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FOREIGN LOANS AND ECONOMIC DEVELOPMENT

PART III

THE I.B.R.D. APPROACH

by

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CONTENTS

I. Introduction:

In the first two parts of the present memo. we have discussed the conditions under which a single loan can have a given net effect on the size of the country's capital at a certain future date. Two variants of the annual annuity were considered; the decreasing and the fixed. We also discussed the effects of the existence of lags in the shape of gestation periods and grace periods. The guiding principle was to define the addition to the country's capital at a given future date, which was chosen initially as the date of full retirement of the debt. Later it was necessary to modify this rule since it did not fit in quite reasonably with the need to evaluate the actual capital resources required to finance a certain target of economic growth. It is one thing to say that a certain loan can add something to the country's resources at the end of its repayment period; and it is another to tell what it is going to add at a given future date irrespective of the length of that period.

As concluded in the previous parts, we still have to investigate the behaviour of the parameters of the economy in order to take full account of long term development aspects. Before doing that however, we have to discuss the implications of yet another approach, developed by the experts of I.B.R.D. In general, this approach is concerned with the effect of debt servicing on long-term growth. The starting point was to consider a given growth rate, g say, already determined by some model. If this rate requires more investments than could be financed by the self-generated savings of the country, the need for borrowing, and further, the size of foreign loans (assumed to be the only source of external finance) will be determined. The main question will, therefore, be: Under what conditions can the country maintain the given rate of growth and at the same time be able to service the ensuing debt?

In other words, the I.B.R.D. experts are interested in the capacity to service the debt under given development requirements. This is a point of view of a creditor rather than a debtor, which is natural if we look at the problem from the side of the Bank. However, it leaves much to be desired. In the first place, the approach does not tell the developing country how to fix its target growth rate, which will eventually be a function of loan conditions. The implicit advice given in the form of the necessity to stop borrowing if repayment is impossible within a reasonably long period in the future (e.g., over a generation) begs the problem. As was shown before, there are cases where a country could have ended with a larger capital stock without, rather than with, the loan. If this is so, we might face a situation in which the capacity to repay exists, but the net outcome is much below the level achievable in the absence of loans. Such situations do exist and they call for a reconsideration of the whole approach, in spite of its seemingly plausible foundations.(1)

Two variants of the I.B.R.D. model will be considered. The first was prepared about ten years ago by Gerald M. Alter for a conference organized at Rio de Janeiro, August 1957; by the International Economic Association. Alter's paper(2) is concerned with the determination of the critical level of the marginal propensity to save, required to ensure the realization of a certain rate of growth, and at the same time delimiting the cumulation of foreign debt within predetermined bounds. This model has been later simplified by Avramovic and his colleagues,(3) and used to study the so-called debt cycle. We shall give in the following sections a summary of these studies, then show how far the whole approach can be relied upon to deal with the real problems of economic development.

To the list of notations given in pages 4-5 of Part I of this Memo., we shall be in need of the following additional notation:

- P = Population
- p = rate of growth of population
- g = rate of growth of G.D.P.
- s_t= average propensity to save in year t
- σ'= per capita marginal propensity to save
- g'= rate of growth of per capita G.D.P.
- L_t= new loans needed in year t.
- D_t= Deficit of balance of payments in year t
- Q_t= Cumulative debt at end of year t
- n = target year for which the cumulative debt, or its ratio to G.D.P. reaches a certain (extreme) level.

(1) M.M.El-Imam: The Contribution of Foreign Loans to Economic Development. April, 1968 - T.U. 34, of specialized course on Financing of Development, organized jointly by I.N.P. and I.D.E.P., Cairo, 1968

(2) G.M. Alter: "The Servicing of Foreign Capital Inflow by Under developed Countries". in, H.S. Ellis and H.C.Wallich (eds): Economic Development for Latin America; Macmillan, N.Y. 1961.

(3) D. Avramovic, et al: Economic Growth and External Debt; I.B.R.D., Johns Hopkins, Baltimore, 1964.

Variables measured on a per capita basis will be denoted by the same symbols primed; e.g., Y', S', I'.

II. Alter's Model:

Alter prefers to work with per capita rather than aggregate variables. With constant rates of population growth, and of income growth, this implies a constant overall rate of growth of G.D.P.:

$$(1 + g) = (1 + g')(1 + p) \tag{1}$$

However, this would generally lead to a variable overall marginal propensity to save, even though the per capita m.p.s. is assumed constant:

$$\sigma_t = \sigma' + \frac{p}{g}(s_{t-1} - \sigma') \tag{2}$$

This means that if the m.p.s. is greater than the a.p.s. in the base year, the overall m.p.s. will be smaller than the per capita.

Starting from the definitions:

$$\sigma' = \frac{S'_t - S'_{t-1}}{Y'_t - Y'_{t-1}}, \qquad g' = \frac{Y'_t - Y'_{t-1}}{Y'_{t-1}} \tag{3}$$

we obtain the difference equation:

$$S'_t = S'_{t-1} + \sigma' g' Y'_o (1 + g')^{t-1} \tag{4}$$

The solution of this equation is:(4)

$$S'_t = S'_o + \sigma' Y'_o \left[(1 + g')^t - 1\right] \tag{5}$$

Multiplying throughout by $P_t = P_o (1 + p)^t$, and denoting, as before, the a.p.s. in the base year by s, we obtain the formula for <u>overall savings</u>

$$S_t = \left[\sigma'(1 + g)^t + (s - \sigma')(1 + p)^t\right] Y_o \tag{6}$$

(4) See Appendix A, equation (A/34)

On the other hand, overall investment required to realize income growth at the given rate can be determined by means of the capital coefficient:

$$I_t = \gamma g Y_o (1+g)^t \qquad (7)$$

Then the deficit of the current account of the balance of payments will be:

$$D_t = I_t - S_t = \left[(\gamma g - \sigma')(1+g)^t - (s - \sigma')(1+p)^t \right] Y_o \qquad (8)$$

If this quantity is positive, the country will be in no position to repay anything of the out-standing debt. In fact it has to borrow that amount together with interest charges on the out-standing debt. If according to the conditions of previous loans, a part of the principal is to be repaid, an equivalent amount should be borrowed.

Thus, the net flow of new loans in year t would be

$$L_t = r.\, Q_{t-1} + D_t \qquad (9)$$

The cumulative debt outstanding at the end of year t will therefore be,

$$Q_t = Q_{t-1} + L_t$$

or

$$Q_t = (1+r)\, Q_{t-1} + D_t \qquad (10)$$

The solution of this difference equation for the case $g \neq r$ is:[5]

$$Q_t = \left[(\gamma g - \sigma') \left\{ \frac{(1+g)^{t+1} - (1+r)^{t+1}}{g - r} \right\} - (s - \sigma') \left\{ \frac{(1+p)^{t+1} - (1+r)^{t+1}}{p - r} \right\} \right] Y_o \qquad (11)$$

If $g = r$, we have:[6]

$$Q_t = \left[(\gamma g - \sigma')(t+1)(1+g)^t - (s - \sigma') \left\{ \frac{(1+p)^{t+1} - (1+g)^{t+1}}{p - g} \right\} \right] Y_o \qquad (12)$$

If $p = r$, a similar formula can be derived,

$$Q_t = \left[(\gamma g - \sigma') \left\{ \frac{(1+g)^{t+1} - (1+r)^{t+1}}{g - r} \right\} - (s - \sigma')(t+1)(1+g)^t \right] Y_o \qquad (13)$$

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(5) Appendix A, equation (A/40)
(6) Appendix A, equation (A/41)

The formula for cumulative debt can be used to answer the question: What are the conditions under which the country can ensure the service of debts, and at the same time go on with the implementation of its long term development activities?

If such conditions can be easily satisfied, there will be no problem in servicing the debt. On the contrary, the problem will be to absorb more loans, in the sense of finding more investment possibilities to be financed by means of these loans. On the other hand, if it is difficult to realize such conditions, it will be difficult, or even dangerous, to get envolved in any debt, however small.

Suppose that the values of the parameters, p, s, r, g and γ are given. Then knowing σ' we can calculate Q for any future date n considered by the planners. This will be a direct application of (11); but it does not help in formulating a decision. Therefore, Alter suggests that a certain restriction on the value of Q, then solve (11) to obtain the value of σ' necessary to realize that restriction.

III. Alternative Constraints on Debt:

The need for putting constraints on debt arises from the possibility that debt might grow on indefinitely, which is a situation impossible to sustain. Starting from a given base year deficit, debt will be growing so long as new deficits are either positive, or negative but smaller than what is necessary to provide for interest charges. At the beginning debt will be growing faster than income. Then there will be two possibilities. either it will remain to do so, or starts to grow at decreasing rates. We execlude the first alternative.

For the second, there must be a point at which the rate of growth of debt will fall down to that of income. In fact, if the rate of growth of debt at a point t is q_t, then it will be found that the condition that the ratio of debt to income is a maximum will be:

$$\frac{\partial}{\partial t}\left(\frac{Q_t}{Y_t}\right) = \frac{1}{Y_t^2}\left[Y_t \cdot q_t \cdot Q_t - Q_t \cdot g \cdot Y_t\right] = o$$

which means that:

$$q_t = g \qquad (14)$$

If the rate of growth of debt diminishes after that, there will be a point at which that rate is equal to zero. This is, in fact the condition that debt will reach a certain maximum at this point.

$$q_t = 0, \qquad \text{or,} \qquad L_{t+1} = 0 \tag{15}$$

After this point the outstanding debt will start to decrease. There will arrive another point at which it will completely vanish:

$$Q_t = 0 \tag{16}$$

Afterwards the country has to worry no more about debts: It will become a net creditor rather than net debtor.

Thus, for a given point of time, n, we can find the value of σ' necessary to achieve any of the above three turning points. Let us therefore rewrite (11) as follows:

$$Q_n = \left[\sigma' \left(\frac{P-R}{p-r} - \frac{G-R}{g-r}\right) + \gamma g \left(\frac{G-R}{g-r}\right) - s\left(\frac{P-R}{p-r}\right)\right] Y_o \tag{17}$$

where,

$$P = (1+p)^{n+1}, \qquad R = (1+r)^{n+1}, \qquad G = (1+g)^{n+1} \tag{18}$$

The critical values of σ' can be obtained as follows:

1. The most restrictive condition is (16). Substituting (17) and solving for σ' we obtain:

$$\sigma' = \left[s\left(\frac{P-R}{p-r}\right) - \gamma g \left(\frac{G-R}{g-r}\right)\right] \Big/ \left[\left(\frac{P-R}{p-r}\right) - \left(\frac{G-R}{g-r}\right)\right] \tag{19}$$

2. A less restrictive condition is to assume that debt will reach absolute maximum in year n. We use (15), which means that:

$$L_{n+1} = r\,Q_n + D_{n+1} = 0$$

Hence,

$$\left[r\sigma' \left(\frac{P-R}{p-r} - \frac{G-R}{g-r}\right) + r\gamma g \left(\frac{G-R}{g-r}\right) - rs\left(\frac{P-R}{p-r}\right)\right] + \left[\sigma'(P-G) + \gamma g\,G - sP\right] = 0$$

This can be rearranged as follows:

$$\sigma' \left(\frac{pP-rR}{p-r} - \frac{gG-rR}{g-r}\right) + \gamma g \left(\frac{gG-rR}{g-r}\right) - s\left(\frac{pP-rR}{p-r}\right) = 0$$

Which can be solved to give:

$$\sigma' = \left[s\left(\frac{pP-rR}{p-r}\right) - \gamma g \left(\frac{gG-rR}{g-r}\right) \right] / \left[\left(\frac{pP-rR}{p-r} - \frac{gG-rR}{g-r}\right) \right] \qquad (20)$$

3. Finally, the mildest condition is to allow debt to reach a maximum rate of increase in year n, hence grow at the same rate as income, as in(14) This means that:

$$Q_{n+1} = (1+g)\, Q_{n+1}$$

or,

$$(1+r)\, Q_n + D_{n+1} = (1+g)\, Q_n$$

In other words:

$$(r-g)\, Q_n + D_{n+1} = 0$$

Substituting as before, we obtain:

$$\sigma' = \left[s \left\{ (p-g)\, P + (g-r)\, R \right\} - \gamma g (p-r) R \right] / (p-g)(P-R) \qquad (21)$$

Commenting on these formulae, Alter states that[7]

"It is perhaps obvious that all other things being equal the target rate of increase of per capita income, compared with the rate that can be achieved in the absence of foreign capital inflow, may be put at a higher level and a larger volume of foreign capital inflow is permitted when:

1. the marginal savings ratio is higher;
2. the incremental capital-output ratio is lower,
3. the rate of population increase is lower,
4. the required rate of return on foreign capital inflow is lower,
5. the degree of independence of foreign capital that must be achieved within a given time period is lower;
6. the time period in which a given degree of independence must be achieved is longer".

No formal proof was given to verify the validity of these conclusions which were merely described as "obvious". What the above formulae indicate are certain critical values of σ', which could be compared with actual values in order to determine the possibility of achieving a certain degree of independence of foreign loans within a given period. However, some numerical examples can be calculated to illustrate the results, and the following is one given by Alter.

(7) op. Cit., p. 149

<u>Example (1):</u> Suppose that,

rate of growth of population, $p = 0.025$
rate of return on foreign capital, $r = 0.045$
initial average savings ratio, $s = 0.085$
capital/output ratio, $\gamma = 3.5$
target year, $n = 24$

It is required to investigate the marginal savings ratio σ' for three alternative targets of the rate of growth g' of per capita income, under the condition that external debt should reach a maximum in 24 years, (formula 20). Having done that, we can substitute in (17) to obtain Q_n, then relate it to Y_0 or to aggregate investment. The results are summarized as follows:

Per capita Rate of Growth g'	Required marginal Savings Ratio σ'	Capital Inflow as Ratio of	
		Initial National Income	Aggregate Net Investment
0.005	0.23	0.23	0.07
0.01	0.28	0.61	0.14
0.02	0.31	1.42	0.22

Thus an increase in the marginal savings ratio from 0.23 to 0.31 (i.e., by about one-third), enables an increase in the rate of growth four times, and capital inflow could be increased over six times, while it remained possible to service it so that it reaches a maximum in 24 years. The rest of Alter's argument relates mainly to the risks befalling the creditor, which is natural for an expert on the supplying side.

IV. <u>Implications of Alter's Model</u>:

For any positive rate of growth of per capita income, we have $g > p$. Further, it is clear from (8), that for D_0 to be positive, we should have, $\gamma g > s$. This means that $- sP > - \gamma gP$, where P is defined according to (18). Thus, if D is to become negative at some point of time, n+1 say, we should have:

$$0 > D_{n+1} > (\gamma g - \sigma')(G - P)$$

Which means that $\sigma' > \gamma g$. It follows that:

$$\frac{s}{\gamma} < g < \frac{\sigma'}{\gamma} \qquad (22)$$

Consider now equation (19) which is given in the same order suggested by Alter (Formula I, p. 159). First we notice that:

$$\frac{G-R}{g-r} - \frac{P-R}{p-r} = (1+r)^n \sum_{i=0}^{n} \left(\frac{1+p}{1+r}\right)^i \left[(1+g')^i - 1\right] > 0 \qquad (23)$$

Hence we multiply both numerator and denominator of (19) by - 1, to ensure that the latter is positive. This means that (19) can be rewritten as follows:

$$\sigma' = \gamma g + (\gamma g - s) \left(\frac{P-R}{p-r}\right) / \left[\frac{G-R}{g-r} - \frac{P-R}{p-r}\right] \qquad (24)$$

The factor multiplying (γ g-s) is positive, which ensures that σ' satisfies (22).

Similar remarks can be made with respect to (20). It is evident that:

$$\frac{gG - rR}{g - r} = g \left(\frac{G-R}{g-r}\right) + R = \left[p+g'(1+p)\right]\left(\frac{G-R}{g-r}\right) + R$$

Further,

$$\frac{pP-rR}{p-r} = p \left(\frac{P-R}{p-r}\right) + R = p \left(\frac{P-R}{p-r}\right) + R$$

Hence

$$\frac{gG-rR}{g-r} - \frac{pP-rR}{p-r} = p\left[\frac{G-R}{g-r} - \frac{P-R}{p-r}\right] + g' (1+p) \cdot \left(\frac{G-R}{g-r}\right)$$

Again we have to multiply both numerator and denominator of (20) by (-1), to ensure that the latter is positive. Thus we can rewrite (20) as follows:

$$\sigma' = \gamma g + (\gamma g-s)\left(\frac{pP-rR}{p-r}\right) / \left(\frac{gG-rR}{g-r} - \frac{pP-rR}{p-r}\right) \qquad (25)$$

Finally, equation (21) can be rewritten as:

$$\sigma' = \gamma g + (\gamma g-s) \left[(g-r)R - (g-p)P\right] / (g-p)(P-R) \qquad (26)$$

Alternatively it can be rewritten as:

$$\sigma' = \gamma g + (\gamma g-s)\left[R - (g-p) \left(\frac{P-R}{p-r}\right)\right] / (g-p) \left(\frac{P-R}{p-r}\right) \qquad (27)$$

Comparing these forms, we find that they take the general formula:

$$\sigma' = \gamma g + (\gamma g - s) X_{jn} \qquad (j=1,2,3) \qquad (28)$$

Given that $\gamma g > s$, it follows that $\sigma' > \gamma g$ if X is positive. The factors X are;

$$\left.\begin{aligned} X_{1n} &= \frac{\frac{P-R}{p-r}}{\frac{G-R}{g-r} - \frac{P-R}{p-r}} \\ X_{2n} &= \frac{\frac{pP-rR}{p-r}}{\frac{gG-rR}{g-r} - \frac{pP-rR}{p-r}} \\ X_{3n} &= \frac{R}{(g-p)\cdot\left(\frac{P-R}{p-r}\right)} - 1 \end{aligned}\right) \qquad (29)$$

It is obvious that both

$$X_{1n} > 0 \quad \text{and} \quad X_{2n} > 0 \qquad (30)$$

Since X_{3n} is the difference between two positive quantities, its sign has to be further investigated . On the other hand two properties can be expected a priori:

1) The factor X is smaller for the milder conditions, which means that, for any $n > 0$:

$$X_{1n} > X_{2n} > X_{3n} \qquad (31)$$

2) The factor X is smaller for larger values of n; i.e., it is a decreasing function of n:

$$X_{j,n+1} > X_{jn} \qquad (j = 1,2,3) \qquad (32)$$

To prove these properties we introduce the following abbreviated notation:

$$\left.\begin{aligned} a &= \frac{P-R}{p-r}, & b &= \frac{G-R}{g-r} \\ A &= \frac{P(1+p) - R(1+r)}{p-r}, & B &= \frac{G(1+g) - R(1+r)}{g-r} \\ c &= A - a = \frac{pP-rR}{p-r}, & d &= B-b = \frac{gG-rR}{g-r} \end{aligned}\right\}$$

$$\therefore\ c = ra + P, \qquad d = rb + G$$

$$C = \frac{P(1+p)p - R(1+r)r}{p - r}, \qquad D = \frac{G(1+g)g - R(1+r)r}{g - r}$$

$$\therefore\ C = (1+r)c + pP, \qquad D = (1+r)d + gG \tag{33}$$

It follows that:

$$X_{1n} = \frac{a}{a-b}, \qquad X_{2n} = \frac{c}{d-c}$$

For the first inequality in (31) to hold, we should have

$$(ad-ac) - (ca-cb) = a(B-b) - b(A-a) = aB - Ab > 0$$

But:

$$aB - Ab = a\left[(1+r)b + G\right] - \left[(1+r)\ a + P\right] b$$

$$= aG - bP$$

$$= (1+g)^{n+1} \sum_{i=0}^{n} (1+r)^{i}(1+p)^{n-i} - (1+p)^{n+1} \sum_{i=0}^{n} (1+r)^{i}(1+g)^{n-i}$$

$$= (1+g)^{n}(1+p)^{n+1} \left[\left(\frac{1+g}{1+p}\right) \sum_{i=0}^{n} \left(\frac{1+r}{1+p}\right)^{i} - \sum_{i=0}^{n} \left(\frac{1+r}{1+g}\right)^{i}\right]$$

$$= (1+g)^{n}\ (1+p)^{n+1} \sum_{i=0}^{n} \left(\frac{1+r}{1+g}\right)^{i} \left[(1+g')^{i+1} - 1\right]$$

For all $i \geqslant 0$, the last expression between square brackets is positive
It follows that:

$$aB > Ab \tag{34}$$

which ensures the first inequality in (31). In other words, the most stringent condition (1) requires a higher m.p.s. than the milder condition (2).

Further, if we subtract from both sides of (34) the quantity aA, we find that:

$$\frac{a}{b - a} > \frac{A}{B - A}$$

This means that properety (2), stated by (32) in satisfied for X_{1n}, determining the m.p.s. for condition (1). To prove that this applies for condition (2), so that:

$$\frac{c}{d - c} > \frac{C}{D - C}$$

we have to prove the validity of a formula similar to (34), namely:

$$cD > Cd \qquad (35)$$

Now,

$$cD - Cd = c\left[(1+r)d + gG\right] - \left[(1+r)c + pP\right] d$$

$$= (ra + P)\ gG - pP(rb + G)$$

$$= rg\ (aG - bP) + (g-p)\ P\ (rb + G)$$

$$= rg\ (aB - Ab) + (g - p)\ Pd$$

All factors in both terms are psotive; hence, (35) is satisfied. Hence property (2) applies, and (32) is satisfied for condition (2).

To show that the second of inequalities (31) is true we evaluate $X_{2n} - X_{3n}$. For this purpose we rewrite them as follows:

$$X_{2n} = \frac{R + pa}{gb - pa}\ , \qquad X_{3n} = \frac{R - (g-p)\ a}{(g-p)\ a}$$

The sign of the difference depends on the sign of its numerator:

$$(g-p)\ a\ (R+pa) - (gb-pa)\left[(R - (g-p)\ a\right]$$

$$= (g-p)\ a\ (pa + gb - pa) + R\ (ga - pa + pa - gb)$$

$$= g\left[(g-b)\ ab + R(a-b)\right] = g\left[a\ (gb+R) - b\ (pa+R)\right]$$

$$= g\ (ad - bc) = g\ (aB - Ab)$$

This means that $X_{2n} > X_{3n}$.

Finally, for $t = n + 1$, we have: