, or operations. However, there is a certain important difference between this elementary unit and the former one with respect to actual measurement. Thus if we consider the purchase of a given commodity by a certain consumer, we have in mind the number of units of that commodity purchased by that single consumer. This means that the relvant unit of measurement relates to the nature of the commodity. But

the actual magnitude relates to an individual consumer. Further, the concept of the price of the given commodity is directly related to the commodity itself. If prices are the same for all individual consumers, then there is no need to specify them according to consumer. The concept of price remains the same although its exact magnitude might change if consumers pay different prices for the same commodity (e.g., in different markets).

The role of time is a multiple one. Thus in dynamic analysis, or in actual measurement, the values of variables change over time. This applies to stocks as well as flows. But from a dimensional point of view, one has to determine the elementary time unit with reference to the operations studied. Thus, the elementary unit of analysis should relate to the period of time necessary for the implementation of one single decision. This raises certain complications for the econometrician:

1. The elementary unit of time differs according to the type of decision.

2. It need not concide with the units used in the actual collection of data.

It is clear that actual statistical observations relate to composite time units, which are found to be convenient for collection of data, especially on economic flows. However this raises certain difficulties with respect to the measurement of variables void of the time dimension, namely stocks and ratios. The solution of these difficulties is found through familiar statistical techniques (e.g., averages). This envolves an aggregation procedure which is of a relatively simple nature. Other aggregation problems, relating to the other types of units are of a more complex nature.

III. Construction of Composite Units

Starting from elementary agents, one can construct a large variety of composite units, according to a well-defined principle. For example, we can group together all consumers within a given market, or economy, or belonging to a given region or a given social group. Again firms belonging to a given branch or sector, or falling within a certain class of size, or using a certain technique, can be grouped together.

Further, the aggregation might be carried out at a series of stages. The first stage, which is concerned with the aggregation of elementary agents into certain groups of agents, can be considered as primary aggregation. The following stages can be considered as cases of secondary aggregation. In each of these stages the aggregation is done over aggregates obtained through primary or secondary aggregates to obtain more composite aggregates.

The same applies also to economic entities. From the elementary units we can construct groups of commodities such as food. Further composite units can be constructed from these latter, e.g. total consumers' goods. While the aggregation of economic agents usually envolves simple processes of summation, the aggregation of economic entities is generally more complicated. For example, it is not possible to add up quantities of different commodities, even though their physical units are the same (e.g. tons). Index numbers are an attempt to approximate quantity and price composites. The abundant controversy around this subject is a clear indication of its complexity, as well as its ambiguity. We have to observe also that the attempt to solve the index numbers problem without reference to the exact context in which they are to be used, is responsible for many of their shortcomings.

Supposing that a convenient unit of time (usually composite) has been chosen, we still have to determine the level of measurement of the other two aspects of economic variables: Their combination yields four categories

1. Variables relating to elementary agents and elementary entities
2. Variables relating to elementary agents and composite entities.
3. Variables relating to composite agents and elementary entities.
4. Variables relating to composite agents and composite entities.

In other words we have to consider cases of no aggregation; of aggregation over entities, agents or both.

IV. Types of Analysis:

It is customary, in economic analysis, to differentiate the micro and the macro levels. Ideally the micro-analysis will be concerned with the interpretation of variables of type(1) although this might be done with reference to variables of other types. On the other

hand macro-economics is essentially concerned with the interpretation of "broad aggregates", i.e., variables of type (4). We have to allocate types (2) and (3) to either branch of analysis. This has to be done according to a precise definition of the two branches, in an exhanstive and a mutually exclusive manner.

According to Bushaw and Clower:(1)

"Broadly speaking, microeconomics is concerned, first, with specifying alternative possible decisions that an individual economic unit might make and, second, with describing the process by which decisions are selected from the admissible alternatives".

This definition emphasizes the decision-making problem at the level of the elementary agent. However, it is not a universal one. Thus, Henderson and Quandt(2) would consider micronmics as: "the study of the economic actions of individuals and well-defined groups of individuals". This latter definition is more in line with the more traditional subdivision of economics into price analysis and income analysis. The term "well-defined groups of individuals" can be seen to mean those groups which are related to an elementary entity. Accordingly this definition implies that microeconomics is essentially the study of elementary entities whether in relation to elementary or composite agents. It follows that macroeconomics would be "the study of broad aggregates such as total employment and national income". Therefore, microeconomics includes categories(1) and (3), while macroeconomics deals with (2) and (4). According to the former definition, microeconomics would deal with elementary agents, categories(1) and (2), while macroeconomics deals with categories (3) and (4). Hence the study of market equilibrium for a single commodity was classified by Bushaw and Clower as macro-economic, while it was included by Henderson and Quandt in their treatment of micro-economics.

(1) D. W. Bushaw and R. W.Clower: Introduction to mathematical Economics, p. 102. Richard Irwin, 1957.

(2) J. M. Henderson and R.E. Quandt: Microeconomic theory - A Mathematical Approach, p.2. Mc graw Hill, 1958.

In justifying the need for a separate discipline for the study of macro-economics, Dernburg and McDugall have emphasized the agents rather than the entities aspect. They stress the fact that:(3)

"Aggregate economic behaviour does not correspond to the summation of individual activities".

What could be included in the ceteris paribus clause in micro-analysis need not remain so in macro-economic problems. In other words the lists of exogenous and endogenous variables are different. According to this point of view, the study of market equilibrium belongs to the field of macro-economics. However, the authors confined their monograph to problems arising in connection with category (4) only. The same emphasis on the aspect of studying the behaviour and activities of individuals as the criterion differentiating micro-economics, appears in many parts of the literature.(4) However, a precise definition of the two disciplines should be based on a better understanding of the role of aggregation.

V. The Need for Aggregation:

The economic theory of any model runs in micro-terms, based on decisions taken by elementary agents (5). This means that we basically possess micro-relationships, i.e., economic relationships explaining variables of type (1). In principle, there is no difficulty in formulating such relationships and discussing their implications so long as the analysis is partial. For a general study, i.e., the study of the whole economy, we can always think of a model which encompasses a whole setup of such micro-relationships. Such a model would in fact be quite cumbersome. Further, there might be some need to concentrate on certain aspects at the aggregate level.

Accordingly it is found convenient to develop economic models on the basis of broad aggregates which attempt at explaining variables of category (4). On the other hand, econometric analysis might

(3) T. F. Dernburg and D.M. Mc Dougall: Macro-Economics - The measurement, Analysis, and Control of Aggregate Economic Activity, p.2 McGraw-Hill 1960.

(4) See, e.g. R.G.D. Allen: Mathematical Economics, p. 694 Macmillan - 1956; C. Abraham and A Thomas: Microéconomie; Decisions Optimales dans L'Entreprise et dans la Nation, p.IX Dunod, 1966

(5) Allen, op. cit.

make it necessary to reconstruct the micro-relations into a macro form before subjecting them to empirical verification. For example a micro-demand-function for a commodity such as wheat might be expressed in terms of individual income, wheat prices and the prices of many related commodities. If we do not possess statistical information at this micro level, we have to replace it by a macro-function, in terms of aggregate income, wheat price and a general price index.

Both in economic analysis and in econometric investigation the question must be raised: What is the exact relationship between the micro- and the macro-relationships? The present conditions are such that the two disciplines co-exist with little to say about their correspondence. Generally speaking, macro-analysis is made with no explicit reference to the underlying micro-theory. Two approaches are open:

1. The economist might refer to certain "first principles" - sometimes to direct observations - in order to build up a certain hypothesis concerning a given macro-relationship. For example the market law of demand has been formulated by Cournot long ago before Marshall introduced his micro-analysis, based on individual decisions.

2. Alternatively, he might go through the existing stock of micro-theory, indicate the relevant factors then elaborates his hypothesis through an implicit (mental) process of aggregation. Thus from an investigation of the factors that affect individual decisions on consumption expenditure, Keynes could derive his macro propensity to consume. He also took into consideration factors that might be at work at the macro-level, such as the effect of income redistribution, to allow for differences between the micro-parameter values.

If the two hypotheses - the micro and the macro - are to be consistent, we have to make an explicit study of the degree by which the one implies the other. Theoretical analysis could do without this imperative condition, but not always with success. The dangers of this crude approach is best illustrated by the fallacy of copying a micro-relationship between the rate of wages and the size of employment into an exactly similar macro-relationship.

The econometrician who attempts to estimate economic models on the basis of micro-theory but using macro-variables faces the same dangers. As we mentioned before, in an attempt to estimate the demand function for wheat, he might build the observable macro-function in exactly the

same form as the basic micro-function, substituting macro- for micro-variables. Before testing the empirical validity of the hypothesis, he should test the new function for consistency with the underlying economic theory. Otherwise he would be investigating a completely new hypothesis without sufficient theoretical investigation. In fact it was the careful attempt to build aggregative econometric models on the basis of economic theory, that led klein to formulate the problem in its proper shape.(6) A number of authors have since joined the controversy, and various criteria and approaches had been suggested.

VI. Approaches to Aggregation:

In general, two main approaches can be distinguished(7):

1. To build up a macro-model which satisfies, the conditions of micro-theory, and makes use of available statistical aggregates.

2. To derive an aggregated model from the micro-theory, then try to construct the statistical aggregates consistent with the derived theoretical ones.

To illustrate the two approaches, let us assume that a micro-function relates the dependent variable y to a number of explanatory variables z_j, such that we have for the i-th individual

$$y_i = a_i + \sum_j b_{ji} z_{ji} \qquad (1)$$

Further, available statistical data define certain aggregates, e.g. of the form:

$$Y = \sum_i s_{oi} y_i \,, \qquad Z_j = \sum_i s_{ji} z_{ji} \qquad (2)$$

Where the s_{ji} are given weights; if they are equal to unity, we obtain natural sums. An aggregate equation similar to (1) in shape and using variables (2) would be:

(6) L.R. Klein: "Macroeconomics and the Theory of Rational Behavior" - Econometrica, vol. 14, pp. 93-108
"Remarks on the Theory of Aggregation" - Econometrica vol. 14, pp. 303-12.

(7) L.R. Klein: Economic Fluctuations in the United States, 1921-1941 pp. 13-14 - John Wiley, 1950.

$$Y = a + \sum_j b_j Z_j \qquad (3)$$

By aggregation we mean the analysis of the conditions under which (3) can be considered as an acceptable representation of the group of equations (1). But according to the second approach, we apply the aggregation of Y to equations (1) to obtain a macro-equation consistent with them:

$$Y = \sum_i s_{oi} a_i + \sum_i \sum_j s_{oi} b_{ji} z_{ji} \qquad (4)$$

The last term need not be directly observable, since it is a function of the (unknown) micro-parameters. Even if the micro-parameters are known a priori we might still be unable to estimate the aggregates thus defined since we do not possess observations on the micro-variables. Another difficulty arises from the fact that when we attempt at building a multi-equational model, the aggregates appropriate for each equation would be different from those relating to another, since the micro-parameters are usually different. It should be also noticed that any deviation from the linear form indicated would complicate the problem.

Using malinvaud's notation(8), let:

Y_o = the set of elements y_i containing the values of the dependent micro-variable.

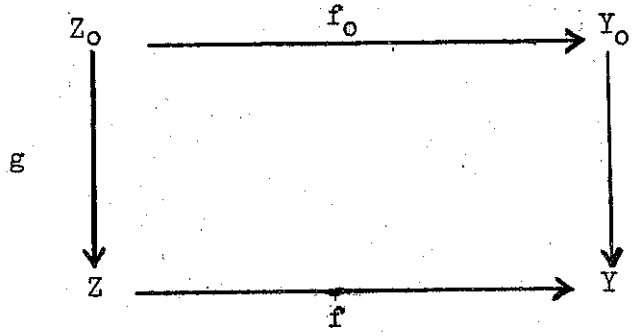
Z_o = the set of elements $\underline{z_i} = (z_{1i}, \ldots\ldots, z_{mi})$ containing the values of explanatory micro-variables.

Y = the set of elements Y containing the values of the dependent macro-variable

Z = the set of elements $Z = (Z_1, \ldots, Z_m)$, containing the values of macro explanatory variables.

Let f_o be the transform (function) of Z_o into Y_o, i.e., the micro-function; and f the transform of Z into Y. Further let g be the transform of the micro-explanatory variables Z_o into Z, and h the transform of Y_o into Y. Then the theory of aggregation means that we have to state the conditions under which the following scheme is consistent:

(8) E. Malinvaud: "L'Aggrégation dans les Modèles Economiques" - pp. 69-146, Cahiers du Séminaire d'Econométrie, Paris, 1956.



(5)

The first approach assumes that f_o as well as g and h are independently given. It investigates the conditions under which a given choice of f would close the scheme. The second approach assumes that bath f_o and h are given, and obtains f as a result of applying h to f_o. The functions g thus derived are then reconciliated to those which normally define Z as functions of Z_o.

It is clear from the above discussion that any solution of the problem is by necessity of an approximative nature. For practical reasons the first approach is preferred. But this need not lead to a clear-cut solution unless we apply the principles of the second approach in order to judge the consistency of the results.

VII. The Treatment of Disturbances:

It has been shown that whenever we have econometric investigation in mind, the first approach would be more convenient, since it recognizes the full implications of the given functions g and h. However, in the econometric formulation of economic relationships due allowance should be given to the specification of the distributions of random disturbances involved. We want to indicate that a complete solution of the aggregation problem should be based on a consideration of the econometric rather than the economic structures of the models envolved.(9)

Suppose that the equation for the i-th individual in period t is as follows:

$$y_{it} = a + b\, z_{it} + u_{it} \qquad (6)$$

where $i = 1,\ldots,N_t$, hence the number of elementary agents is variable over t. Equation (6) assumes similar economic structures for simplification. To complete the specification of the econometric structure, let us assume that for all i and all t:

$$E(u_{it}) = 0, \qquad V(u_{it}) = \sigma^2 \qquad (7)$$

(9) See section I above.