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The Applications of Linear Programming Models for Planning the Optimal Structure of foreign trade

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Forword

We held a series of discussions on model building for foreign trade planning at the International Economic Relations Centre at the I.N.P. Cairo. As part of it I presented a model for optimizing the regional distribution of exports and imports which should be applied practically for the A.R.E. foreign trade. While this experiment is still going on I will summarize and elaborate on experiences we gained in the German Democratic Republic in building and applying linear programming models for foreign trade planning, which might be of interest for those who are also working in this field.

I am much obliged to all who took part in the discussions, especially the staff of the I.E.R.C., for getting better ideas on problems of model building in developing countries, especially in the A.R.E..

And there is another question to be mentioned. We always consider such models as auxiliary instruments for planning. A model, even highly-sophisticated, is without any use, if the expert or the planner has not elaborated on it in the correct lines, e.g. when determining the objectives and goals, when fixing the conditions which are limiting the economic growth, estimating some inputs etc. Also when using modern tools like models of optimization the role of the planner and the knowledge of the experts are decisively and even growing. Besides we want to stress that the social development and the economy are so manifold and include so many different factors and tendencies which cannot completely be included in models. Thus one has always to check the results coming from the computer and comparing them with the real life before deciding on the future plan.

In the following we will elaborate on models of optimization built in the G.D.R. and applied for planning in the foreign trade sector. Especially we will deal with problems one will be facing in the practical approach.

1. Basic ideas for conctructing linear programming models for planning foreign trade.

At the beginning of both either the research work or practical steps in applying models for economic planning we should always rise the question what will be the aim in building the model, what will be the effect of using the model in practical planning.

As regards foreign trade planning, we are aiming at finding the best set of the commodity structure of exports and imports and its regional distribution on areas and countries in order to maximize the foreign exchange receipts and to increase the efficiency of foreign trade. Thus we are searching for optimal plan targets whichcover our long-term and short-run objectives and are in conformity with the internal and external conditions. And here it might be useful to apply models as auxiliary intruments for drawing up the foreign trade plan and for determining plan-figures.

There are several programming techniques which have been developed in the 50-ies and 60-ies and which have proved to be useful for economic planning on macro as well as on micro level, such as linear programming, nonlinear programming, dynamic programming. The linear programming technique seems to be the most developed one for economic planning till now and here the socialist countries have practical experiences, too. For constructing linear programming models for foreign trade planning we will refer to the characteristics of this method.

At first there must be a linear objective function of such a type:

$$f(X) = C_j X_j$$

which aims either at a maximization or at a minimization.

At second there is a set of linear constraints

$$\sum_{j=1}^{n} a_{ij} X_j \leq b_i \qquad i = 1, 2, \ldots, m$$

which limit the variables of the model, and last not least all variables must be non-negative, i.e.

$$X_i \geq 0 \qquad i = 1, 2, \ldots, m$$

The decisions on which the planner in the field of foreign trade is elaborating and the frame to which he is often bound seem to be like these characteristics so that applying linear programming models in planning suggests itself.

Let us elaborate on it in details.

Suppose, there is a number of commodities, which shall be exported or imported. The external prices we will get for the exports or have to pay for imports are different from country to country so that we are looking for the best distribution of the exports and imports. But when following such criteria we have to take into consideration some limiting conditions which may exist both internally and on external markets, such as

(a) limitations in the capacity for producing the exportables,

(b) the necessity of fulfilling the domestic demand in products which are also exportables,

(c) usually the external demand for our exports is not unlimited and there may exist upper bounds,

(d) there may be a maximum supply possibility for commoditie we want to import,

(e) from trade and payment agreements the necessity of following certain balance requirements may arise and we have to follow a fixed trade-policy.

Now we will construct a model which will include these conditions and criteria and the application of which will contribute to the finding of the best solution of the above mentioned task.

When k is the index for export commodities (k = 1, 2, ..., n), g the index for import commodities (g = 1, 2, ..., s) and r the index for countries/areas (r = 1, 2, ..., m) then we are looking for optimal export quantities/values for each commodity to each country ($x_{k,r}$) and for the optimal items of our imports ($y_{g,r}$).

The following table gives insight in to the variables (outputs) of the calculation, we are looking for.

Country / Commodity	1	2	3	....	m
exportable k = 1	x_{11}	x_{12}	x_{13}	....	x_{1m}
" 2	x_{21}	x_{22}	x_{23}	....	x_{2m}
" 3	x_{31}	x_{32}	x_{33}	....	x_{3m}
	.	.	.		.
	.	.	.		.
	.	.	.		.
n	x_{n1}	x_{n2}	x_{n3}	....	x_{nm}
importable 1	y_{11}	y_{12}	y_{13}	....	y_{1m}
g = 2	y_{21}	y_{22}	y_{23}	....	y_{2m}
3	y_{31}	y_{32}	y_{33}	....	y_{3m}
	.	.	.		.
	.	.	.		.
	.	.	.		.
s	y_{s1}	y_{s2}	y_{s3}	....	y_{sm}

And here are the inputs of the model necessary for determining the optimal distribution of the exports and imports.

When e_k stands for the quantity or value of each commodity k, available for export and i_g stands for the quantity/value of commodity g to be imported for covering the internal demand, one can formulate:

$$(1) \qquad \sum_{r=1}^{m} x_{kr} \leq e_k \qquad k = 1, 2, \ldots, n$$

which means that the export of commodity k to all markets added together can only be equal to the predetermined export capacity e_k or less than it.

$$(2) \qquad \sum_{r=1}^{m} y_{gr} = i_g \qquad g = 1, 2, \ldots, s$$

The import quantities of commodity g from all markets should be equal to the internal demand for such imports. If i_g is to be understood as the minimum limit for imports of commodity g, one can also formulate:

$$(2\,a) \qquad \sum_{r=1}^{m} y_{gr} \geq i_g \qquad g = 1, 2, \ldots, s$$

Both are limitations from the internal side of the country. On the external markets there may exist a minimum demand ($\underline{b}_{kr}$) for our exports, which at least should be covered, and a maximum sale's possibility. Thus one can formulate

$$(3) \qquad x_{kr} \leq \bar{b}_{kr} \qquad \begin{matrix} k = 1, 2, \ldots, n \\ r = 1, 2, \ldots, m \end{matrix}$$

$$(4) \qquad x_{kr} \geq \underline{b}_{kr} \qquad \begin{matrix} k = 1, 2, \ldots, n \\ r = 1, 2, \ldots, m \end{matrix}$$

and for the importables analoguously

$$(5) \qquad y_{gr} \leq \bar{b}_{gr} \qquad \begin{matrix} g = 1, 2, \ldots, s \\ r = 1, 2, \ldots, m \end{matrix}$$

$$(6) \qquad y_{gr} \geq \underline{b}_{gr} \qquad \begin{matrix} g = 1, 2, \ldots, s \\ r = 1, 2, \ldots, m \end{matrix}$$

where $\underline{b}_{gr}$ = minimum import obligations for certain commodities g from certain markets r.

and $\bar{b}_{gr}$ = maximum supply possibility for commodity g from market r.

By formula (4) and (6) we have at the same time included the non-negativity condition for all variables.

And now we have to formulate balance limitations, e.g. in such a way

$$(7) \qquad \sum_{k=1}^{n} p_{kr} \, x_{kr} - \sum_{g=1}^{s} d_{gr} \, y_{gr} = S_r \qquad \text{or}$$

$$(7a) \qquad \sum_{k=1}^{n} p_{kr} \, x_{kr} \geqq E_r$$

$$\sum_{g=1}^{s} d_{gr} \, y_{gr} \leqq I_r \qquad r = 1, 2, \ldots, m$$

which means that the total value of exports to country r minus the total value of imports from country r should cover a saldo S_r, which can be understood as surplus or deficit (7).

or

One can bind the export value to a certain predetermined amount E_r and the import value to I_r, each on country/area basis (7a).

Of course we will include these limitations only for countries/areas where we are bound to follow such restrictions or where we are aiming at realizing a certain surplus or deficit. In practice the laying down of such limitations and the margins depend on the trade-policy

targets, the type of trade and payment aggreements (bilateral or multilateral payment balance equalization, settlement in clearing currency or in terms of convertible currency) and on the concrete situation of the payments in the given period. | Now we have to determine the objective function.

The main criteria for determining the x_{kr} and y_{gr} will be selling the exports to such markets where we get best prices and purchasing the imports from markets, where we pay the lowest prices and this will be formulated as the objective function of the model.
When p_{kr} stands for the export price per unit of commodity k by country r and d_{gr} for the import price we have to pay for commodity g in country r, then the objective function looks as follows:

$$\sum_{k=1}^{n} \sum_{r=1}^{m} p_{kr} \; x_{kr} - \sum_{g=1}^{s} \sum_{r=1}^{m} d_{gr} \; y_{gr} \longrightarrow \max.$$

which aims at a maximization of our foreign exchange receipts and a minimization of the foreign exchange expenditures.

From the linear programming point of view the model is complete now. As regards the practical application for foreign trade planning, the above mentioned model should be considered as the simplest type for optimizing foreign trade. It is a basic model from which a series of specific models for planning foreign trade can be derived. In the following we will elaborate on such models we have applied in the G.D.R., putting special emphasis on problems we had been facing and how to solve them.

2. Models of optimization for the foreign trade sector

2.1 Model for optimizing the regional distribution of exports and imports.

We have stated at the beginning that one aim in foreign trade planning is finding the best regional distribution of exports and imports and determining the optimal regional structure. In the G.D.R. we started applying linear programming models just in this field, as the subject of optimization is limited here and problems which may arise at the beginning of using such a technique can easier be solved.(1)

The basic type of the Regional Optimization Model (R.O.M.) can be seen in Appendix 1. It is similar to the model explained before. The 1-st and 2-nd equations, namely

$$(1) \qquad \sum_{r=1}^{m} x_{kr} = e_k \qquad k = 1, 2, \ldots, n$$

$$(2) \qquad \sum_{r=1}^{m} y_{gr} = i_g \qquad g = 1, 2, \ldots, s$$

are characteristic ones and make it obviously that the export volume of the commodities (e_k for $k = 1, 2, \ldots, n$) and the import volume of the commodities (i_g for $g = 1, 2, \ldots, s$) have to be predetermined

(1) Here we made use of earlier studies on this problem carried through by Trzeciakowski (A model of optimization of current directions of policy in foreign trade, Gospodarka Planowa, Warsaw 1960) and Kronsjö (Iterative pricing for planning foreign trade, Economics of Planning, vol. 3 No. 1, 1963).

and are not a subject of optimization here. The R.O.M. is aimir at the optimization of the regional structure of foreign trade with full adherence to national economic and trade policy objectives and the limitations imposed by the concrete internal and external conditions.

There will be an optimal distribution of the exports and imports on regions when the foreign exchange receipts are a maximum and the foreign exchange expenditures are at the lowest level within the bounds previously mentioned. In principle, the objective function

$$\sum_{k=1}^{n} \sum_{r=1}^{m} p_{kr} \; x_{kr} - \sum_{g=1}^{s} \sum_{r=1}^{m} d_{gr} \; y_{gr} \longrightarrow \text{maximum}$$

is fulfilling the above mentioned criteria as the net foreign exchange receipts will be maximized or, to be precise, a possible surplus will be maximized, a deficit minimized. Often we are especially interested in getting some hard currencies and we don't want to maximize bilateral currency receipts. Therefore we can include a preference coefficient in the objective function giving weights to certain markets/countries and maximizing the foreign exchange components in a differentiated manner. The objective function then looks as follows:

$$\sum_{k=1}^{n} \sum_{r=1}^{m} t_r \; p_{kr} \; x_{kr} - \sum_{g=1}^{s} \sum_{r=1}^{m} f_r \; d_{gr} \; y_{gr} \longrightarrow \text{maximum}$$

where t_r is a coefficient which stimulates the exports with certain markets and

f_r will slow down imports from certain markets.

One can also include different terms of delivery (fob, cif) and different terms of payment in the objective function. For example when there is a significant difference in the prices on cash basis and on credit terms, we should differentiate in the objective function between cash receipts/payments and payment on credit terms per commodity and country.

From G.D.R. experience applying the R.O.M. has proved to be useful for finding the best regional distribution of our exports and imports and besides for rationalizing the planning procedure.

Up till now there are more than 50 examples of practical application of the R.O.M. We gained good experiences in applying this technique on enterprise level but also on central state level. Sometimes the complete export and import nomenclature of a General Organization or a Foreign Trade Agency had been calculated in its optimal regional distribution. On the other side we also built some special models for exports or for imports only according to the concrete circumstances and tasks of foreign trade planning. When having applied the R.O.M. on macro economi level-either by the State Planning Commission or by the Ministry of Foreign Trade-we were aiming at optimizing the total foreign exchange receipts/expenditures. But here we had been facing some other problems and new difficulties, which mainly resulted from the new dimension (size) of the model. Suppose, there are more than 1.000 commodities to be exported or imported to/from about 50