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SOME GENERAL REMARKS ON THE SUCCESSIVE STAGES METHOD IN PLANNING WITH AN ADHOC CONSIDERATION OF THE TIME LAG EFFECTS ON THE ECONOMY'S DEVELOPMENT

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Some General Remarks On the Successive Stages Method In Planning With An Adhoc Consideration Of The Time Lag Effects On The Economy's development (1)

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(1) A more detailed analysis of both the method of planning in stages, and the method of simultaneous planning can be found in my forth-coming research - work: " National Planning In Stages Verus Simultaneous National Planning".

In the sequence I shall give a brief general outline of what is meant by planning in stages and analyse the effect of introducing in the analysis the concept of a time-lag, specially its effect in the case of an expanding economy. I shall not, however, comment or discuss the advantages and disadvantages of the method of national planning in stages in this rather short paper. This will be found together with a detailed analysis of that method, as well as a detailed analysis of the Simultaneous national planning method with comments on it, in my forth-coming research-work : National planning in stages Versus Simultaneous National Planning, The method of Planning in stages (two stages of more) consists mainly of choosing at the first stage, total production and total investment, looked at as aggregates for the years of the program or the plan. At the Second stage the choice is of the composition of production for each year separately. In the first stage the assumption of the existence of one single gestation period and one single price level can be given. The first stage aims at determining the future course of production by choosing an optimum program of investment for the future as a whole. One of the principles upon which an optimum program of investment may rest is the maximization of utility over time, also the maximization of justice between present and future generations can be taken into considerations.

In the second stage, the choice of the composition of production for a year or for a specific period of time can be done. Some economists' contention is that this choice ought to be based on the principle of maximizing the growth of national income at world market prices, with the side condition of the given volume of investment resulting from stage one.

If we consider an open economy, and assume constant prices, it will be found that in the determination of a maximum of Linear expressions, boundary conditions ought to be added, in form of limitations imposed on the rate of increase in any single line of production.

<u>In the following model :-</u>

V^h = volume of production of sector h

C^h = volume of final consumption inside sector h

e^h = volume of exports of sector h (imports are indicated by negative values)

$V^{hh'}$ = volume of deliveries from sector h to sector h', for current production in sector h'

$W^{hh'}$ = volume of deliveries from sector h to sector h' for investment in sector h'

Y = national income.

The following relations are assumed to exist:-

$$v^h = c^h \; , \; c^h + \sum_{h'} v^{hh'} + \sum_{h'} w^{hh'} \; , \quad h = 1 \ldots H \qquad (1)$$

Equation one is a balance-equation. There are many (H) as there are rectors or Commodities Considered.

$$v^{hh'} = \varepsilon^{hh'} \; v^{h'} \qquad h,h' = 1\ldots H \qquad (2)$$

The above is an input-output relation, where the coefficient $\varepsilon^{hh'}$, indicates how much more of good h is needed, if the production of h' is raised by one unit There are H^2 such relations and Coefficients.

$$w_t^{hh'} = \frac{x^{hh'}}{\theta h'} \left(v_t^h + \theta^{h'} - v_t^h \right) \qquad \begin{matrix}(a)\\(3)\end{matrix}$$

The above relationship indicates the necessary investment, during period t in good h, for an expansion $v_t^{h'} + \theta^{h'} - v_t^{h'}$ of the production of good h' between time t and time $t + \theta^{h'}$ where $\theta^{h'}$ is the gestation period characteristic of industry h'. $X^{hh'}$ may be called the partial Capital Coefficient of industry h' with regard to good h.

If total investment needed for a unit increase in production $v^{h'}$ amounts to $\theta^{h'} \sum w^{hh'} = \sum_h x^{hh'} = x^{h'}$ (b) (3)

Then those partial Capital coefficients add up to what is called the Capital coefficient for industry h'

The investments in all sectors add up to the pre-designed or pre-planned volume of investment S:-

$$\sum_h \sum_{h'} w^{hh'} = S \qquad (4)$$

The definition of Y (national income) is as follows :-

$$Y = \sum_h v^h - \sum_h \sum_{h'} v^{hh'} \qquad (5)$$

The demand for consumers goods is assumed to be

$$C^h = \gamma^h (Y - S) + \hat{C}^h \qquad h = 1 \ldots H \qquad (6)$$

Where γ^h are marginal propensities to consume and $\hat{C}^h$ are other constants characteristic of the preferences of the Consumers. Those two coefficients, however, have to satisfy the following conditions :-

$$\sum \gamma^h = 1, \quad C^h \geq 0 \qquad \sum \hat{C}^h = 0$$

S and Y are determined in a macro-economic model in the first stage of planning. A model in which only these variables for all time limits appear. It is assumed that a given value for S_t for year t has resulted from the first stage.

In the second stage the above equations (1 to 6) can be applied to choose at the beginning of year t the volumes of production $v_t^h + \Theta^h$ which are aimed at the end of the gestation period for each industry. If there is an equal gestation period for all industries a maximum is then required of

$$y_t + \Theta = h\,(vt^h + \Theta - \Sigma_{h'}\ v_t h'_{+} h'_{\Theta}) \qquad (7)$$

Where Θ is the gestation period.
Taking equation (2) into consideration, the above expression can be transformed into :-

$$y_t + \Theta = \Sigma_h\ v^h{}_{t+\Theta}\,(1 - \Sigma_{h'}\quad h'h) = \Sigma_h\ v_t{}^h + \Theta\,\ell^{\,Oh} \qquad (8)$$

The size of the total investment in year t is an imposed side condition :-

$$\Sigma_h x^h\,(v_t{}^h + \Theta - v_t{}^h) = \Theta\, S_t \qquad (9)$$

using (3^a) and (3^b)

The interpretation of (8) and (9) can be as follows each unit increase in production $v_t{}^h + \Theta$ costs x^h units of savings, and yields a contribution ℓ^{Oh} to national income $y_t + \Theta$. Hence the sector to expand according to the stages method is the one for which $\ell\,\frac{Oh}{x^h}$ is highest. Evidently this sector is the one which yields the maximum comparative advantage to the country in relation to the only scarce factor assumed which is capital.

If limitations are set to the expansions of any single sector, which may be called $\bar{v}_t^h + \Theta$, then the sector with the highest $\ell\,\frac{Oh}{x^h}$ is expanded until its limit $\bar{v}_t^h + \Theta$. If there are other investment funds left over, then the industry with the second highest $\frac{\ell Oh}{x^h}$ is expanded until its limit and so on.

The problem is solved in this manner and the $\bar{v}_t^h + \theta$ can be determined.

If the θ^h are different, then the $v_t^h + \theta h$ (10) should be written $\sum_h \frac{x^h}{\theta^h} (v_t^h + \theta^h - v_t^h) = s_t$ and belong to different y_t. Instead of maximizing $y_t + \theta$ we may now maximize the total addition to income at various time points, to be made by the investments financed from the Savings of year t and appearing in (10). Now we come to the point of the effects of introducing the time lag. It can be shown that the introduction of a time-lag slows down the expansion's rate of the economy. We may first suppose that there is a specific time-lag between investment and the growth of capital-stock, as well as a specific capital-output ratio and constant saving rate. If :

K = Capital-Stock.
y = income.
j = investment started.
S_t = amount of saving.
K = Capital-output ratio $\frac{I}{y}$
σ = Saving ratio (or coefficient)
g = gestation period.

The following equations can constitute a model.

$K_t = Ky_t$ aggregated production function (1)
$S_t = \sigma y_t$ savings equation. (2)
$K(t) = k(t-1) + j(t-g)$ capital stock at time t (3)
$S_t = J_t$ condition for equilibreum between savings and investments. (4)

There can be different solutions of the model out of which are these:-

$$k\ (t) = K\ (t-1) + J\ (t-g)$$

or $$J\ (t-g) = K\ (t) - K\ (t-1)$$

or $$J\ (t) = K\ (t+g) - K\ (t+g-1)$$

or $$\sigma y(t) = K_Y(t+g) - K_Y(t+g-1)$$

or $$\sigma\beta y(t) = Y(t+g) - Y\ (t+g-1)$$

or $$Y\ (t+g) - Y(t+g-1) - \sigma\beta\ (t) = 0$$

or $$(E^g - E^{g-1} - \sigma\beta)Y(t) = 0$$ E can [illegible] called a shift operator.

There is one real positive root, which is the maximal root. That is to say that there is no other $\lambda > \lambda_0$ where λ_0 is the real positive root. It is greater than one and bounded by the inequality $1 < \lambda_0 < 1 + \sigma\beta$. That inequality implies that the growth rate of the economy decreases when the idea of the gestation lag is introduced. In the field of economic policy, that means that if the capital-coefficient is given and to realize a given rate of growth in the case where time-lags are assumed, the saving coefficient ought to be higher.

Now, if we assume that there are "n" different producing sectors in the economy, which are vertically integrated, and that the gestation periods of investment differ from sector to sector. The time-lags for each sector can be represented by g_1 ..... g_n. Those lags can be arranged as follows $g_1 < g_2 < g_3 < g_4$ .....$< g_n$ where g_n is the longest time lag and g the shortest

The definition of the various variables and parameters can be as follows :-

Y_j (t) = income of sector j at period t (i = 1, ..., u)

Y (t) = $\sum_{g}^{n} = 1 \; Y_j$ (t)

k_j (t) = capital stock of the j^{th} sectoral time period t (j = 1,....., n).

i_j (t) = investment activity started in the j^{th} sector at time (t) .

K_j = capital-output ratio for the j^{th} sector.

β_j = $\frac{1}{K_j}$

λ_i = the propostion of the i^{th} sector's share of total investment.

σ = propostion of income saved and invested.

g = gestation period.

Mathematically the following equations can follow :-

$$\begin{aligned} k_1 (t) &= k_1 (t - 1) + i_1 (t - g_1) \\ k_2 (t) &- k_2 (t - 1) + i_2 (t - g_2) \\ &\text{------------------------------} \\ k_n (t) &= k_n (t - 1) + i_n (t - g_n) \end{aligned} \qquad (1)$$

$$\begin{aligned} k_1 (t) - k_1 (t - 1) &= i_1 (t - g_i) \\ k_2 (t) - k_2 (t - 1) &= i_2 (t - g_2) \\ &\text{------------------------------} \\ k_n (t) - k_n (t - 1) &= i_n (t - g_n) \end{aligned} \qquad (2)$$

$$\begin{aligned} Y_1 (t) - Y_1 (t - 1) &= \beta_1 \, i_1(t - g_1) \\ Y_2 (t) - Y_2 (t - 1) &= \beta_2 \, i_2(t - g_2) \\ &\text{------------------------------} \\ Y_n (t) - Y_n (t - 1) &- \beta_n \, i_n(t - g_n) \end{aligned} \qquad (3)$$

$$
\begin{aligned}
Y_1 (t + g_1) - Y_1 (t + g_1 - i) &= \beta_1 \; i_1 (t) \\
Y_2 (t + g_2) - Y_2 (t + g_2 - 1) &= \beta_2 \; i_2 (t) \\
\hline
Y_n (t + g_n) - Y_n (t + g_n - 1) &= \beta_n \; i_n (t)
\end{aligned}
\qquad (4)
$$

$$
\begin{aligned}
(E^{g_1} - E^{g_1 - 1}) \, Y_1 (t) &= \beta_1 \; i_1 (t) \\
(E^{g_2} - E^{g_2 - 2}) \, Y_2 (t) &= \beta_2 \; i_2 (t) \\
\hline
(E^{g_n} - E^{g \cdot 1}) \, Y_n (t) &= \beta_n \; i_n (t)
\end{aligned}
\qquad (5)
$$

$$
\begin{aligned}
\left\{ (E^{g_1} - E^{g_1 - 1}) \, \delta_1 \right\} Y(t) &= \beta_1 \; i_1 (t) \\
\left\{ (E^{g_2} - E^{g_2 - 2}) \, \delta_2 \right\} Y(t) &= \beta_2 \; i_2 (t) \\
\hline
\left\{ (E^{g_n} - E^{g_n - 1}) \, \delta_n \right\} Y(t) &= \beta_n \; i_n (t) \qquad i = 1, \ldots, n
\end{aligned}
\qquad (6)
$$

$$
\begin{aligned}
\left\{ (E^{g_1} - E^{g_1 - 1}) \, \delta_1 \right\} Y(t) &= \beta_1 \lambda_1 \, i (t) \\
\left\{ (E^{g_2} - E^{g_2 - 1}) \, \delta_2 \right\} Y(t) &= \beta_2 \lambda_2 \, i (t) \\
\hline
\left\{ (E^{g_n} - E^{g_n - 1}) \, \delta_n \right\} Y(t) &= \beta_n \lambda_n \, i (t)
\end{aligned}
\qquad (7)
$$

If however we add up then :-

$$
\left\{ (E^{g_n} - E^{g_n - 1}) \, \delta_n \right\} + \ldots + \left\{ (E^{g_1} - E^{g_1 - 1}) \, \delta_1 \right\} Y(t)
$$

$$= \left(\sum_{j=1}^{n} \lambda_j \beta_j\right) i(t)$$

$$= \left(\sum_{j=1}^{n} \lambda_j \beta_j\right) \sigma y(t)$$

or $\left\{ \left(\delta_n \left(E^{g_n} - E^{g_n - 1}\right) + \delta_{n-1}\left(E^{g_{n-1}^{-1}} - E^{g_{n-1}-1}\right) + \dots + \delta_1 \left(E^{g_1} - E^{g_1 - 1}\right) - \left(\Sigma \lambda_j \beta_j\right)\sigma \right\} y(t) = 0$

a simpler form of the above equation can be deduced if the g's are equally spaced at an interval of one time period.

$$\left\{ \delta_n E^{g_n} - (\delta_n - \delta_{n-1}) E^{g_n - 1} - \dots \delta_1 E^{g_1 - 1} - (\Sigma \lambda_j \beta_j)\sigma \right\} Y(t) = 0$$

In a three sector economy's case with time lags equal to 2,3,4, time periods then the following equation of the 4th degree can be considered :-

$$\left\{ \delta_3 E^4 - (\delta_3 - \delta_2) E^3 - \dots \delta_1 E - (\Sigma \lambda \beta)\sigma \right\} Y(t) = 0$$