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UNITED ARAB REPUBLIC

THE INSTITUTE OF NATIONAL PLANNING

Memo. No. 379

(being Memo.101 in the new series of Professor Frisch)

MODEL 1. INVESTMENT REQUIREMENTS UNDER A GIVEN TIME SHAPE OF CURRENT FINAL DEMAND

by

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7 December 1963

CAIRO
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Memo. 101 - 7 December 1963
To
The Ministry of National Planning
and
The Institute of National Planning
from
Professor Ragnar Frisch

The numbering of my new series of memoranda starts on 101 to avoid confusion with the several memoranda I have written to the former "National Planning Committee"

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Acknowledgements

Dr. Nazih Deif of the Ministry of National Planning and Dr. Salah Hamid of the Institute of National Planning have cooperated on an analysis of the investment requirements that are due to a given time shape of current final demand. Some preliminary pilot computations in this connection have been carried out on the IBM 1620 electronic computer which is now available in the Operations Research Center of the Institute of National Planning. The purpose of these computations was to shed light on problems related to the five year plan whose execution is to start on 1 July 1965.

At the request of Dr. Nazih and Dr. Salah I have worked out a systematic and formalized model which - on the basis of the investment requirements - can lead to a determination of the time shape of the whole constellation of the economy over the planning years (to the degree of detail included in the model). And this model has been applied to actual data available.

My work on this model - to be called Model 1 - could not have been accomplished if I had not had at my disposal the results of the theoretical, factual and computational investigations undertaken by Dr. Nazih and Dr. Salah. I must take this opportunity of saying that it has been extremely gratifying to see the great competence with which they have utilized the way of thinking which I have tried to build up in the Cairo milieu on my several previous visits.

In working out this memorandum I have also profited by pertinent remarks made by Dr. Labib Shoker, Deputy Minister of Planning, and by Dr. Mahmoud El Shafie, Undersecretary of State for Planning.

My friend and colleague Assistant Professor Tore Johansen of the Oslo University, at present with the Institute of National Planning, Cairo, has with his habitual carefulness gone through my manuscript. He has also attended to the proofreading.

1. Introduction

Let X_h^t be a measure of the total domestic production -i.e. the domestic output- in sector No.h in the year t of the plan. The origin of the time scale is chosen as the year t=0 where the plan is definitely decided upon. Then, t=1 means the first year of the execution of the plan, t=2 means the second year of the execution of the plan, and so on. For brevity I will say that t as thus defined indicates the calendar year.

Let T (say T=5) be the number of years considered in the plan. And let n (say n=11) be the number of domestic production sectors considered.

We shall here assume fixed and scale independent input coefficients in the current account operations. Let X'^{t}_{hk} be the input coefficient from the delivering sector h to the receiving sector k in the year t. Then the amount of input needed from h to k in current account operations in year t will be equal to

$$(1.1) \qquad X^{t}_{hk} = X'^{t}_{hk} \; X^{t}_{k} \qquad \begin{matrix}\text{(h=any delivering sector)} \\ \text{(k=any receiving sector)}\end{matrix}$$

The formula (1.1) with given X'^{t}_{hk} means that one assumes that there is <u>no substitution possibilities</u> among input elements from different sectors.

In my Oslo Institute memorandum of 13 May 1961 " A survey of types of economic forecasting and programming and a brief description of the Oslo Channel Model", dedicated respectfully to the Accademia Nazionale dei Lincei as a token of gratitude, I discussed (in Section 3) the concept of ring structure, which permits to take account of substitution possibilities. This can be done without destroying the linearity of the model. But this refinement is by (1.1) not considered in the present memorandum.

In addition to the total domestic output X^{t}_{h} from sector h we also consider

(1.2) $\quad A^{imp.t}_{h}$ = total imports of the <u>kind</u> of goods that are or might conceivably be produced by the domestic sector h

Further we consider the sum

(1.3) $\quad X^{t}_{h} + A^{imp.t}_{h}$ = total availability of the kind of goods that are or might conceivably be produced by the domestic sector h

The use of the individual particles of this total availability of the h-goods can be classified in the following six categories

I - Input into domestic production sectors ("cross deliveries","intermediate demand"), i.e. $\underset{k}{\$} X^{t}_{hk}$, c.f. the definition (1.1) [1)]

II - Private consumption

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1) \$ indicates summation sign.

(1.4)
- III - Government use of goods and services in current operations (not for investment purposes)
- IV - The accumulation (positive, negative or zero) of stocks of goods (whether in the private or in the public sector)
- V - Gross exports, to be denoted $A_h^{exp.t}$
- VI - The use of goods and services for the construction of fixed real capital (whether in the private or in the public sector), to be denoted J_h^t

The sum of the four categories II, III, IV, V in (1.4) we denote

(1.5) F_h^t = Final current demand = sum of the four categories II, III, IV, V in (1.4)

(1.6) The difference $A_h^{nét.t} = A_h^{exp.t} - A_h^{imp.t}$ is the net export of the kind of goods that are or might conceivably be produced in the domestic sector h.

Since any particle of total availability must belong to one and only one of the six categories (1.4) we have by definition, in any year t

$$X_h^t + A_h^{imp.t} = \underset{k}{\$} X_{hk}^t + F_h^t + J_h^t \quad \text{(for any sector h and any year t)} \tag{1.7}$$

The left member in (1.7) is the availability side and the right member the user side.

In the present model we do not consider complementary imports into the receiving sector k. In other memoranda the complementary imports were denoted B_k^t. If it is not wanted to discuss in particular the complementary aspect of the problem,[1] it is quite feasible and logical to assume $B_k^t = 0$. But then we must interpret $A_h^{imp.t}$ as all imports, both complementary and competitive. This is the viewpoint adopted in the present memorandum.[2]

1) And if we do not aim at a thoroughgoing programming analysis (which we don't in the present memorandum).
2) The sum $\$_h X_h^t$ will then denote the total domestic production minus all complementary imports.

The ratio of imports to the total availability in sector h is denoted q_h^t, i.e.

(1.8) $$A_h^{imp.t} = q_h^t \, (X_h^t + A_h^{imp.t}) \qquad \text{(for all h and t)}$$

which also can be written

(1.9) $$A_h^{imp.t} = Q_h^t \; X_h^t \qquad \text{(for all h and t)}$$

where

(1.10) $$Q_h^t = \frac{q_h^t}{1-q_h^t}$$

Since by definition X_h^t and $A_h^{imp.t}$ are both non negative (and not both equal to zero), the coefficient q_h^t must be a number between 0 and 1, limits included. The case $q_h^t = 0$ means that there is no import of the h-kind of good, i.e. $A_h^{imp.t} = 0$. The case $q_h^t = 1$ means that all the h-kind of goods are imported, i.e. $X_h^t = 0$. Therefore, the difference

(1.11) $$1 - q_h^t = \text{selfsufficiency coefficient}$$

indicates the degree to which the country is selfsufficient with regard to the h-kind of goods.

While the range of the coefficient q_h^t is between 0 and 1, that of Q_h^t is between 0 and + ∞ . Since the limiting case $Q_h^t = + \infty$ is excluded in the Egyptian data pertaining to the present model, this limiting case will not produce any computational difficulty.

The selfsufficiency coefficient (1.11) for the h-kinds of goods-and the corresponding coefficients q_h^t and Q_h^t that express the same idea-should not be confused with the coefficient that indicates the ratio of <u>complementary</u> imports into the receiving sector k. As has already been said the complementary import aspect is not considered in the present model. If wanted, it may be introduced <u>after</u> the model has been solved. One may then simply ask how large a portion of $A_h^{imp.t}$ (the import of the h-kinds of goods) that corresponds to complementary import inputs into any particular or into all the receiving sectors[1] $k=1,2 \ldots n$.

The reason why so much interest has been focused on the selfsufficiency coefficients is the concern about the foreign exchange balance and a desire to protect this balance. This concern can, I think, be more effectively taken care of by introducing the foreign exchange balance as a separate variable in the model (as was done in my several earlier memoranda). This procedure is more rational than to consider the selsufficiency coefficients, because it takes account of all the indirect effects in the economy. If all indirect effects are taken account of, one may well find that the foreign exchange balance can be better protected by admitting less selfsufficiency of some particular kind of goods. Even if we think of the average selfsufficiency in the whole economy it may be true that selfsufficiency may not be the best way to protect the foreign exchange balance, namely if total exports may be increased more than total imports by admitting a smaller degree of selfsufficiency.

The consideration of selfsufficieny coefficients for individual kinds of goods may be justified for other reasons, not connected with the concern about the foreign exchange balance. For instance the concern about the nation being able to assure a sufficient supply of strategically vital goods even in the case of a war or other crises.

Whatever the reason, the present model assumes that the selfsufficiency coefficients for each kind of goods are politically given. They will be described in the form of the Q_h^t coefficients defined by (1.10).

The time shape of final current demand, i.e. the magnitudes F_h^t as function of time will also be assumed as given.

1) Any such amount - computed afterwards - would have to be added to $\sum_k X_k^t$ in order to give " the total domestic production" as distinct from total value added.

2. The investment requirements

Investment requirements may be due either to a desire to bring about advantages infraeffects, or they may be due to the need for bringing about capacity effects.

The infraeffect is the effect which investments may have in changing the input coefficients X'^{t}_{hk} or changing other coefficients in the model. Such changes are important because through them large savings of costs and of scarce resources may be achieved. In a country where determinate effort are being made towards rapid economic development (say the doubling of national income in ten years) the infra effect is of a paramount importance. An advantageous change of the coefficients of the model may even be a conditio sine qua non for obtaining the goal one is striving at. The introduction of the infra effect will make the model non linear and thus considerably increase the computational difficulties. This complication is therefore not considered in the present simple model. (The infra effect and the corresponding computational problem is considered more explicitly in several of my memoranda from the Oslo Institute of Economics).

The capacity effect is the effect which investments may have on the capacity of production in the domestic delivering sectors. This effect is considered in the present model. It is not done in the more elaborate and satisfactory way that was followed in my previous work on the Cairo Channel Model (described in my memoranda to the former National Planning Committee), but in the form that one tries to determine in an approximate way the requirements for investments that follow from a politically given time shape of the final current demand F^{t}_{h}. (Only one investment channel for each sector will be considered.) Subsequently the analysis is made exact by showing that certain supplementary assumptions must be added in order to make the solution determinate.

It takes time to complete the execution of an investment project. It is therefore necessary to distinguish sharply between the starting of the execution of a project, and the sinking of investment goods during the execution of this project in a certain number of years that follow after the starting. This sinking will have to continue until the project is completed. A third concept is the capacity emerging. In the more elaborate Cairo Channel Model which was developed previously, several capacity emerging years were considered because the capacity increase may emerge little by little until the total capacity increase is reached. This complication is not considered in the present model. We simply assume that the total capacity emerges suddenly in one year, namely in the year immediately following the last sinking year.

To summarize: the starting year, the sinking years and the capacity emerging year are three time concepts that must be clearly distinguished. The difference between a sinking year and the starting year of the project in question will be called a sinking delay.

In what follow only one capacity increasing channel, i.e. No. g, will be considered for each delivering sector.

Let c_g be the construction time, i.e. the number of sinking years that have to be considered when an investment is started in the investment channel g, that is in the channel through which the capacity in sector g is increased. This means that for a starting which takes place in the channel g in the year S of the plan we must consider the sinkings which this starting entails in the following years:

(2.1) $S + 0,\ S + 1,\ S + 2 \ldots S + (c_g - 1)$

of the plan. The number of years written in (2.1) is equal to c_g which corresponds to our definition of c_g as the number of sinking years we have to consider in the channel g.

If we assume that the total capacity emerges in the beginning of the year that follows immediately after the last sinking year, we see that

(2.2) $t = S + c_g$ is the capacity-emerging year

From this follows that if we want the increase in capacity to occur in the given calendar year t, the required starting year will be

(2.3) $S = t - c_g$

Now consider the <u>volume</u> of startings and sinkings(measured for instance in money values under a constant system of prices).

We define

(2.4) J_{hg}^{sS} = the volume of sinkings of h-goods into channel g s years after starting, when starting takes place in year S.

$(0 \leqq s < c_g)$

The total volume of sinkings which is necessitated by the starting in channel g in the year S is

(2.5) $$H_g^S = \sum_{s=0}^{c_g-1} \sum_{h} J_{hg}^{sS}$$

In (2.5) h runs over all sectors.

The total volume (2.5) will be called the <u>size</u> of the project, or the project " in its full dress", or again the starting variables for the project. If there is no starting in the channel g in the year S, then $H_g^S = 0$. If there <u>is</u> a starting in channel g in the year S, then H_g^S is different from zero and the magnitude H_g^S will indicate <u>how big</u> the starting in channel g is in this year.

Let us express the individual sinkings as <u>fractions</u> of the total size of the projects. This leads to considering the coefficients J'^{sS}_{hg} defined by

(2.6) $$J^{sS}_{hg} = J'^{sS}_{hg} \; H^{S}_{g}$$ (For any h,g,s and S)
(s=sinking delay)
(S=starting year)

The coefficients J'^{sS}_{hg} we may call the investment input coefficients, or better the <u>sinking</u> coefficients, to indicate explicitly that it is a question of coefficients that express how much goods and services that need to be sunk in the year (S+s) in order to execute the project.

An apostroph will be consistently used to express a coefficient and the affixes on this coefficient will be the same as the affixes on the absolute volume figure whose size it is wanted to express. Cf. (1.1) and (2.6).

If we insert (2.6) in the right member of (2.5) we get

(2.7) $$H^{S}_{g} = \sum_{s=o}^{c_g-1} \sum_{h} J'^{sS}_{hg} \; H^{S}_{g}$$

If the project is to be undertaken we will have $H^{S}_{g} \neq o$. Therefore we may divide in (2.7) by H^{S}_{g} and thus get

(2.8) $$\sum_{s} \sum_{h} J'^{sS}_{hg} = 1$$ (for any g and any S)

In (2.8) the summation over s runs over all the sinking delays where sinking due to the investment starting considered (i.e. investment started in the calendar year S in channel g) <u>actually occurs</u>. We may if we like even let the summation over s in (2.8) run from $-\infty$ to $+\infty$ and so to speak leave it to the coefficients J'^{sS}_{hg} <u>themselves</u>

to watch the values of s for which they are to be zero. They are zero for any negative sinking delay, and for any sinking delay that occurs <u>after</u> the completion of the project (i.e. for any sinking delay that is equal to or larger than c_g).

When using any set of numerical data one should always apply (2.8) as a <u>check formula</u> to verify that the numerical data are consistent. (2.8) is a necessary (but not a sufficient) condition which the J' coefficients must satisfy in order to be consistent.

As a special case the coefficient J'^{sS}_{hg} may not depend on the starting year S but only on the sinking delays, i.e. on the number of years that have elapsed <u>after</u> the starting. In this case the complex of <u>two</u> affixes sS is replaced by the <u>single</u> affix s. In this case we use the notation J'^{s}_{hg} (s=t-S).

In this case (2.6) reduces to

(2.9) $$J^{sS}_{hg} = J'^{s}_{hg} H^{S}_{g} \qquad \text{(s=t-S in the case (2.9))}$$

This expresses the sinking that takes place in the year s+S due to a project that was started in channel g s years earlier, namely in the year S.

In the present model the assumption (2.9) is made for all h and all g, in the numerical work, but this is only an incidental feature of the computations. In principle it is no need to make this assumption. In order to assure generality for other possible application I shall, in the sequel, use the general formulation (2.6).

The total sinking of the h kinds of goods that takes place in a given calendar year t - i.e. the term J^{t}_{h} in (1.7)- is due to a number of different startings in the years <u>preceeding</u> t. We get

(2.10) $$J^{t}_{h} = \sum_{g} \sum_{s} J^{s,t-s}_{hg}$$

where the $J^{s,t-s}_{hg}$ are defined by (2.4).

We are now ready to study what is meant by investment requirement. In a more thoroughgoing analysis (as the one underlying the Cairo Channel Model and the much more complete Oslo Channel Model) one studies explicitly the concept of <u>production capacity</u> in each sector and one imposes the condition that the total production in any delivering sector h in a given year t must never go beyond the capacity that <u>exists in this sector in the year t</u>. Only through the time consuming investment process which we have just discussed can capacity be increased.[1] In the present memorandum this is taken care of in a special way which leads up to a determination of the way in which the production in each domestic sector depends on all the time shapes of final current demand for the h-kind of goods, i.e. on F_h^t. Cf. also the remarks in the beginning of this section.

This politically given F_h^t - it will in practice mean an <u>increasing</u> F_h^t - will necessitate an increasing total availability ($X_h^t + A_h^{imp.t}$). And since by (1.9) - with politically given Q_h^t - the import will follow the domestic production, the final result will be that the increasing F_h^t must lead to increasing domestic production X_h^t in the sector h. And this in turn will necessitate investment to bring the <u>capacity of production</u> in sector h at the level needed.

From the year (t-1) to the year t the total domestic production will increase by an amount $(X_h^t - X_h^{t-1})$. If the capacity for producing X_h^{t-1} has previously been provided for, we are now facing the necessity of providing for an <u>addition to capacity</u> which will emerge in the calendar year t and which is of such a size that we will in year t have sufficient capacity to be

1) The number of shifts is assumed given.

able to produce domestically X_h^t.

Let g be the investment channel through which the capacity in sector h is increased. Since we assume that there is only one channel for each sector we could have used the same numbering, i.e. we could have spoken of the investment channel No.h or the sector No.g. For the subsequent handling of the formulae it is convenient to have both letters, h and g, available.

If the capacity due to an investment in the channel g is to emerge in the year t, the starting must be made in the year $(t-c_g)$. How large should this starting at $(t-c_g)$ be ? The total capital which will finally be invested when all the ensuing sinkings are completed, is H_g^{t-cg}. In some sectors -i.e. in some channels- there is more capital invested per unit of output than in others. Let C_g^S be the capital to output ratio in channel (sector) g in the year S. This means that if we are going to make an investment large enough to be able to satisfy an increase in output equal to $(X_g^t - X_g^{t-1})$ we must insert a capital equal to $C_g^{t-c_g}(X_g^t - X_g^{t-1})$. This therefore must be the size of the starting we are now considering. In other words, we must have

(2.11) $$H_g^{t-c_g} = C_g^{t-c_g} (X_g^t - X_g^{t-1}) \quad \text{(for any t)}$$

Writing as before S for the starting year, i.e. putting in (2.11) $S=t-c_g$, and hence $t=S + c_g$, (2.11) takes the form

(2.12) $$H_g^S = C_g^S (X_g^{S+c_g} - X^{S+c_g-1}) \quad \text{(for any S)}$$

This is the investment starting which it is required that we make in channel g in the calendar year S.

In the numerical work in connection with the present model it was assumed that C_g^S was independent of S, but this is only an incidental feature of the