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THE ECONOMETRIC APPROACH TO STUDYING BUSINESS CYCLES: THE GENERAL MODEL AND SPECTRAL ANALYSIS

BY

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PREFACE

In the name of Allah, most Gracious, most Merciful. Praise be to Allah, the Cherisher of the worlds. May His blessings and peace be on his prophet and messenger Muhammad, the lord of the faithful.

Studying business cycles can be purely theoretical. In fact, there are many theoretical explanations to economic fluctuations ranging from under-consumption to multiplier-accelerator interaction. However, the scientific approach hardly suggests deduction alone to be the only criterion for evaluating scientific principles.

Business cycles can also be studied in a pure empirical way. The work of Burns and Mitchell is a good example. Empiricists do not depend on economic theory nor do they relate their own analysis to any theoretical structure. They take all available data as the first step. Then, they proceed with some smoothing processes to discover economic fluctuations.

A third school stands between the theoreticians and the empiricists. This school follows the econometric approach. It takes from theory the logic of all structural and behavioral relations. It takes from mathematics the technique of expressing these relations in terms of a formal model. Finally, it takes from statistics the method of relating models to data and making any inference or significance tests.

Because of different problems relating to space and scope, we will review the shortcomings of the emperical approach in brief, and explain the econometric approach of studying economic fluctuations. For the sake of clarity, a small explanatory model will be added to show some of the mechanics involved in using the general model. The second Part of the paper introduces spectral analysis as another technique of studying fluctuations.

The introduction of spectral analysis is done in the easiest possible form. Due to the strange terms of spectral analysis which are foreign to economists, and due to the high level of the required mathematical background, this technique appears to be rather difficult.

However, economists who can follow mathematical analysis if every step is writtern down and explained without much detail will be able to benefit from this simplification. Therefore, an exposition to spectral analysis will be introduced first as a necessary background. We will try in this exposition to follow the mathematical form step by step. Nevertheless, we will avoid too rigorous proofs and theorems since the literature is already rich with them. Finally, a brief account of how such a technique can be used for studying fluctuations will be provided.

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I. THE SHORTCOMINGS OF THE EMPERICAL APPROACH(1)

Koopman's criticism of Empiricism is directly related to the work of Burns and Mitchell(2). We will review his criticism in brief and discuss the empiricist's answer to it.

The empirical approach of Burns and Mitchell lacks guidance from theoretical consideration, while "theoretical preconceptions" about the nature of observations cannot be dispensed with. This complete divorce from theory is clear in the following:

1 - In Chapters 9-12 they search for possible changes in cyclical behavior overtime. The average measure of cyclical behavior is computed from seven series chosen with no systematic discussion of the reasons of their selection.

2 - They do not study behavior in terms of the behavior of groups of economic agents whose modes of behavior, in the institutional and technological environment, are the ultimate determinants of the levels of economic variables and their fluctuations. They study behavior in a more mechanical sense of certain measurable joint effects of several actions and responses. This eliminates the benefits that might be received from economic theory. It also divorces the study of fluctuations from the explanation of the levels or trends around which the variables may fluctuate.

3 - There is no organizing principle to determine on what aspects of the observed variables attention should be concentrated, except for applying the formal definition of the cycle.

4 - The study is not related to the problem of prediction, its possibilities and limitations.

5 - Although Burns and Mitchell are aware of the problem of randomness, they do not discuss it in terms of definite distributional hypotheses. Their variance tests applied to durations, amplitudes, and time lags are not rigorous. The measure of these variables need not be independent over successive cycles.

Resorting to economic theory is also a practical necessity. Without resort to the theory in some systematic way, no conclusions relevant to the guidance of economic policy can be made.

Empiricists answer these criticisms on two bases:(3)

1 - The aggregate has an existence apart from its constituent parties. The behavior characteristics of its own are not deducible from the behavior of the particles. Therefore, seeking a basis for economic dynamics in the analysis of the economizing behavior of the individual may not be necessary or even particularly desirable.

2 - Social usefulness of economic policy is hardly a relevant criterion in the evaluation of economic research.

Our answer to the first point is simple. Aggregates do not "act" independently of their constituents. Even if their behavior is not directly related to their "Micro" units, a first step is necessary before dealing empirically with aggregates. There should be a theory which explains the economizing behavior of aggregate units. Then, this theory should be incorporated with any empirical work. Until such theory exists the empiricists should either use the existing theory or develop the necessary theory themselves.

The answer to the second point is also simple. Intelligent criteria are undoubtedly needed to evaluate economic policy. Economic research is the only source for such criteria. Therefore "social usefulness" is a relevant basis of evaluating research, but not an ultimate one.

II. THE GENERAL MODEL APPROACH

The econometric approach includes the simultaneous equation model approach, and the use of spectral analysis in studying economic fluctuations. The latter approach is discussed below, and this part takes up the former one.

In order to have a good understanding of the econometric approach it is necessary to understand its classification of variables and equations. Variables are classified into the following categories:

1. The Exogeneous or Independent Variables: There are three principles according to which the exogeneous variables are determined.(4) The first is the departmental principle. It treats the variables which are wholly or partially outside the scope of economics, like population and time, as independent. The second is the causal principle. It regards as exogeneous the variables which influence the remaining exogeneous variables but are not influenced thereby. The third principle is time which classifies lagged (predetermined) variables as exogeneous.

2. The Endogeneous or Dependent Variables: They are those variables which are mutually dependent and determined within the system.

Equations can be classified as follows:(5)

1. Definitional Equations: They express definitions or identities which hold exactly in the system. As an example:

$Y = C + I$

Y : income
C : consumption
I : investment
W : total wage bill
w : wage rate
L : labor

$W = w\,L$

2. Stochastic Equations: They do not hold exactly. They are randomly determined. Three kinds of stochastic relations can be found:

a. Behavioral : which expresses a behavioral relationship, as

$C = a + b\,Y + u$

a : constant
b : marginal propersity to consume
u : error term

b. technological: which expresses a technical or institutional constrain as

$O = a\,L^{\alpha}\,K^{\beta}\,u$

α, β : parameters
K : capital
O : total output

c. adjustment: which refers to the adjustment process taking place when there is a disequilibrium in a particular market, as

$I - S = M^d - M^s + u$

M^d: money demand
M^s: money supply
S : savings

The error term is included in the stochastic equations to count for the unknown variables and the errors of observations(6). To give the model a probablistic nature, we generally assume that the effects of the unexplained variables and errors happen at random.

The econometric approach has three components(7). First, a specification of the process by which the independent variables are generated. Second, a specification of the process by which the observed distrubances generated. Third, a specification of the relationship connecting these to the observed independent variables. A given body of economic observations, cross section sample or time series, may be viewed as a sample from the population. Once we specify a parent population, the rules of statistical inference can be applied to develop a rational method of measuring a relationship of economic theory from a given sample.

Therefore, the quantification of economic theory is not a mechanical task. It is not a matter of measurement without theory. It is merely a specification of the probabilistic mechanisms that link economic observations to economic theory.

After constructing the structural equations of the model, we can solve the system for the endogeneous variables. This solution will be in terms of the exogeneous and predetermined variables, and the disturbances. This solution is called the reduced form. It can be used in forecasting the values of the dependent variables given the independent ones.

In order to examine the dynamic properties of the model, i.e., under what circumstances cycles, growth, etc., are found, we have to construct the "final or Timbergen equations".(8)

In each reduced form equation, the lagged variables, except for the one explained by the equation, are algebraically eliminated. Then, we get a difference equation in one endogeneous variable expressed as a function of a number of its lagged values, of parameters, of exogeneous variables, and of disturbances. If the exogeneous variables, the parameters, and the disturbances are held constant, the dependent variable will be a function of only its past values. This time path solution is called the solution of the final equation. It can be examined to see under what conditions it contains cycles, growth, decline,..., etc.

In order to explain the approach, a simple model follows as an example.

Let us assume we are interested in the time path of the price of a certain commodity, whose market equations are:

$$Q_t^d = \alpha_{11} P_t + \alpha_{12} P_{t-1} + \alpha_{13} Y_t + U_1 \quad (1)$$

$$Q_t^s = \alpha_{21} P_t + \alpha_{22} P_{t-1} + \alpha_{23} Y_t + U_2 \quad (2)$$

$$Q_t^d = Q_t^s \quad (3)$$

where:

P : price.

Q : Q^d : quantity demanded, Q^s quantity supplied

Y : Income

U_i : disturbance variable

α_{ij}: parameters

t : time subscript.

Then, we have three equations in three dependent variables: Q_t^s, Q_t^d, and P_t, and one lagged variable, one exogeneous variable, and an error term.

The Reduced form:

By using (3), (1) and (2) are equal. Then:

$$\alpha_{11} P_t + \alpha_{12} P_{t-1} + \alpha_{13} Y_t + U_1 = \alpha_{21} P_t + \alpha_{22} P_{t-1} + \alpha_{23} Y_t + U_2 \quad (4)$$

By solving (4) for P_t:

$$P_t(\alpha_{11} - \alpha_{21}) = P_{t-1} (\alpha_{22} - \alpha_{12}) + Y_t (\alpha_{23} - \alpha_{13}) + (U_2 - U_1) \quad (5)$$

$$P_t = \frac{\alpha_{22} - \alpha_{12}}{\alpha_{11} - \alpha_{21}} P_{t-1} + \frac{\alpha_{23} - \alpha_{13}}{\alpha_{11} - \alpha_{21}} Y_t + \frac{U_2 - U_1}{\alpha_{11} - \alpha_{21}} \quad (6)$$

Let:

$$\frac{\alpha_{22} - \alpha_{12}}{\alpha_{11} - \alpha_{21}} = \alpha \quad (7)$$

$$\frac{\alpha_{23}-\alpha_{13}}{\alpha_{11}-\alpha_{21}} = \beta \qquad (8)$$

$$\frac{U_2 - U_1}{\alpha_{11}-\alpha_{21}} = V_t \qquad (9)$$

By substituting (7), (8), (9) in (6):

$$P_t = \alpha P_{t-1} + \beta Y_t + V_t \qquad (10)^*$$

By the same logic:

$$P_{t-1} = \alpha P_{t-2} + \beta Y_{t-1} + V_{t-1} \qquad (11)$$

By substituting for P_{t-1} from (11) in (10):

$$P_t = \alpha\left[\alpha P_{t-2} + \beta Y_{t-1} + V_{t-1}\right] + \beta Y_t + V_t \qquad (12)$$

$$= \alpha^2 P_{t-2} + (\beta Y_t + \alpha \beta Y_{t-1}) + (V_t + \alpha V_{t-1}) \qquad (13)$$

Then, we can express P_{t-2} as in (10) and then substitute for it in (12), Repeating this operation s times will render:

$$P_t = \alpha^{s+1} P_{t-s-1} + \sum_{r=0}^{s} (\alpha^r \beta) Y_{t-r} \sum_{r=0}^{s} \alpha^r V_{t-r} \qquad (14)$$

By letting s go to infinity, and assuming that $\lim_{r \to \infty} \alpha^r = 0$, then

$$P_t = \sum_{r=0}^{\infty} (\alpha^r \beta) Y_{t-r} + \sum_{r=0}^{\infty} \alpha^r V_{t-r} \qquad (15)$$

In this case the system is said to be stable, and its final equation is (15). In other words, P_t is a function of some level of income and disturbances. If $\lim_{r \to \infty} \alpha^r \neq 0$, the final equation will show instability.

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* Needless to say, this is a difference equation of the first order.

If s = t-1, equation (14) will be:

$$P_t = \alpha^t P_0 + \sum_{r=0}^{t-1} (\alpha^t \beta) Y_{t-r} + \sum_{r=0}^{t-1} {}^r V_{t-r} \tag{16}$$

where P_0 is some initial value of P.

If the exogeneous variable Y is indefinitely sustained at $\bar{Y}$, then P will approach its equilibrium value $\bar{P}$ which is equal to:

$$\bar{P} = \sum_{r=0} (\alpha^r \beta) \bar{Y} \tag{17}$$

$$= \sum_{r=0}^{t-1} (\alpha^r \beta) \bar{Y} + \sum_{r=t}^{\infty} (\alpha^r \beta) \bar{Y} \tag{18}$$

$$= \sum_{r=0}^{t-1} (\alpha^r \beta) \bar{Y} + \alpha^t \sum_{r=0}^{\infty} (\alpha^r \beta) \bar{Y} \tag{19}$$

By comparing with 17,

$$\bar{P} = \sum_{r=0}^{t-1} (\alpha^r \beta) \bar{Y} + \alpha^t \bar{P} \tag{20}$$

By subtracting (20) from (16).

$$P^{\circ} = \alpha^t P_0^{\circ} + \sum_{r=0}^{t-1} (\alpha^r \beta) Y^{\circ} + \sum_{r=0}^{t-1} \alpha^r V_{t-r} \tag{21}$$

where $P^{\circ} = P_t - \bar{P}$

and $Y^{\circ} = Y_t - \bar{Y}$

and $P_0^{\circ} = P_0 - \bar{P}$.

In other words the enodgeneous and exogeneous variables are expressed as derivatives from their equilibrium values.