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An Introduction To The Input-Output Model

By

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Historical background

Consistent thinking is no doubt "a moral obligation"(1) and if it is so for intelligent human beings it is certainly much more so for rational economists. But even a rational well disciplined economist is often at a loss when confronted with the highly complicated mechanisms which govern and rule economic decisions . This feeling of uncertainty is even more vivid when he is faced with the unsurmountable difficulties which he has to overcome when dealing with the so-called underdeveloped economies. These difficulties are further enhanced by such limitations, as the lack of statistics, which the latter economies put upon the tools with which he is acquainted and hopes to discover the "invisible hand" which turns the economic wheel in the more advanced economies. In such case, the ideal state of consistent thinking, although remains a desired goal, could not be lived up to all the time. But the rational economist, nevertheless, is in constant search for the proper analytical tool which keeps him within the vicinity of that thinking.

In fact the awareness of the economist of the considerable specialization within each sector of the economy and among the different economies, besides leading to the formulation of such elaborate theories as the comparative cost theory, made him always timid to explore the mysterious sphere of economic interdependency. Furthermore it led him to adopt a certain analytical tool, one which came to be known as partial analysis. This partial analysis was a predominate feature of economic analysis up until a very short time ago. This is despite the equal awareness of the fact that the different sectors of the economy, whatever their degree of specialization may be, could not work in complete isolation from the other sectors of the economy. This awareness is clearly illustrated by the example of the "production of a pin" which Adam Smith forwarded to illustrate the extreme specialization within the economy.

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(1) This term is borrowed from a delightful memorandum written by Prof. Ragnar Frisch and published by the National Committee, Cairo. June 19, 1960.

Partial analysis, however, is an indispensible tool in economic understanding particularly if we desire to enhance our knowledge about the operation of the market economy. Marshall's partial equilibrium system in fact could be looked upon as a simplification of the equilibrium theory. However, it does not claim to be looking at the economy as an entity; but rather it traces the changes within individual sectors assuming, of course, that other things will remain unchanged. The changes in the behavior of the consumers and the producers of a particular commodity are examined, with prices and outputs of that commodity being the crucial points. This type of relationship is presented in a set of supply and demand functions, with changes in the level of income or of outputs of the consuming sector appearing as shifts in the demand functions. Similar shifts in the supply functions of the using sector may appear with changes in other uses of its inputs.(1) In fact, what the partial analyst would generally do is to isolate the effects of certain variables and attempts to detect the results of their interaction while assuming the other variables in the economy to be constant. While this approach is commendable in the sphere of physical sciences it is less meaningful in economics as the other variables rarely stay without changing. This is due, of course, to the complicated nature of the economic mechanism and the fact that there exists a great deal of interdependence among the different economic sectors.

The idea of interdependence in the economy, as we have mentioned before, was recognized by Adam Smith, and even by earlier economists.(2) The "Tableau Economique" in which François Quesnay (3) attempted to demonstrate the flow of wealth through the economy is but a definite recognition of that idea. In fact interindustry models are generally traced back to those "tableaux". However, it is generally maintained (4) that the present development in the field of interindustry economics is largely indebted to the inspiration provided by the work of "Leon Walras" who developed a complex and abstract model which made that phenomenon its main theme. He recognized the fact that "everything in the economy is dependent upon everything else" and his model was built with the aim of examining the requists for general equilibrium. It contained a number of equations which described the cost structure in the production sectors, the consumers incomes and expenditures as well as the total supply and demand of the different commodities and primary inputs. He took into consideration the substitutability of one product by the other as well assuming that industries would compete for the factors of production, two points to be dropped

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(1) Chenery and Clark, "Interindustry Economics", p. 2.
(2) As early as 1759, a draft of what later developed into Wealth of Nations existed in Smith's notes but it was not until March 9, 1776, did the work come from the press. See Robert B. Downs, "Books that Changed the World," p. 42.
(3) François Quesnay published his tables in 1758.
(4) Chenery and Clark, "Interindustry Economics", p. 2.

later, as we shall see, by Prof. Leontief. The Walrasian system was criticised later as being "an example of the emptiness of economic theory at this level of abstraction, since few interesting conclusions about economic reality can be drawn from the formal properties of the model, and the system as formulated does not lend itself to empirical verification." (1)

Other economists who contributed in this field are Pareto and Cassel who aimed at perfecting the model of Walras. However, it was always felt that their models had to be simplified in order that their functional relations could be statistically determined. At this point one should not underestimate the contribution made by Prof. Ragner Frisch of Norway in his paper "Circulation Planning" which appeared in Econometrica in 1934 in which he discussed the case of a closed economic system as well as that of an open one, both with determinants which were equal to 0. (2) In that article he put forward a set of equations which resembled in essence those of the Input-Out Model.(3)

However, it was Prof. Wassily Leontief who thought of these ideas not merely as theoretical sophistications but as an inspiration for his great empirical work. (4) Unlike Prof. Frisch he discussed the case of an open system with non-vanishing determinant. Furthermore he was the first to attempt an empirical application of the general equilibrium theory. Although his work on an empirical model for the American economy started in 1931, his first results were published in 1936.(5) It was Prof. Leontief who simplified the Walrasian model to the extent at which it could be statistically determined. As we mentioned before, he dispensed with the limitations Walras put upon the factor supplies as well as that of substitution among inputs. In doing that he accepted the original assumption of Walras of fixed "coefficients of production." The Leontief model eliminates the effects of prices on the composition of consumer demand, and the demand for intermediate goods as well as that for the factors of production and in doing that it "precludes many of the adjustments characterizing the Walrasian Concept of general Equilibrium.(6)

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(1) Ibid, p. 3.
(2) Ragner Frish, "Some Basic Notions of Input-Output Analysis", National Planning Committee, Cairo, 1957, p. 8.
(3) Ibid, p.7.
(4) W. Duane Evans and Marvin Houffenberg,"The Interindustry Relations study for 1947 U.S. Department of Labour Satistics."
(5) Leontief's basic ideas were published in his article "Quantitative Input-Output Relations in the Economic System of the United States, The Review of Economics and Statistics" Vol. 18, August 1936.
(6) Chenery and Clark, "Interindustry Economics", p. 3.

As it could be observed the approach of a Leontief analyst would be in sharp contrast to that of a partial analyst. While the later studies the effects of changes in certain magnitudes and attaches no importance to the effects of these changes on the rest of the economy, the Leontief analyst considers the study of these changes his prime object. He takes on the other hand the equilibrium adjustment of input properties as given and does not attach great importance to the assumption of maximizing behavior.(1) Moreover he does not think of the changes in the supply and demand of the different commodities as a result in the changes in prices but rather due to changes in the demand function of the different sectors which are the result of changes in the levels of production. In the short run, the Leontief analyst would think of the changes in demands as having direct influence on output rather than on prices.

The aggregative income analyst, however, has much more in common with the Leontief Analyst. Uniform aggregate behaviors which generally could be determined statistically are corner stones in their analysis. While this is so the partial analyst attempts to rationalize, on the basis of theoretical justifications, the changes involving certain isolated magnitudes. They both (i.e. the former) make assumptions about the autonomous elements in their systems and try to reach the effects of changes in these elements on the level of production or income through a set of coefficients indicating the structure of the economy in the case of the Leontief model or through a set of propencities indicating the response of the rest of the economy to these changes in the case of the aggregate income approach.

However, it must be remembered that a major point of difference still exists between the two analysts and that is the extreme importance which the Leontief analyst attaches to the interdependence in the economy, a point which the aggregative income analyst takes only into consideration but does not exert any effort to bring it into the forefront.

Since the publication of the input-output tables for the American economy for the years 1919 and 1929, remarkable progress had been made, both in techniques of constructing and utilizing the tables. For a while, the field of Input-Analysis was limited to a number of specialized economists. Although, still largely so the technique has become an obligatory course in economics and many economists are eager to learn more about it. Governments in different parts of the world have become aware of the usefulness of that technique in designing their economic policy and we find that an increasing number of them have constructed tables for their economies or are in the process of doing so. This, of course, is one of the remarkable results of the Leontief contribution to economic theory.

Recent developments in economics and statistics, however, provided us with alternative models. Linear programming techniques, although overcome the assumption of fixed input coefficients,(2) have so far

(1) Ibid p. 4.
(2) Chenery and Clark, p. 3.

been applied to the case of single plants. Nevertheless it presents a lineal continuation of thinking and offers other choices in the field of interindustry models.

As we will see later the input-output model has been criticised on several grounds. Some of these are valid and others have no firm ground upon which to stand. The most common among the latter type is the attempt on behalf of some economists to be little Leontief's great contribution on the account that it was, after all, an old Walrasian idea; but as someone said "everything of importance has been said before by someone who did not discover it."

Economic models, a positive step for consistent thinking

Economic models have become an essential tool in economic analysis. In fact, the latter depends a great deal on how the available data are set in such models. Although the economic analyst is limited in forming such models by the restricted availability of the required data more than by his ability to form them, at least theoretically, yet it is the formation of such models that lead to the discovery of the inconsistencies in the available data as well as the gaps in them. It has also become a recognized fact that in order to carry out a certain economic analysis one has to investigate and describe in a comprehensive manner the features of the economy which are of relevance to that type of analysis. These features are often expressed in terms of magnitudes of certain variables which among themselves describe the prevailing structure of the economy. However, in doing that certain definitional relations have to be satisfied. For instance the gross production from sector i couldbe expressed as an equal sum of the demand on the production of that sector for inter mediate consumption plus the demand for final use. This definitional relation could be represented by the following simple equation:

$$X_i = N + Y$$

where N is the demand for intermediate consumption and Y is the demand for final use. But this is only one sector and only one relation. The economy, however, has other sectors and other magnitudes and all are governed by other definitional relations. To be consistent, one must make sure that these relations are fulfiled and not only one of them. If several macroeconomic magnitudes are estimated or planned independently such a requirement would necessitate considerable attention and care that it is safe to state that without the aid of a comprehensive economic model it would be next to impossible to reach any consistent conclusions.

But the definitional relations are not the only relations which should be fulfilled. An economy, at any point of time, has a certain underlying structure and any consistent analytical approach should take this structure into consideration. It would be absurd to think of a future development program which is incompatible with the prevailing structure of an economy. However, this does not mean that

one should think of a certain structure in a static sense but only to indicate that any deliberate change in that structure should be compatible with the underlying one. In any case there is always a set of structural relations which exist in the economy, the interrelations amongst which are highly complicated, a fact which makes the reliance on an economic model an indispensable tool if we ever attempt to examine intimitely the structure of the economy.

To illustrate the structural relations let us assume, on the other hand, that in order to produce one unit from sector j we need to have certain deliveries from other sectors plus a certain delivery from imports and other deliveries from primary factors like labor and profits on capital. This could be expressed as follows: $Xj = \Sigma_i Xij + Mj + Vj$ $(i = 1,2, \ldots .n)$ (2) where Xij indicates the delivery from sector i to sector j and Mj indicates the inputs from imports into sector j andVj indicates the inputs of primary factors or the value added in sector j.

In the meantime these deliveries could be related to the production of sector j through a set of coefficients which indicate the proportion of each input needed for the production of one unit from sector j. This is assuming that there is a linear relation between the production of the sector and the inputs needed for that production. In other words we assume that the proportion of the inputs will not vary by a change in the size of production. These relations could be expressed as follows:

$$Xj = \Sigma_i\ aij\ Xj + mj\ Xj + VjXj \quad (i = 1, 2, \ldots n) \qquad (3)$$

where aij indicates the proportion of the input from sector i which is necessary for the production of one unit from sector j and mj indicates the proportion of inputs from imports which are needed for the production of one unit from sector j, and Vj indicates the proportion of primary inputs or value added created by a production of one unit from sector j.

This is only a simple illustration of a structural relation which describes the structure of only one sector of the economy. But there are many other sectors and there are as many relations of this type as there are sectors. Also there are many other types of structural relations and any future development could not be conceived independently of the prevailing structure of the economy.

By now it has become clear that unless we systematize the economic concepts, whether these are definitional or structural, in a comprehensive model we may never be able to reach constructive conclusions regarding economic problems which we have to deal with and even if we reach any such conclusions we will not possess a tool for testing their logical consistency. Of course an economic model should not be conceived as a magic tool capable of answering all our questions and delivering answers with complete accuracy. Such models generally contain assumptions and reduce the complicated economic relation into simplified formulas and seek reasonable approximation of results.

In fact this very nature of the economic models makes the question of what sort of specifications are relevant and necessary and what simplifications are permissible in the model we intend to use for a particular problem, a prominent one indeed. There is not one economic model to solve every economic problem, but rather for every problem a suitable model to deal with it. In this article we are only dealing with one particular model, i.e. the Input-Output model.

The underlying concepts of the input-output model

The Input-Output Model and its utilization has been extensively discussed by many writers and the literature on that subject is increasing rapidly.(1) However, as these articles intend to put before the reader, who may be entrusted by such work, the experience of the writer in the field of construction and utilization of the Input-Output model in a developing economy, we find it necessary to refer, in as simplified way as possible, to the basic concepts of that method as well as the mathematical aspects (2) involved in its utilization. We will be dealing mainly with the simplest input-output model or that which is refered to as the "open Leontief model." It is called an "open system" because it treats certain sectors of the economy as exogenous sectors. In other words the magnitudes of those sectors are generally considered as given and not derived from a solution of the model. As the stock of capital needed for the creation of certain productive capacities is included in that exogenous part of this system, it is therefore considered a static model because we will be unable to reach a solution which tells us the amount of capital requirements which satisfies our particular target. These points., however, may be discussed in some details later but reference to them was made only to indicate that there are other complicated forms of the input-output model. Nevertheless it is safe to state that these forms could be treated as generalizations of this simple model.

As we have mentioned before, the input-output method is another form of general equilibrium analysis. It is based on the cost structure of the different types of production and like all the other economic models it rests upon certain assumptions about economic behavior and observes certain definitional relations among its variables. But unlike many others, the input-output model is a multi-sector model and aims mainly at illustrating the interdependency among the different sectors of the economy. One can hardly conceive of any change in the demand for the production of one sector without this change having a chain of direct as well as indirect reactions on the other sectors of the economy. Let us assume for instance that with an anticipated increase in per-capita income we were able to project, through utilization of such tools as income elasticity of demand, certain increase in the demand for industrial commodities. Let us also assume that all industries could be grouped into one sector and that we know the inputs needed for the production of that sector. Table 1 makes it clear that in order to produce industrial goods, the industry sector would require inputs from the other sectors. It will have to buy agricultural products, energy, industrial products

(1) See "Input-Output Bibliography", by Charlotte E.Taskier of the Harvard Economic Research Project, a United Nations Publication.
(2) This will be discussed in future articles.

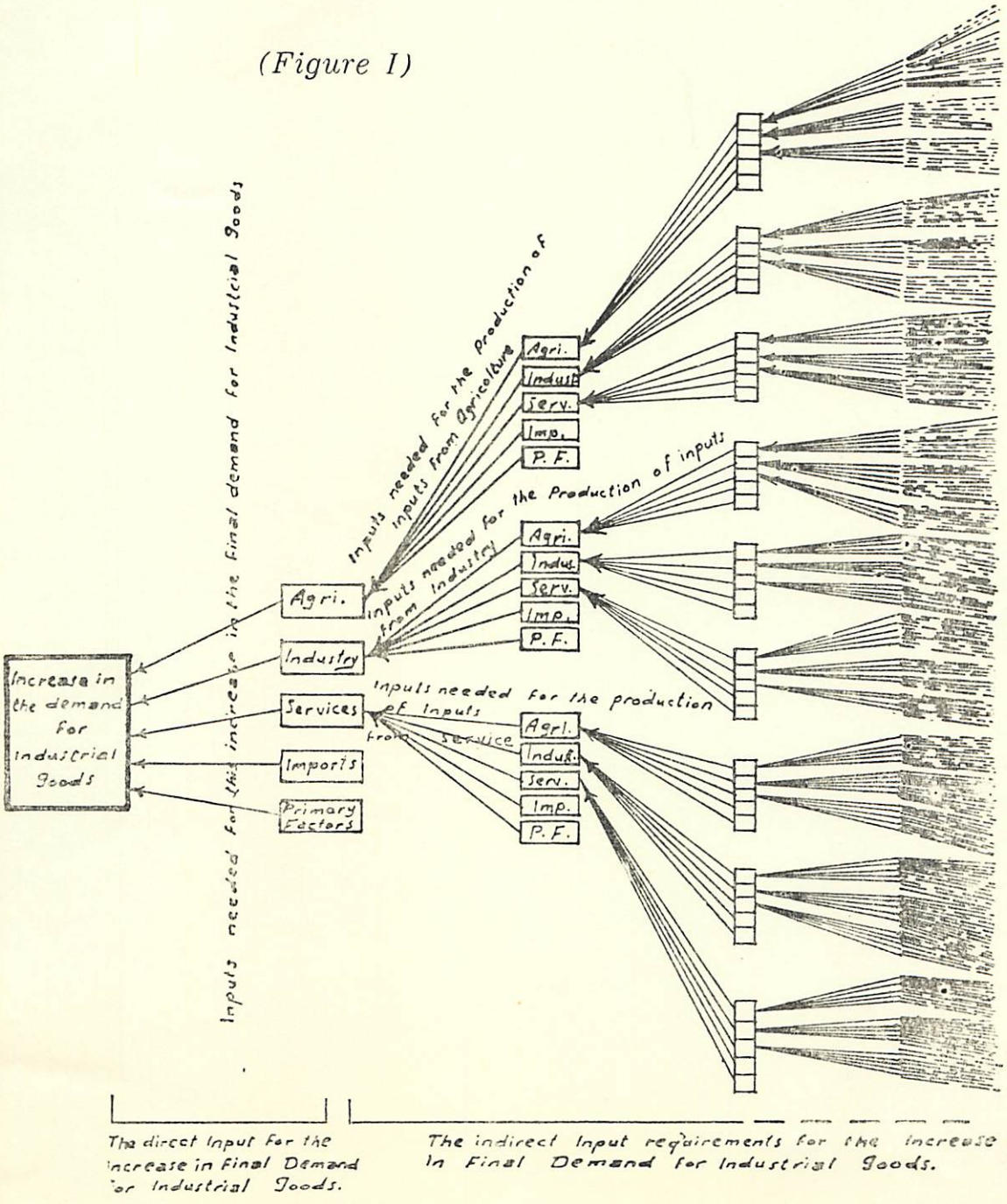
from itself, transportation facilities, construction activities, and services and some imported goods as well as primary inputs. These purchases represent the direct input requirements for the production of the sector industry. But this is not the whole story as these purchases represent in themselves increases in the demand for commodities from the sectors concerned. And those increases in demand mean increases in production and in its turn the increase in the production of every sector requires a variety of inputs of raw materials, intermediate goods and imports. In fact, the very essence of interdependency among the dirrerent sectors of the economy is responsible for the creation of a series of reactions which are generally extended to an ever increasing number of sectors. Although the impact of these reactions become progressively weaker(1) yet their total is significantly large to be ingored. The complicated nature of these direct and indirect reactions could be easly conceived in the attached illustration. (Figure 1)

However, it must be remembered that this complicated mechanism which the above illustration made crystal clear, is only the product of an increase in the demand for the production of one sector. Now it is left to the imagination of the reader to consider the case where the demand for other commodities increase simultaneously as they generally do in practice. Here we are faced with the dilemma of quantifying these reactions and as one can see it is beyond any human mind to trace such reactions except through a systematic tool of analysis. In fact, this is the greatest asset of the input-Output table. It could tell us, through a set of coefficients, which are generally refered to as "the coeffecients of direct and indirect requirements", the total requirements from each sector to satisfy a certain increase in the demand for some commodities. This, being the case, the input-output table, could provide us with answers to a variety of questions which are of great importance particularly in the field of national planning. For instance what would be the effect of a particular investment program in road construction or irrigation. These effects could betranslated as total input requirements from both domestic production and imports, labor requirements, value added created and others. In such case an investment fund could be conceived as expenditure on goods and services, including labor, from the other sectors and if so it could be thought of as increases in the demand for those goods and thus we can. through the above mentioned coefficients, trace their effects.(2) As can be seen the inputs required for such investments will not be the type designed to satisfy the consumers needs directly but rather through being an

(1) This fact will be refered to in a later article when we discuss the iterative method of solving an input-output problem.

(2) This point will be examined in greater detail later when we discuss the application of the table to certain problems in Egypt.

(Figure I)


Increase in the demand for Industrial Goods
Agri.
Industry
Services
Imports
Primary Factors
Inputs needed for the increase in the final demand for Industrial Goods
Inputs needed for the production of inputs from Agriculture
Inputs needed for the Production of inputs from Industry
Inputs needed for the production of Inputs from Service
Agri.
Indus.
Serv.
Imp.
P.F.
The direct Input for the increase in Final Demand for Industrial Goods.
The indirect Input requirements for the increase in Final Demand for Industrial Goods.

essential part for the production of the finished goods. In this case, the application of such criterions as the income elasticity of demand in order to predict the requirements for such commodities would be extremely difficult and may even be inconceivable. The input-output table, on the other hand, provides us with a particularly suitable device for such prediction. This is due, of course to the very nature of the table which does not overlook the complexity of the relationships among the different sectors but rather emphasizes it. The coefficients of "direct and indirect requirements" in this case are the only answer. The table could inform us also of the repercussions of certain government policies, such as increase in taxation, more government investment or increase in public works. All these actions could be conceived as having direct effects on the demand for the different commodities and their repercussions on the different sectors of the economy could thus be determined. As will be described in a later article in greater details the input-output table provides us with unique tool for calculating the effects of an import substitution policy. This is of paramount importance particularly in a developing country where industrialization generally tends to substitute imports by local production. In this case it is extremely important to calculate the net savings in imports which such policy may lead to.(2) This could be easily done through the input-output table as it enables us to calculate the direct and indirect requirement of imports for any particular demand. In short, the input-output method is most suited for solving the problems where the complex characteristic of interdependency is involved.

The input-output model and the concept of national income accounts

Those who are familiar with national income accounts and its underlying concepts will feel at home when being introduced to the input-output method. They both provide us with a system of accounts which cover the whole economy and, in fact, they are complementary to each other. However, there is a basic difference between the two methods, the reason for which could be traced to the ultimate purpose which each aims to serve. As for national income accounts, it aims at measuring the value of goods and services produced by the entire economy within a specified period of time, normally one year. The national income accountant, therefore, is interested in the final results of the economic activities and hence, he only considers the net effects of all the dealings which take place within the economy. To do that he avoids double counting and therefore deliberately disregards transactions between the different productive sectors. It is there, in fact, where the basic difference between that method of accounting and the input-output method lies. The latter being a main body of the interindustry analysis which concerns itself mainly with

(2) This adoption of such method may be offered as a good tip to the economists of the Ministry of Industry. The indirect effects are so important that they should be taken into account.

the interrelations arising from production, attaches a distinct importance to the flow of goods and services from one productive sector to the other. It is this interdependence among the different sectors which the input-output table aims to bring to the limelight and in doing that double counting becomes an accepted phenomenon. Also as we indicated before the input-output model is most helpful in dealing with the problems where the structural interdependency is the crucial factor and where the more simplified framework of national income accounts could be of very little help.

To illustrate the relationship between the input-output table and the national income accounts we have utilized the data of the 1954 input-output table for the Egyptian Economy to construct the three tables given below. Table 1 shows clearly how in calculating the G.N.P. we avoided double counting. In both sides of the table our starting point was the gross value of production. In the left side the value added in the three sectors was calculated simply by subtracting from the gross value of producion in each sector, the value of inputs required for the production of that sector. For instance the value of any particular input, say industrial goods has to be subtracted from the value of the gross production of the sector which consumed it as it has already been calculated in the production of its own sector. This fact leads us to realize how in measuring the G.N.P. we are really more concerned with the value of the final goods produced by the whole economy. This end could be pursued by constructing an account similar to that given in the left side of Table 1 which shows the value of the sales of every sector to final demand purposes. As it is clear this was reached at by subtracting from the gross value of production from each sector that part of its production which was sold for intermediate consumption purposes. If we add up the value added from the three sectors which appear in the left side of the table and also add the final sales from the three sectors which appear in the right side of the table we should find that the two totals are equal. However, this is not so in our example (1) simply because we have to deal with imports and also with a certain portion of the value added which appear in the final demand sectors of the input-output table, i.e. Table 2. In fact we had to follow this course in order to show how the figures in our three tables are related and how the gross national product could be calculated from them. Now in order to equate the two sides of Table 1 we should add to the total of the value added in the three sectors the part of the value added which appear in the input-output table within the final demand sectors. This will give us a total of value added of 1,054,109.(2) As regards the left side of the table we should add to the total of the final sales the value of the imports needed for final demand purposes and also the same figure of value added which appear in the final demand columns of the input-output table.

(1) An illustrative example with no imports invloved could be constructed to show the total of the value added is equal to the final sale.

(2) This figure is in thousand Egyptian pounds. In other words it means 1,054 million pounds.

TABLE 1

(Figures in thousand Egyptian Pounds)

GROSS NATIONAL INCOME			GROSS NATIONAL PRODUCT		
AGRICULTURE			AGRICULTURE		
Gross Production		400,814	Gross Production.....		400,814
Inputs from			Intermediate Sales to		
Agriculture ...	47,481		Agriculture ...	47,481	
Industry	15,170		Industry	205,790	
Services	148,734		Services.....	92	
Imports	16,500				253,363
		227,885			
Value Added.....		172,929	Final Sales		147,451
INDUSTRY			INDUSTRY		
Gross Production.....		898,765	Gross Production		898,765
Inputs from			Intermediate Sales to		
Agriculture ..	205,790		Agriculture ...	15,170	
Industry.....	253,615		Industry	253,615	
Services.....	99,358		Services	48,288	
Imports	66,966				317,073
		625,729			
Value Added		273,036	Final Sales		581,692
SERVICES			SERVICES		
Gross Production.....		554,561	Gross Production		554,561
Inputs from			Intermediate Sales to		
Agriculture ..	92		Agriculture ...	148,734	
Industry	48,288		Industry	99,358	
Services	28,659		Services	28,659	
Imports	7,313				276,751
		48,352			
Value Added.....		470,207	Final Sales.....		277,810
			Total Final Sales.		1.006,953
			+ Imports for Final demand ..		82,105
Total Value added in the three Sectors		916,172	+ Value added in the final demand sectors		137,935 1,226,993
+ Value added in the final demand Sectors.....		137,935	- Total Value of Final demand imports....		172,884
TOTAL Value Added (Gross National Income)		1,054,109	(Gross National Product).		1,054,109