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INPUT OUTPUT ANALYSIS
A PILOT EXAMPLE

by

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An Introduction:

Dr. Nazih Deif and his group, of the ministry of the national planning, has constructed recently, an Input/Output table for the year 1962.*

The Operations Research group has the opportunity of working out in collaboration with Dr. Deif, the different applications of that I/O table.

The purpose of this paper is to present these results, to be taken as an example of what an I/O table can provide us.

It is hoped that this pilot application will be considered when more elaborate I/O table is constructed.**

This paper is devided into three sections. The first section is a brief description of the I/O table used. The second section is a theoritical formulation of the applications of the I/O tables, while the third section contains the numerical results.

Section I. The I/O table

In the I/O table provided by Dr. Deif, the sectors considered had been aggregated into 13 sectors. The components of these sectors are given in table 1.

The I/O technical coefficients are given in table 2. The coefficients in each cell are devided into two components: the first component corresponds to inputs from local production while the second component coresponds to imported inputs.

The primary factors had been devided into three parts:

1- Wages.

2- Customs and duties.

3- Residual.

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* A separate paper will be given by Dr. Deif explaining in detail the structure of that table.

** There is now, in the Operations Research Center; under the supervision of Dr. Aglan; a project for constructing an elaborate I/O table.

Table 3 is a reformulation of table 2 where the import components in each column had been aggregated to form the import row no 14.

Table -1-

The sectors used in the I/O table and their components .

Sector No	Name of the sectors and its components.
1	Agriculture, including, foresting, animal breeding husbandry and animal products.
2	Minning and Quarying, including petroleum extraction.
3	Food, beverages and tobacco products.
4	Ginning and Pressing
5	Textiles and Clothing industries, including foot wear.
6	Chemical industries; basic chemicals, pharmthetical products and printing and publishing.
7	Non-metalic industries; cement, ceramics, bricks.
8	Basic metal industries.
9	Engineering industries; electric and non-electric machinary, transport equipment and rolling stock, and other metal products.
10	Other industries; wood products, furnitures, paper products, automobile tyres and other rubber products.
11	Energy and Fuel; electricity and Petroleum refinary products.
12	Construction; rail, road, houses and industrial plant construction.
13	Services; transportation, communication, Suez Canal, housing, water sewage, health, education (private sector), trade and finance, personal services(including hotels), recreation and professional services.

The I/O technical coefficients

(Local Production and Import I/O coefficients)

Table (2) 1.

1	2	3	4	5	6	7
0.000 474 0.193 073 0.193 547	---- ---- ----	0.069 934 0.438 509 0.508 443	---- 0.909 037 0.909 037	0.010 209 0.008 932 0.019 141	---- 0.001 289 0.001 289	---- 0.025 735 0.025 735
---- ---- ----	---- ---- ----	---- 0.000 734 0.000 734	---- ---- ----	---- ---- ----	0.016 753 0.014 175 0.030 928	---- 0.055 147 0.055 147
---- 0.011 690 0.011 690	---- ---- ----	0.034 875 0.107 930 0.142 805	---- ---- ----	---- 0.000 851 0.000 851	0.050 258 0.036 082 0.086 340	---- ---- ----
---- 0.001 738 0.001 738	---- ---- ----	---- 0.012 298 0.012 298	---- ---- ----	---- 0.203 318 0.203 318	---- ---- ----	---- ---- ----
---- 0.001 580 0.001 580	---- ---- ----	---- 0.007 526 0.007 526	---- 0.008 918 0.008 918	0.022 544 0.318 162 0.340 706	---- 0.006 443 0.006 443	---- ---- ----
0.020 063 0.036 493 0.056 556	0.010 169 0.040 678 0.050 847	0.001 101 0.010 095 0.011 196	---- 0.000 595 0.000 595	0.018 715 ---- 0.018 715	0.125 000 0.081 186 0.206 186	---- 0.055 147 0.055 147
---- ---- ----	0.003 390 ---- 0.003 390	---- ---- ----	---- ---- ----	---- ---- ----	---- ---- ----	---- 0.062 500 0.062 500
---- ---- ----	---- ---- ----	---- ---- ----	---- ---- ----	---- ---- ----	---- 0.006 443 0.006 443	---- 0.011 029 0.011 029

Table (2) 2.

9	0.000 948 0.002 844 0.003 792	0.006 780 0.027 118 0.033 898	0.001 285 0.000 551 0.001 836	0.001 189 0.000 595 0.001 784	0.004 679 0.023 828 0.028 507	0.0 0.0 0.0
10	—— 0.001 264 0.001 264	0.006 780 0.010 169 0.016 949	—— 0.001 836 0.001 836	—— —— ——	—— 0.021 268 0.021 268	0.0 0.0
11	0.002 212 0.014 218 0.016 430	0.003 390 0.010 169 0.013 559	0.002 937 0.010 830 0.013 767	0.000 595 0.005 351 0.005 946	0.001 701 0.022 118 0.023 819	0.0 0.0 0.0
12	—— 0.000 474 0.000 474	—— 0.010 169 0.010 169	—— 0.001 101 0.001 101	—— —— ——	—— 0.000 851 0.000 851	0.0 0.0
13	—— 0.019 273 0.019 273	—— 0.037 288 0.037 288	—— 0.030 470 0.030 470	—— 0.047 562 0.047 562	—— 0.030 625 0.030 625	0.0 0.0
$\sum_k X'_{kh}$	0.282 647	0.135 591	0.621 880	0.972 058	0.629 953	0.
B'_h	0.023 697	0.030 509	0.110 132	0.001 784	0.057 848	0.
$B'_h + \sum_k X'_{kh}$	0.306 344	0.166 100	0.732 012	0.973 842	0.687 801	0.
W'_h	0.219 589	0.172 881	0.043 686	0.008 323	0.135 262	0.
r'_h	0.005 213	0.003 391	0.113 069	0.000 595	0.027 222	0.
ϵ'_h	0.468 854	0.657 628	0.111 233	0.017 240	0.149 715	0.
G.T	1.000 000	1.000 000	1.000 000	1.000 000	1.000 000	1.

Table (2) 3.

8	9	10	11	12	13	G.T.
— — —	— — —	— 0.032 609 0.032 609	— — —	— — —	— 0.000 321 0.000 321	0.015 775 0.198 693 0.214 468
.133 779 .086 957 .220 736	— — —	— — —	0.051 980 0.160 891 0.212 871	— 0.010 050 0.010 050	— 0.000 801 0.000 801	0.003 673 0.008 158 0.011 831
— — —	— — —	0.008 696 0.069 565 0.078 261	— — —	— — —	— — —	0.009 009 0.027 992 0.037 001
— — —	— — —	— — —	— — —	— — —	— — —	— 0.021 497 0.021 497
— — —	— 0.003 322 0.003 322	— 0.008 696 0.008 696	— — —	— 0.006 533 0.006 533	— 0.004 532 0.004 532	0.002 049 0.034 256 0.036 305
— .003 344 .003 344	0.001 107 0.047 619 0.048 726	0.050 000 0.145 652 0.195 652	— 0.018 564 0.018 564	— 0.015 075 0.015 075	— 0.022 737 0.022 737	0.011 638 0.026 214 0.037 852
— — —	— — —	— 0.004 348 0.004 348	— — —	— 0.127 538 0.127 538	— 0.001 122 0.001 122	0.000 039 0.009 279 0.009 318
.043 478 .033 445 .076 923	0.025 471 0.129 568 0.155 039	— 0.013 043 0.013 043	— — —	— 0.083 794 0.083 794	— 0.004 763 0.004 763	0.001 392 0.010 401 0.011 793

Table (2) 4.

0.026 756 0.013 378 0.040 134	0.102 015 0.297 719 0.399 734	0.019 565 0.050 000 0.069 565	0.008 663 0.009 901 0.018 564	---- 0.038 693 0.038 693	---- 0.056 667 0.056 667	0.004 833 0.010 555 0.015 388
---- ---- ----	---- 0.012 182 0.012 182	0.160 870 0.113 043 0.273 913	---- ---- ----	---- 0.044 724 0.044 724	---- 0.002 981 0.002 981	0.002 938 0.013 571 0.016 509
0.073 579 0.043 478 0.117 057	0.005 537 0.028 793 0.034 330	0.006 522 0.030 435 0.036 957	0.030 941 0.084 158 0.115 099	---- 0.016 080 0.016 080	---- 0.013 558 0.013 558	0.003 982 0.022 038 0.026 020
---- 0.003 344 0.003 344	---- 0.005 537 0.005 537	---- 0.002 174 0.002 174	---- 0.002 475 0.002 475	---- ---- ----	---- 0.003 442 0.003 442	---- 0.001 392 0.001 392
---- 0.080 268 0.080 268	---- 0.046 346 0.046 346	---- 0.043 478 0.043 478	---- 0.074 257 0.074 257	---- 0.193 467 0.193 467	---- 0.273 301 0.273 301	---- 0.124 188 0.124 188
0.264 214	0.571 086	0.513 043	0.350 246	0.535 954	0.384 225	0.508 234
0.277 592	0.134 130	0.245 653	0.091 584	----	----	0.055 328
0.541 806	0.705 216	0.758 696	0.441 830	0.535 954	0.384 225	0.563 562
0.183 946	0.172 591	0.141 304	0.133 663	0.311 558	0.204 808	0.167 028
0.073 579	0.015 504	0.054 348	0.025 990	----	----	0.032 555
0.200 669	0.106 689	0.045 652	0.398 517	0.152 488	0.410 967	0.236 855
1.000 000	1.000 000	1.000 000	1.000 000	1.000 000	1.000 000	1.000 000

The I/O technical coefficients
(Local Production + Import row).

Table (3). 1

2	3	4	5	6	7
0.000 000	0.438 509	0.909 037	0.008 932	0.001 289	0.025 735
0.000 000	0.000 734	0.000 000	0.000 000	0.014 175	0.055 147
0.000 000	0.107 930	0.000 000	0.000 851	0.036 082	0.000 000
0.000 000	0.012 298	0.000 000	0.203 318	0.000 000	0.000 000
0.000 000	0.007 526	0.008 918	0.318 162	0.006 443	0.000 000
0.040 678	0.010 095	0.000 595	0.000 000	0.081 186	0.055 147
0.000 000	0.000 000	0.000 000	0.000 000	0.000 000	0.062 500
0.000 000	0.000 000	0.000 000	0.000 000	0.006 443	0.011 029
0.027 118	0.000 551	0.000 595	0.023 828	0.001 289	0.003 676
0.010 169	0.001 836	0.000 000	0.021 268	0.015 464	0.128 676
0.010 169	0.010 830	0.005 351	0.022 118	0.047 680	0.073 529
0.010 169	0.001 101	0.000 000	0.000 851	0.003 866	0.003 676
0.037 288	0.030 470	0.047 562	0.030 625	0.063 144	0.091 912
0.030 509	0.110 132	0.001 784	0.057 847	0.216 495	0.036 765
0.172 881	0.043 686	0.008 323	0.135 262	0.139 157	0.180 149
0.003 391	0.113 069	0.000 595	0.027 222	0.054 124	0.011 030
0.657 628	0.111 232	0.017 241	0.149 715	0.313 144	0.261 029
1.000 000	1.000 000	1.000 000	1.000 000	1.000 000	1.000 000

Cont. Table (3) . 2

h / k	8	9	10	11	12
1	0.000 000	0.000 000	0.032 609	0.000 000	0.000 000
2	0.086 957	0.000 000	0.000 000	0.160 891	0.010 050
3	0.000 000	0.000 000	0.069 565	0.000 000	0.000 000
4	0.000 000	0.000 000	0.000 000	0.000 000	0.000 000
5	0.000 000	0.003 322	0.008 696	0.000 000	0.006 533
6	0.003 344	0.047 619	0.145 652	0.018 564	0.015 075
7	0.000 000	0.000 000	0.004 348	0.000 000	0.127 538
8	0.033 445	0.129 568	0.013 043	0.000 000	0.083 794
9	0.013 378	0.297 719	0.050 000	0.009 901	0.038 693
10	0.000 000	0.012 182	0.113 043	0.000 000	0.044 724
11	0.043 478	0.028 793	0.030 435	0.084 158	0.016 080
12	0.003 344	0.005 537	0.002 174	0.002 475	0.000 000
13	0.080 268	0.046 346	0.043 478	0.074 257	0.193 467
B'_h	0.277 592	0.134 130	0.245 653	0.091 584	0.000 000
W'_h	0.183 946	0.172 591	0.141 304	0.133 663	0.311 558
Duties	0.073 579	0.015 504	0.054 348	0.025 990	0.000 000
Resid.	0.200 669	0.106 681	0.045 652	0.398 571	0.152 488
G.T.	1.000 000	1.000 000	1.000 000	1.000 000	1.000 000

Section II. Theoritical Formulation.

I/O analysis have been discussed elaboratory in the literature. We shall follow, however Ragner Frisch's representation which is given in many of his writings*.

We feel that it will be useful if we explain here in some details and in a systematic way the different formulae which has been given by R. Frisch.

Let k denotes the delivering sector and h denotes the receiving sector. The symbols k and h take the values from 1,...,n.

Let X_{kh} be the deliveries (in volume figure) from sector k to sector h. $(k,h = 1,\ldots,n)$.

Let B_h, W_h, r_h and ϵ_h represent respectively the deliveries to sector h from the imports, wages, customs & duties and residual.

If X_h is the total production of sector h then we have the following relations.

$$\sum_{k=1}^{n} X_{kh} + B_h + W_h + r_h + \epsilon_h = X_h \qquad (h=1,\ldots,n) \ \ldots (1)$$

Let D_k be the final demands in sector k.
By definition we shall have

$$\sum_h X_{kh} + D_k = X_k \qquad (k=1,\ldots,n) \ \ldots (2)$$

Assuming constant technical coefficients, then we shall have

$$\frac{X_{kh}}{X_h} = X'_{kh} = \text{const.}$$

$$\frac{B_h}{X_h} = X'_h = \text{const.} \qquad \ldots (3)$$

$$\frac{W_h}{X_h} = W'_h = \text{const.}$$

* See for example : Memo No 3 To the National Planning Committe Cairo by Ragnar Frisch, Dec. 1957, " Advance list of Some Formulae for Decomposition In Interflow Tables.

$$\frac{r_h}{X_h} = r'_h = \text{const.}$$

$$(h=1,\ldots,n)$$

$$\frac{\epsilon_h}{X_h} = \epsilon'_h = \text{const.}$$

It is obvious that :

$$\sum_k X'_{kh} + B'_h + W'_h + r'_h + \epsilon'_h = 1$$

From 2 & 3 we see that

$$\sum_{h=1}^{n} (\delta_{kh} - X'_{kh}) X_h = D_k \quad (k = 1,\ldots,n) \quad \ldots (4)$$

where $\delta_{kh} = 1$ for $k = h$

$= o$ for $k \neq h$

Putting

$$(\delta - X')_{kh} = \delta_{kh} - X'_{kh} \quad \ldots (5)$$

then equations 4 take the form $\sum_h (\delta - X')_{kh} \cdot X_h = D_k \quad (k=1,\ldots,n)$ (6)

Notice that $(\delta - X')_{kh}$ is an n by n matrix. It is simply the n by n input/output technical coefficients matrix substracted from the n by n unit matrix.

This resulting matrix is shown in table 4.

From equation 6 it is seen that.

$$X_h = \sum_{k=1}^{n} (\delta - X')^{-1}_{hk} D_k \quad (h=1,\ldots,n) \quad \ldots (7)$$

where $(\delta - X')^{-1}_{hk}$ is the inverse of the matrix $(\delta - X')_{kh}$.

The inverse is given in table 5.

Denote

ΔD_k = Change in the final demand from sector k.

ΔX_h = The corresponding change in the total production of sector h. From equations 7 we see that

$$\Delta X_h = \sum_{k=1}^{n} (\delta - X')^{-1}_{hk} \Delta D_k \quad \ldots (8)$$

Equations 8 enable us to determine the change in the total production of sector h corresponding to given changes in the final demands of sector k. $(k=1,\ldots,n)$.

These changes in the final production ΔX_h will entail corresponding changes in the imports, wages, customs & duties and residual. To get these corresponding changes multiply the changes ΔX_h by the corresponding technical coefficients.

Thus the change ΔB_h in the imports delivered to sector h due to changes ΔD_k in the final demands of sectors k $(k=1,\ldots,n)$ is given by:

$$\Delta B_h = B_h' \Delta X_h = B_h' . \sum_{k=1}^{n} (\delta - X')^{-1}_{hk} \Delta D_k \qquad (h=1,\ldots,n) \ldots (9)$$

Similarly the changes Δw_h, Δr_h, ΔE_h due to changes ΔD_k is given by

$$\Delta W_h = W_h' . \Delta X_h = W_h' . \sum_{k=1}^{n} (\delta - X')^{-1}_{hk} \Delta D_k \qquad (h=1,\ldots,n) \ldots (10)$$

$$\Delta r_h = r_h' . \Delta X_h = r_h' . \sum_{k=1}^{n} (\delta - X')^{-1}_{hk} \Delta D_k \qquad (h=1,\ldots,n) \ldots (11)$$

$$\Delta \epsilon_h = \epsilon_h' . \Delta X_h = \epsilon_h' . \sum_{k=1}^{n} (\delta - X')^{-1}_{hk} \Delta D_k \qquad (h=1,\ldots,n) \ldots (12)$$

Defining the value added as

$$V_h = W_h + \epsilon_h \qquad \ldots (13)$$

We see that the corresponding change in ΔV_h will be given by:

$$\Delta V_h = (W_h' + \epsilon_h') \Delta X_h = (W_h' + \epsilon_h') \sum_{k=1}^{n} (\delta - X')^{-1}_{hk} \Delta D_k \qquad (h=1,\ldots,n) \ldots (14)$$

From equations 9,...,12 & 14 we see that the matrices $B_h' (\delta - X')^{-1}_{hk}$, $W_h' (\delta - X')^{-1}_{hk}$, $r_h' (\delta - X')^{-1}_{hk}$, $\epsilon_h' (\delta - X')^{-1}_{hk}$ and $V_h' (\delta - X')^{-1}_{hk}$, can be computed once for all. Each h,k element of any of these matrices will simply be the change in the corresponding primary factor's component of the sector h due to a unit change in the final demand of sector k.

$$(\delta - X')_{kh}$$

Table (4) 1.

k \ h	1	2	3	4	5	
1	+0.806 997	0.000 000	-0.438 509	-0.909 037	-0.008 932	-0.0
2	0.000 000	+1.000 000	-0.000 734	0.000 000	0.000 000	-0.0
3	-0.011 690	0.000 000	+0.892 070	0.000 000	-0.000 851	-0.0
4	-0.001 738	0.000 000	-0.012 298	+1.000 000	-0.203 318	0.00
5	-0.001 580	0.000 000	-0.007 526	-0.008 918	+0.681 838	-0.00
6	-0.036 493	-0.040 678	-0.010 095	-0.000 595	0.000 000	+0.91
7	0.000 000	0.000 000	0.000 000	0.000 000	0.000 000	0.00
8	0.000 000	0.000 000	0.000 000	0.000 000	0.000 000	-0.00
9	-0.002 844	-0.027 118	-0.000 551	-0.000 595	-0.023 828	-0.00
10	-0.001 264	-0.010 169	-0.001 836	0.000 000	-0.021 268	-0.015
11	-0.014 218	-0.010 169	-0.010 830	-0.005 351	-0.022 118	-0.047
12	-0.000 474	-0.010 169	-0.001 101	0.000 000	-0.000 851	-0.003
13	-0.019 273	-0.037 288	-0.030 470	-0.047 562	-0.030 625	-0.063

Cont. Table (4) 2.

k \ h	8	9	10	11	12
1	0.000 000	0.000 000	-0.032 609	0.000 000	0.000 000
2	-0.086 957	0.000 000	0.000 000	-0.160 891	-0.010 050
3	0.000 000	-0.000 000	-0.069 565	0.000 000	0.000 000
4	0.000 000	0.000 000	0.000 000	0.000 000	0.000 000
5	0.000 000	-0.003 322	-0.008 696	0.000 000	-0.006 533
6	-0.003 344	-0.047 619	-0.145 652	-0.018 564	-0.015 075
7	0.000 000	0.000 000	-0.004 348	0.000 000	-0.127 538
8	+0.966 555	-0.129 568	-0.013 043	0.000 000	-0.083 794
9	-0.013 378	+0.702 281	-0.050 000	-0.009 901	-0.038 693
10	0.000 000	-0.012 182	+0.886 957	0.000 000	-0.044 724
11	-0.043 478	-0.028 793	-0.030 435	+0.915 842	-0.016 080
12	-0.003 344	-0.005 537	-0.002 174	-0.002 475	+1.000 000
13	-0.080 268	-0.046 346	-0.043 478	-0.074 257	-0.193 467