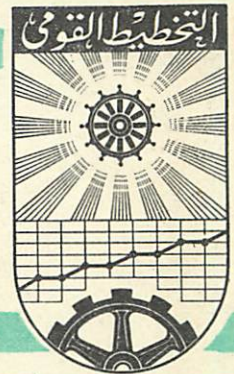


# UNITED ARAB REPUBLIC

## THE INSTITUTE OF NATIONAL PLANNING



Memo. No. 1110

PRACTICAL USE OF ECONOMIC-MATHE-  
MATICAL MODELS IN PLANNING PRACTICE  
(Applicable to input-output balance)

By

Dr. B. ZAITSEV

June 1975

PRACTICAL USE OF ECONOMIC-MATHEMATICAL  
MODELS IN PLANNING PRACTICE

(Applicable to input-output balance)

The computers are used in the work of the USSR Gosplan in two main directions:

- implementation of direct plan estimates;
- modelling of economic processes and plan estimates based on economic-mathematical models.

The direct plan estimates are built up on the basis of simple algorithms containing no logical operations. An example may be the estimates of requirements in material resources for production and operation needs that contain only operations of multiplying a vector by a matrix and aggregation. The direct plan estimates are connected with processing of big volumes of data (dozens and hundreds of thousands of numbers) and their implementation covers about 70% of machine time of computers available at the USSR Gosplan disposal. The said estimates are practically made for all sections of national economic plan.

It is known that the economic-mathematical models can not give an exhaustive description of actual economic processes. An attempt to take into account all details leads to bulky constructions and inevitably entails computing difficulties. Therefore, the structure of model takes into account only main essential features of described process and each of the models is designated for solution of certain problems. The system of interconnected models is required for multiform description of economic processes. In the meantime the USSR Gosplan has a general idea of such system of models, however applied practically

are only individual models experimentally tested, for solution of which there is a relevant mathematical apparatus and mainly necessary plan-economic information. It is natural that while using a complex of models that cover only a part of plan-economic problems a problem of organic coordination of planning economic-mathematical methods with so-called "traditional" methods of plan compilation arises. All economic-mathematical models to be practically applied in the USSR Gosplan are built up on the principle of maximum use, as initial information, of "traditional" plan estimates results with compulsory use of modelling results in further plan estimates.

All information-norm facilities of economic-mathematical models are worked out in accordance with general principles of forming the norm facilities of national economic plan and based on the estimates and substantiation of indicators to be contained in relevant sections of the state plan. The basic method of substantiation of the norms to be used in economic-mathematical models is a techno-economic designing with due account to the factors influencing the change of their value in the planned period, particularly the factors of technological progress. Considered hereby is an experience of work of advanced enterprises and production corporations, world achievements, possibilities that appear due to use of new design solutions, new structural materials and technological methods.

The major source of information for the plan input-output balance is the reported input-output balances that are made out by the Central Statistics Agency in monetary and physical terms as per nomenclature agreed upon with the USSR Gosplan. The said balances to the full extent are made out regularly once in 5-6 years.

Moreover, the reported balances with an enlarged sectoral nomenclature corresponding to the nomenclature of enlarged dynamic models to be used in the USSR Gosplan are annually made out. The enlarged balances are also supplemented with comprehensive production and distribution balances for the most important commodities in physical terms.

With respect to the fact that the USSR Gosplan is the directive body which decisions on plan problems are obligatory to everyone, it is in a position apart from its scientific-research institutes to involve all scientific-research organizations of the country in working out and realizing the economic-mathematical models. The said scientific-research organizations in conformity with the USSR Gosplan plan-economic assignment shall work out economic-mathematical models, form initial information and, sometimes carry out estimates on models and submit their results to the USSR Gosplan. As a rule, the proposed economic-mathematical models and initial information thereto undergo a compulsory expertise in the Gosplan departments. The estimates on models are usually made in the main Computing Centre of the Gosplan being responsible for introduction of economic-mathematical methods and computers in the planning, for implementation of operative plan estimates at the computers for the USSR Gosplan.

However, the major part of economic-mathematical models to be practically applied is worked out and prepared to introduction in the Gosplan.

In general, the process of realization of economic-mathematical models in the USSR Gosplan may be represented as follows:

Content of work	Plan-economic assignment	Economic-mathematical models, algorithms and methods of problems solution	Formation of initial information	Estimates on models	Analysis of results and correcting of initial data
Executives	USSR Gosplan Departments	USSR Gosplan Departments (Main Computing Centre scientific-research organizations (Scientific-Research Institute)	USSR Gosplan Departments (Main Computing Centre Scientific Research Institute)	USSR Gosplan Main Computing Centre	USSR Gosplan Departments

The models to be practically used in the USSR Gosplan on contents of problems to be solved may be divided in the following main groups:

- national economic models (mainly input-output balance models);
- sectoral planning models (optimization of production plans, development and distribution of sectors, forecasts of demand for products of a sector, etc.);
- models of solving individual plan problems (forecast of population number, demand for consumer goods, etc.).

The input-output balances to be used in the USSR Gosplan differ from each other by a degree of nomenclature details (enlarged or comprehensive), applied measuring units (physical, monetary, physical-monetary), range of coverage of economic processes (national economic, regional, sectoral).

At the preliminary stages of national economic plan preparation used are the enlarged dynamic models of input-output balance that are built up for not more than 30 sectors in monetary terms. The simplified model is given in Appendix No. 1. On the basis of this model at the stage of working out the major trends of national economic development for the planned period the national economic plan central department in collaboration with other central departments makes alternative estimates with a view of working out a preliminary hypothesis on major national economic and input-output proportions of economic development in conformity with the draft plan social and economic conception. The estimates on model are made consecutively commencing from the first year of considered period. The estimates include determination of production and investment volumes in the sectors proceeding from the stipulated structure of final net production (estimated are various alternatives of final production that reflect different hypotheses of meeting the final demand).

The above model is the simplest model of national economic development. The USSR Gosplan uses also more complicated models, particularly, dynamic models that take into account in their structure the logs "capital investments-production increase" as well as dynamics of development of capital investments and production capacities. The basic ratios of these models are built up according to the following principle. The volume of capital investments each year of the planned period is determined as a sum of expense elements (it is shaded in the drawing) for construction of projects started in various years.

	t-3	t-2	t-1	t			
$M_{t-3}$	$\alpha_1$	$\alpha_2$	$\alpha_3$	$\alpha_4$			
$M_{t-2}$		$\alpha_1$	$\alpha_2$	$\alpha_3$	$\alpha_4$		
$M_{t-1}$			$\alpha_1$	$\alpha_2$	$\alpha_3$	$\alpha_4$	
$M_t$				$\alpha_1$	$\alpha_2$	$\alpha_3$	$\alpha_4$

6.

$$Q_t = Q'_{t-3} \alpha_4 + Q'_{t-2} \alpha_3 + Q'_{t-1} \alpha_2 + Q'_t \alpha_1$$

or  $Q^t = M_{t-3} q_{t-3} \alpha_4 + M_{t-2} q_{t-2} \alpha_3 + M_{t-1} q_{t-1} \alpha_2 + M_t q_t \alpha_1$

where:

$Q^t$  - capital investments in the sector in  $t$  year;

$Q'_{t-1}, Q'_{t-2}, \dots$  - capital investments in the projects that construction started in  $t-1, t-2$  years;

$\alpha_1, \alpha_2, \dots$  - coefficients of development of construction estimated cost (i.e. share of construction expenses falling upon each year of construction period);  $\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 = 1$

$M_{t-1}, M_{t-2}, \dots$  - production capacities of enterprises that construction started in years  $t-1, t-2, \dots$

$q$  - specific capital investments. (capital-output ratio)

Development of newly production capacities under construction is determined according to analogous principle. Let's consider an example when the enterprise in any sector is constructed within four years (at an average). Let's assume that a certain part of production capacities is being developed in the last

year of construction period ( $\beta_1$  - coefficient of capacities development) and full development of capacities is completed within the next two years. Then the amount of developing new production capacities in year  $t$  is determined as a sum of elements (it is shaded in the drawing):

	$t-5$	$t-4$	$t-3$	$t-2$	$t-1$	$t$		
$M_{t-5}$	$\alpha_1$	$\alpha_2$	$\alpha_3$	$\alpha_4$				
				$\beta_1$	$\beta_2$	$\beta_3$		
$M_{t-4}$	$\alpha_1$	$\alpha_2$	$\alpha_3$	$\alpha_4$				
				$\beta_1$	$\beta_2$	$\beta_3$		
$M_{t-3}$	$\alpha_1$	$\alpha_2$	$\alpha_3$	$\alpha_4$				
				$\beta_1$	$\beta_2$	$\beta_3$		

$$\Delta M^t = M^{t-5}\beta_3 + M^{t-4}\beta_2 + M^{t-3}\beta_1$$

$M^{t-5}, M^{t-4}, \dots$  - production capacities, construction started in years  $t-5, t-4, \dots$

$\beta_1, \beta_2, \dots$  - coefficients of developing production capacities (they are put in by an increasing result, i.e.

$$\beta_3 = 1.00).$$

In general, major ratios of model:

- production and distribution balance

$$X_i^t - \sum_j a_{ij}^t X_j^t - \sum_j b_{ij}^t Q_j^t = y_i^t \quad (1)$$

- capital investments balance

$$Q_j^t = \sum_{k=1}^{\tau_j} M_j^{t-(\tau_j-k)} \alpha_j^{t-(\tau_j-k)} \alpha_j^k \quad (2)$$

- production capacities balance (provided the loading is full)

$$X_j^t = \bar{M}_j^t + \sum_{m=0}^{\tau_j-1} M_j^{t-\tau_j+m} \beta_j^{\tau_j-m} \quad (3)$$

where new symbols are as follows:

$\tau_j$  - average construction period of enterprises in sector  $j$  (in years);

$\bar{M}_j^t$  - completely developed capacities in sector  $j$   
by year  $t$  with due account to removal;

$\mathcal{Y}_j$  - by average continuity of production capacities  
development period in sector  $j$

The simplest alternative of model of input-output balance optimization may be received by introduction of some corrective <sup>objective</sup> functions in formula (1) and (3) as well as purposeful function

$$\alpha^T \rightarrow \max \quad (T - \text{last year of planned period}) \quad \text{or} \quad \sum_{i,t} C_i^t \alpha^t \Delta y_i^t \rightarrow \max$$

where  $C_i^t$  - weight coefficient. Formula (1) introduces therein  $y_i^t = y_{i \min}^t + \alpha^t \Delta y_i^t$ ,  $y_{i \min}^t$  -

- minimum permissible final net product for sector  $i$  in year  $t$ ;  $\Delta y_i^t$  - elementary set of products that determines the structure of final net product increase in year  $t$ .

Formula (3) introduces therein a symbol of inequality:  $\chi_j^t \leq \bar{M}_j^t + \dots$

Unknown variable quantities in the said model are the production volumes  $\chi^t$ , velocity of final net product increase  $\alpha^t$ , capital investments in sector  $Q^t$ , size of production capacities and commencing dates of new enterprises construction  $M$ .

The above model may be used for solution of the following problem as well. There is a reported input-output balance as well as a list of projects that may be implemented in the plan period. It is required to determine the most expedient priority of projects implementation proceeding from available resources of capital investments, and, if the said resources are not sufficient, to reveal also the projects that shall be expeditiously implemented in the next plan period. To solve the above problem introduced is a conception of production methods

that mean, in this case, output of products in each sector at the existing enterprises and newly ones to be under construction in conformity with available projects. If the production methods are illustrated through  $S$  ( $S = 1, 2, \dots, n$ ), then  $X_j^{t,s=1}$  means, for example, a production volume in sector  $j$  in year  $t$  at existing enterprises;  $X_j^{t,s=2}$  - production volume in sector  $j$  in year  $t$  at the enterprise which construction started in the previous plan period and that shall be commissioned in the considered period;  $X_j^{t,s=3}$  - production volume in sector  $j$  in year  $t$  at enterprise No. 1 (Project No. 1) which construction started in the considered period;  $X_j^{t,s=4}$  - ditto but for enterprise No. 2 (Project No. 2) and so on. Each method is characterized by own set of norms to be used in the model.

With due account to the above this model may be specified as follows:

$$(1) \sum_s X_j^{t,s} - \sum_s \sum_j a_{ij}^{t,s} X_j^{t,s} - \sum_s \sum_j b_{ij}^{t,s} Q_j^{t,s} = y_i^t$$

$$(2) Q_j^{t,s} = \sum_{k=1}^{T_j^s} M_j^{s,t-(T_j-k)} Q_j^{s,t-(T_j-k)} \alpha_j^{s,k}$$

$$(3) \sum_s X_j^{t,s} \leq \bar{M}_j^t + \sum_s \sum_{m=0}^{Y_j^s-1} M_j^{s,t-Y_j^s+m} \beta_j^{s,Y_j^s-m}$$

$$(4) \sum_s \sum_j Q_j^{t,s} \leq \bar{Q}^t, \text{ or } \sum_t \sum_s \sum_j Q_j^{t,s} \leq \bar{Q}$$

$\bar{Q}^t$  - amount of capital investments available in the national economy in year  $t$  for production development,

$\bar{Q}$  - ditto for the period as a whole.

$$(5) \sum_s \sum_j \ell_j^{t,s} X_j^{t,s} \leq L^t \quad \text{where } \ell_j^{t,s} \text{ - requirement in skilled manpower per unit of production in sector } j \text{ with production method } S \text{ in year } t;$$

$L^t$  - availability of skilled workers in year  $t$ .

$$(6) \quad y_i^t = y_{i, \min}^t + \alpha^t \Delta y_i$$

$$(7) \quad \alpha^T \rightarrow \max \quad \text{or} \quad \sum_i \sum_t c_i^t \alpha^t \Delta y_i^t \rightarrow \max$$

It is natural that the content of purposeful formula (7) in the model may be changed depending on the targets set for the national economy in the considered period that will result in some correctives and other terms.

The above model facilitates to receive a sectoral structure of production and capital investments, evaluate requirements in manpower, select out of the total list of projects the most expedient priority of projects implementations. The following conditions are automatically observed, such as top-priority of completing the commenced construction sites, increase of loading level of existing production capacities.

As a result of problem solution obtained are mean weighed norms that take into account realization of planned projects of sectors development and, consequently, being planned norms. Therefore, application of such approach facilitates essentially a problem of building up the plan input-output balance providing to correct the reported data with the help of information that contains in the projects. It is natural hereby that the information contained in the projects shall reflect requirements of the model, i.e. apart from the size of production capacities each project shall stipulate requirements in raw and other materials (including fuel and electric power) to be consumed for current production (coefficients  $a_{ij}$ ) and for capital investments, time-periods and coefficients of their development, coefficients of production capacities development.

The structure of input-output balance enlarged models and methods of their use differ subject to the problems to be set for a user of models. In particular, the complex of models may be used for coordination of long-term, medium-term and annual plans for the purpose of simplifying. For instance, the basic outlines of national economic development for 1990 are determined according to one of the models proceeding from which the basic indicators for ending years of five-year period (1980, 1985) are determined according to other model, the indicators for each year of the first five-year period (1976, 1977, 1978, 1979) are determined according to the third model. The said complex of models is worked out and used in the USSR Gosplan for operative estimates in the regime of plan preparation (See description of models in Appendix No. 2).

The enlarged monetary models are used at the preliminary stages of plan preparation. The results of alternative estimates according to such models after expertise and correcting in the Gosplan departments are a basis for working out comprehensive statistical input-output balances being of particular interest to the Gosplan, for they show monetary as well as commodity proportions in physical terms. (See Appendix No. 3). The model of the above balance includes 14 groups of equations: balances of products in physical and monetary terms with respect to Ministries, agencies and sectors, equations of commodity turnover, capital investments, manpower, etc. The commodity nomenclature of physical-monetary balances includes about 260 major commodities for perspective plans (medium-term and long-term) and about 700

commodities for annual plans. The balance contains also the data on products distribution by final product elements, national income, manpower required for provision of planned production level (See Balance Diagram). The initial data for estimating the balanced plan according to the model of physical-monetary input-output balance are the indicators for national income, export, import and overhaul repairs (See Diagram of Balance Estimate). The estimates of input-output balance are made at the preliminary stages of plan preparation for determining the balanced levels of products output in physical and monetary terms.

At the initial stages of plan preparation the estimates are made according to the formula:  $(E-A)^{-1}Y = X$  where E- single matrix; at the completion stages- according to the formula  $(E-A)X=Y$ . To correct the plan in the process of its elaboration the mixed estimates are also made, when production volumes are determined for one products according to known values of final commodity and vice versa the final commodity is determined for other sectors.

The major result of working out the input-output balance being practically used in the USSR Gosplan is the estimates of coefficients of full expenses  $B=(E-A)^{-1}$ . Estimated are coefficients of material expenses  $(b_{ij})$  as well as coefficients of spendings of capital investments, labour, fixed assets according to the following formulas:

$$Q_j = \sum_i b_{ij} q_i ; \quad F_j = \sum_i b_{ij} f_i$$

$$T_j = \sum_i b_{ij} t_i ;$$

where

$Q_j, T_j, F_j$  - coefficient of full spendings of capital investments, manpower, fixed assets;

$q_i, t_i, f_i$  - coefficient of specific capital investments, labour spendings, fund intensity. The reference books of these norms are compiled for commodity nomenclature of input-output balance and contain the data on value and commodity structure of coefficients of direct and full spendings.

They are sent to the USSR Gosplan departments and used there for preliminary plan estimates, operative correcting of plan projections, economic analysis. The use of those reference books at solution of individual plan problems guarantees accounting for direct as well as indirect input-output (inter-commodity) relations. The latter is very important, for indirect spendings of resources surpass rather often in several times (sometimes in ten and more times) direct spendings, and a list of commodities in indirect spendings surpasses considerably a list of commodities to be directly consumed in the given production.

The analysis of economic processes with use of coefficients of full spendings facilitates to detect tendencies that may not be seen while accounting for only direct production relations. For instance, while analysing development of any sector a tendency to increase of production fund intensity may be detected that may be evaluated as negative phenomenon requiring certain arrangements. However, such a conclusion will be premature, for the fund intensity may considerably decrease in adjustment (directly or indirectly) sectors. The analysis of dynamics of full spendings coefficients may detect a cont-