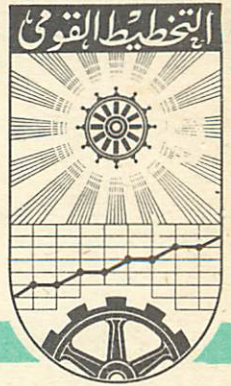


UNITED ARAB REPUBLIC



THE INSTITUTE OF NATIONAL PLANNING

نسخة ففقد

Memo. No 901

ANALYSIS OF A TWO-CHANNEL QUEUING MODEL

(A Simulation Approach)

by

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"Opinions Expressed and Positions Taken by Authors
are Entirely Their Own and do not Necessarily Reflect the
Views of the Institute of National Planning".

THE PROBLEM:

In a 2-CHANNEL SYSTEM of service facilities, following characteristics are pertinent:

- Arrivals are scheduled regularly at times 1,2,...., hours; but the actual arrival points are independently normally distributed around the scheduled arrival points as mean and with standard deviation 0.25 unit.
- The distribution of the service times is hyperexponential with means 1.50 and 2.25
- The queue discipline is first-come, first-served. When both servers are free, an arriving item is dealt with through worker 1

It is required to:

- Estimate the average waiting time per customer
- Estimate the maximum waiting time
- Estimate the standard error of the average waiting time
- Analyze the relation between the mean waiting time and the mean service time

The tool of analysis is the MONTE CARLO SIMULATION TECHNIQUE

PROCEDURE:

1. Start with an empty system
2. Start simulating the system for 200 arrivals, as follows:
 - a- Generate appropriately distributed random quantities:
 - Having normal distribution for arrival times
 - Having Hyperexponential distribution for service timesThe parameters for distributions are defined in the problem formulation
 - b- Construct the simulation table, for each arrival calculate:
 - Admission time to the system
 - Waiting (or queuing) time
 - Delivery time from the system
 - Total time spent in the system
3. Study the system characteristics, in reference to the simulation table, as follows:
 - Divide the arrivals into ten sectors, each of twenty arrivals
 - For each sector calculate:
 - Proportion of zero queuing time
 - Mean queuing time
 - Maximum queuing time
 - Mean service time
 - Amongst the results for different sectors, determine:
 - Global mean queuing time
 - Standard error of the global mean queuing time
 - Relation between mean queuing time and mean service time
 - Calculate the global maximum waiting time

GENERATION OF APPROPRIATELY DISTRIBUTED RANDOM NUMBERS:

A- Arrival Pattern:

As previously stated, the arrivals are scheduled regularly at times 1,2,...., but the actual arrival points are indepently normally distributed around the scheduled arrival points as mean and with standard deviation 0.25.

So, in order to generate numbers resembling arrival times, we have to pick them at random from a normal distribution with mean = 1 and standard deviation = 0.25

The procedure of generation is well documented in a previous work by the same author* and the COMPUTER PROGRAM (coded in FORTRAN II) is included in APPENDIX I for conveniency.

Two hundred quantities are so generated. They are also shown

B- Service Pattern:

Services may occur at rates μ_i with probabilities p_i , where μ_i is the parameter of the i the population ($i = 1,2,....,s$) such that $\mu_i \neq \mu_j$ and $\sum_{i=1}^s p_i = 1$. Such a mixture of exponential variates is said to follow a HYPEREXPONENTIAL DISTRIBUTION (in which a case $S = 2$)

In our case $\mu_1 = 1.50$ and $\mu_2 = 2.25$

Methodology of generation is also previously documented* and the COMPUTER PROGRAM is included in APPENDIX II

Here to, two hundred quantities are generated and results are included.

* Refer to Memos.No 903 , 904

SIMULATION TABLE CONSTRUCTION:

Start with an empty system, then for each arrival:

1. Try to assign first worker to service it, otherwise
2. Try to assign second worker to service it, otherwise
3. Let it wait till one of them is free
4. Calculate the admission time to the system, that is when the worker starts servicing the arrival
5. Calculate the queuing (waiting) time, if any
6. Calculate the delivery time (Time service is terminated)
7. Calculate the total time spent in the system

The procedure was FLOW-CHARTED (as shown), then translated into a FORTRAN II coded program, labeled PROGRAM I in the group of programs presented hereafter. The simulation table is also shown

SYSTEM PROPERTIES AND MAIN CALCULATIONS:

The simulation table is used to trace system properties as follows:

- The table is divided into ten sectors, each of twenty arrivals
- Within each sector, following characteristics are figured:
 - Proportion of zero queuing time
 - Mean queuing time
 - Maximum queuing time
 - Mean service time
- The two global properties are calculated:
 - Global mean queuing time
 - Global maximum queuing time

The needed procedure is shown, together with PROGRAM II, hereafter. Results are also shown.

CALCULATION OF STANDARD ERROR OF MEAN QUEUING TIME

As a result of PROGRAM II, mean queuing times for different sectors (TWM(J)) were calculated, also global mean queuing time (TWMM) was calculated. Formula for standard error is:

$$VAR = \frac{1}{N} \left(\frac{\sum_{J=1}^N (TWM(J) - TWMM)^2}{N} - \left(\frac{\sum_{J=1}^N (TWM(J) - TWMM)}{N} \right)^2 \right)$$

$$STEPR = \sqrt{VAR}$$

Where : N is the number of sectors

The procedure is FLOW-CHARTED, and PROGRAM III is the appropriate one.

The standard error has to be tested for significance.

RELATION BETWEEN MEAN QUEUING TIME AND MEAN SERVICE TIME:

We want to find out how far the mean queuing time and mean service time are correlated. In such a case a LINEAR REGRESSION analysis is to be performed. Upon the results of an F test, we can decide how far the quantities are related.

For detailed explanation of the procedure refer to any statistical textbook. As a quick idea, we first get the slope of line (b) and the interception from the Y axis (a). Then we have to test b by analyzing the variance for the sum of squares for the differences between actual values and corresponding ones on the line. Then it has to be divided into two parts:

- I- Due to regression SSREG
- II- Due to residual SSR

$$F = \frac{SSREG}{SSR} (N-2)$$

$$SST = \frac{\sum (TWM - \overline{TWM})^2}{\sum (TSM - \overline{TSM})^2}$$

$$SSREG = \frac{(\sum (TSM - \overline{TSM})(TWM - \overline{TWM}))^2}{\sum (TSM - \overline{TSM})^2}$$

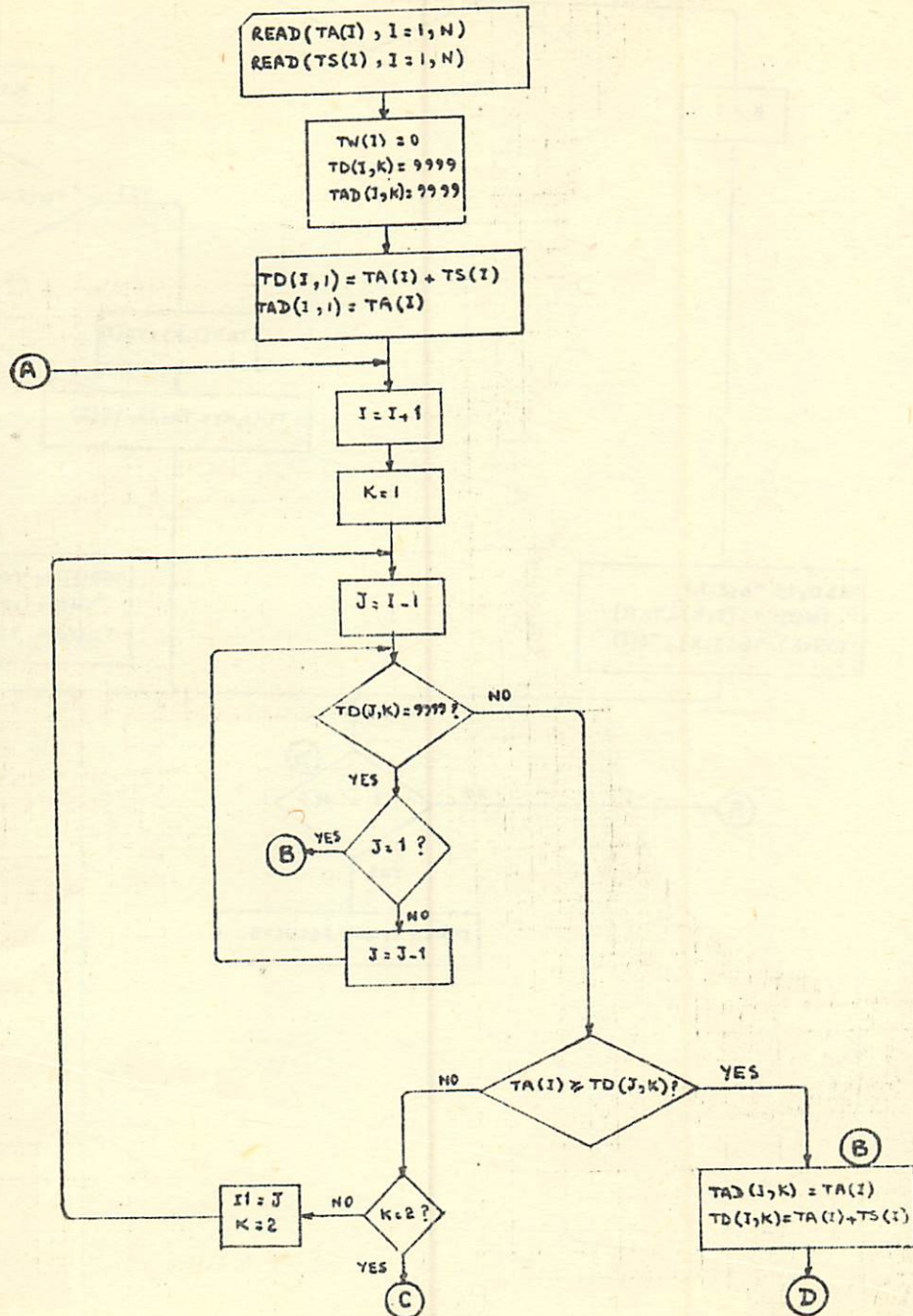
$$SSR = SST - SSREG$$

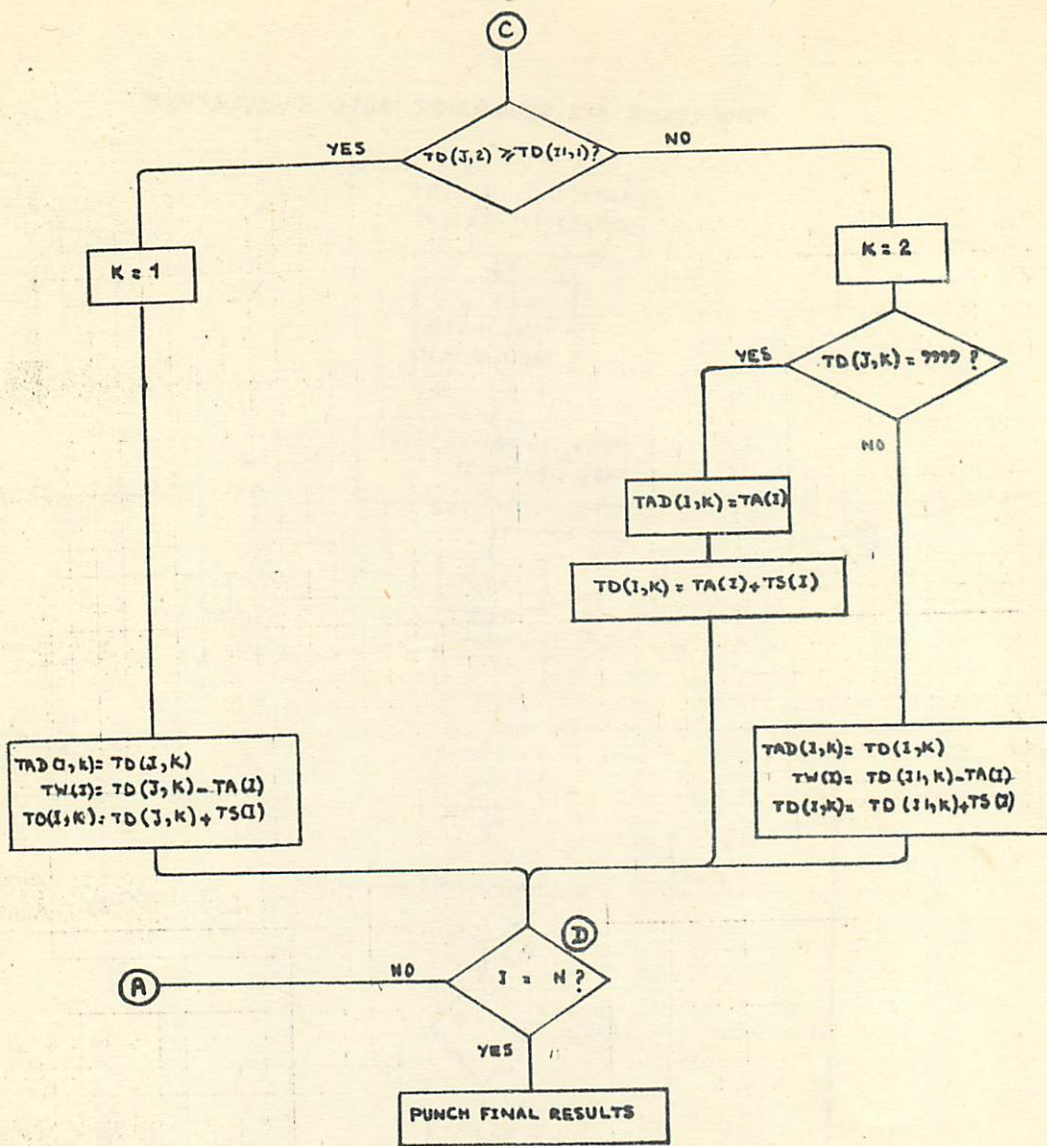
The following table is to be formed:

SOURCE OF VARIATION	DEGREES OF FREEDOM	SUM OF SQUARES	MEAN SUN OF SQUARES	F TEST
Due to Regression	L1 = 1	SSREG	$\frac{SSREG}{1}$	$\frac{SSREG}{SSR} (N-2)$
Due to Residual	L2 = N - 2	SSR	$SSRM = \frac{SSR}{N-2}$	
Total	L3 = N - 1	SST		

The appropriate program is PROGRAM IV

PROCEDURE FOR SIMULATION TABLE CALCULATIONS






```

C C C
PROGRAM 1
A PROGRAM TO FORM SIMULATION TABLE
APPLIED TO 200 ARRIVALS ONLY
DUE TO IBM-1620 (CARD SYSTEM) LIMITED STORAGE CAPACITY
DIMENSION TA(200),TS(200),Tw(200),TC(200,2),TAC(200,2)
DIMENSION TSS(200)
5 READ1,N
  CO1CM=1C,N,1C
  L=M-5
10 READ2,(TA(I),I=L,M)
  CO12M=1C,N,1C
  L=M-5
12 READ3,(TS(I),I=L,M)
  CO15I=1,N
  Tw(I)=C
  CO15K=1,2
  TC(I,K)=9999
15 TAC(I,K)=9999
  TC(1,1)=TA(1)+TS(1)
  TAC(1,1)=TA(1)
  TSS(1)=TS(1)
  J=1
  CO150I=2,N
  K=1
18 J=I-1
20 IF(TC(J,K)-9999.)5C,25,25
25 IF(J-1)25,2C,25
30 GOTO55
35 J=J-1
  GOTO20
50 IF(TA(I)-TC(J,K))7C,55,55
55 TAC(I,K)=TA(I)
  TC(I,K)=TA(I)+TS(I)
  GOTO15C
70 IF(K-2)75,13C,13C
75 K=2
  I1=J
  GOTO18
130 IF(TC(I,2)-TC(I1,1))135,14C,14C
135 K=2
  IF(TC(J,K)-9999.)136,136,136
136 TAC(I,K)=TA(I)
  TC(I,K)=TA(I)+TS(I)
  GOTO15C
138 TAC(I,K)=TC(J,K)
  Tw(I)=TC(J,K)-TA(I)
  TC(I,K)=TC(J,K)+TS(I)
  GOTO15C
140 K=1
  TAC(I,K)=TC(I1,K)
  Tw(I)=TC(I1,K)-TA(I)
  TC(I,K)=TC(I1,K)+TS(I)
150 TSS(I)=Tw(I)+TS(I)
  CO160I=1,N
160 PUNCH3,I,TA(I),TS(I),TAD(I,1),TAD(I,2),TC(I,1),TC(I,2),Tw(I),TSS(I)
  X)
1 FORMAT(12)
2 FORMAT(1CX,1C(F7.2))
3 FORMAT(13,5X,8(F9.2))
GOTO5
END

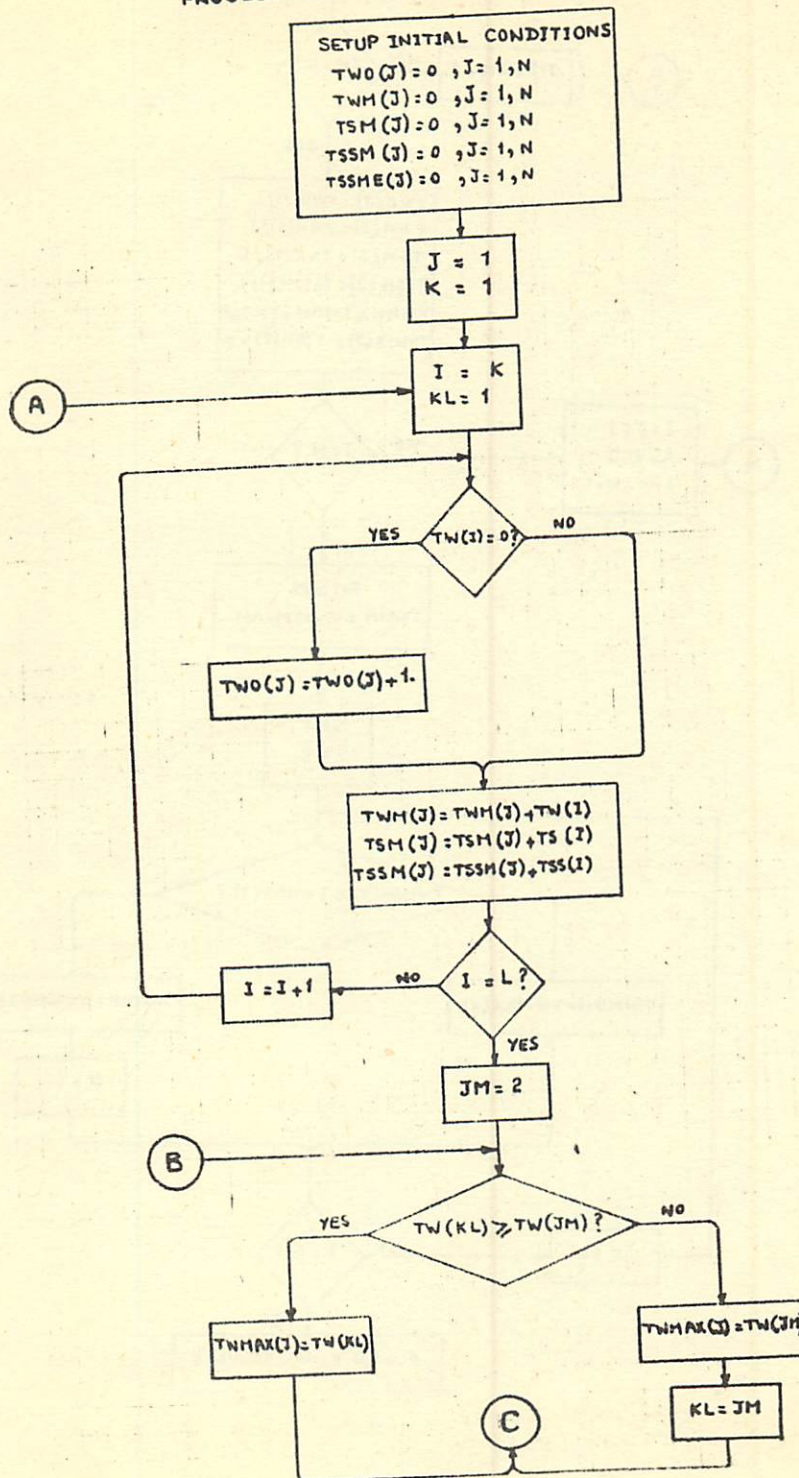
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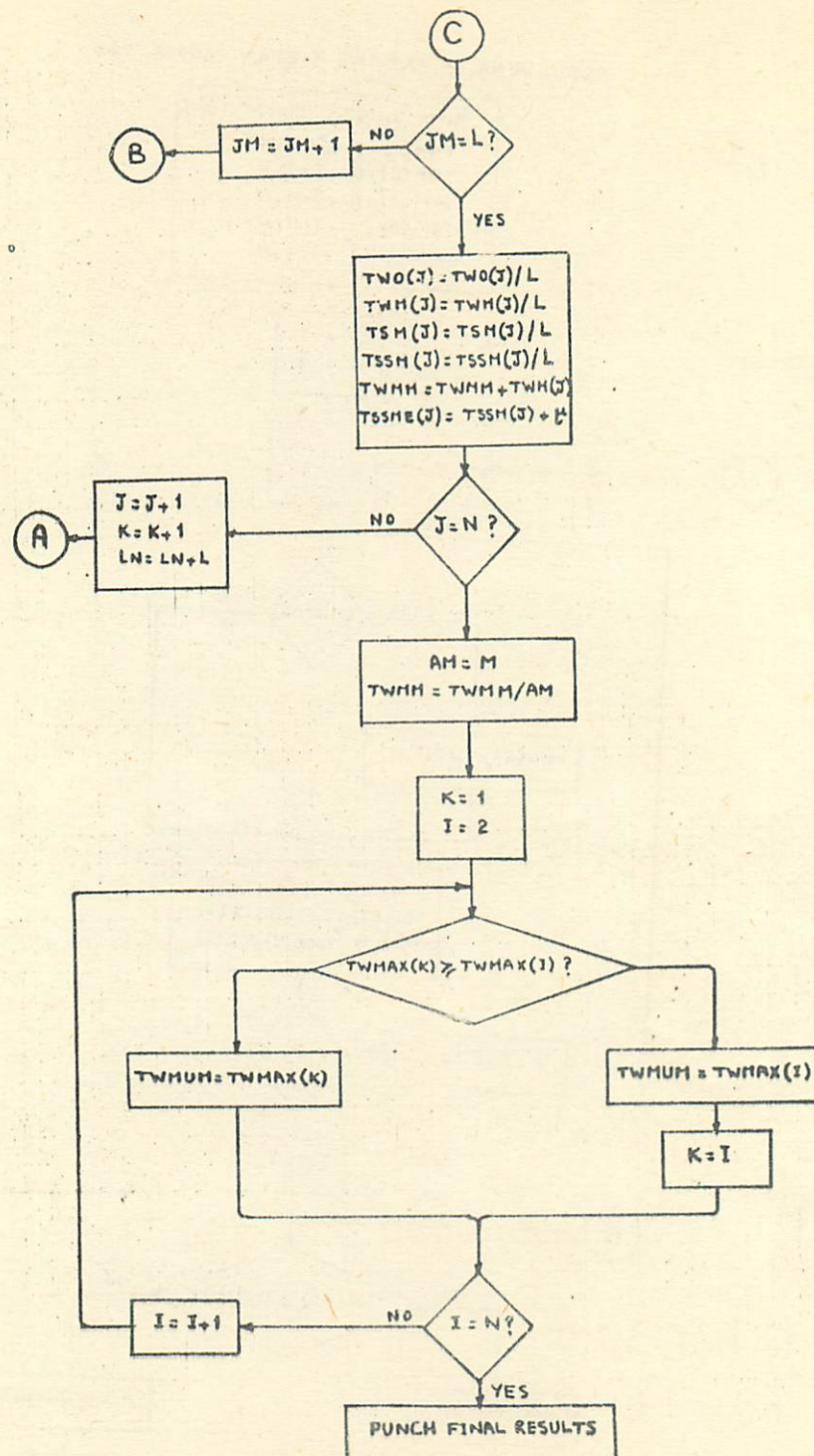

LIST OF ABBREVIATIONS IN PROGRAM 1

N : Number of Arrivals
TA(I) : Arrival Time for Item I
TS(I) : Service Time For Item I
TW(I) : Waiting Time For Item I
TAD(I,K): Admission Time of Item I To Facility K
TD(I,K) : Delivery Time of Item I From Facility K
TSS(I) : Total Time Spent In the System for Item I

PROCEDURE OF TRACING SYSTEM PROPERTIES

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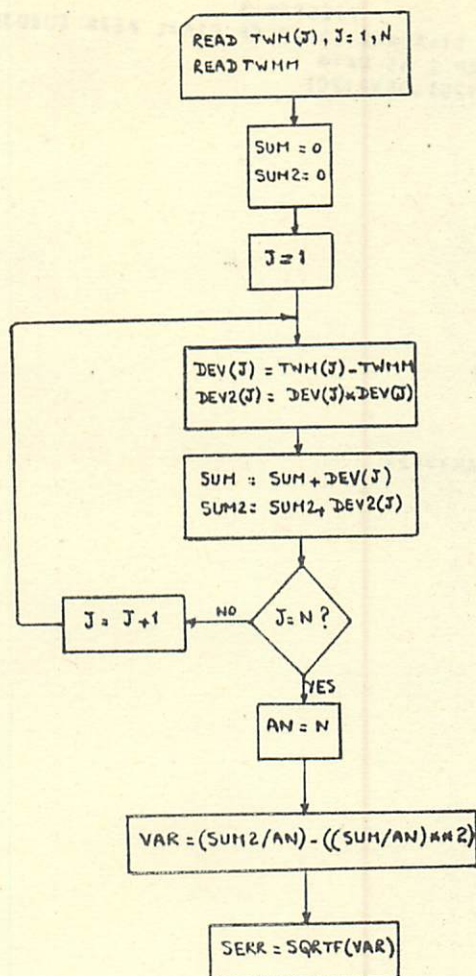
C                                     PROGRAM 2
C      A PROGRAM TO CALCULATE NECESSARY STATISTICS
C      USES SIMULATION TABLE AS THE MAIN DATA
      DIMENSION TWO(20),TWM(20),TSM(20),TSSM(20),TSSME(20),TWMAX(20)
      DIMENSION TW(200),TS(200),TSS(200)
5  RFAC1,NF,L,CMEH
      N=NF/L
      TWMM=C
      DO1CJ=1,N
        TWO(J)=C
        TWM(J)=C
        TSM(J)=C
        TSSM(J)=C
10  TSSME(J)=C
      DO115I=1,NF
115 REAC3,TS(I),TW(I),TSS(I)
      K=1
      LN=L
      DO50J=1,N
        KL=1
        DO25I=K,LN
          IF(TW(I))20,15,20
15  TWO(J)=TWO(J)+1.
20  TWM(J)=TWM(J)+TW(I)
      TSM(J)=TSM(J)+TS(I)
25  TSSM(J)=TSSM(J)+TSS(I)
C
      K1=K+1
      FIND MAXIMUM WAITING TIME
      DO40JM=K1,LN
        IF(TW(KL)-TW(JM))35,30,20
30  TWMAX(J)=TW(KL)
      GOTO40
25  TWMAX(J)=TW(JM)
      KL=JM
40  CONTINUE
C
      FIND AVERAGES
      AL=L
      TWO(J)=TWO(J)/AL
      TWM(J)=TWM(J)/AL
      TSM(J)=TSM(J)/AL
      TSSM(J)=TSSM(J)/AL
      TWMM=TWMM+TWM(J)
      TSSME(J)=TSSM(J)+CMEH
      K=K+L
      LN=LN+L
20  PUNCH2,J,TWO(J),TWM(J),TSM(J),TSSM(J),TSSME(J),TWMAX(J)
      AN=N
      TWMM=TWMM/AN
      PUNCH4,TWMM
      K=1
      DO70I=2,N
        IF(TWMAX(K)-TWMAX(I))65,60,60
60  TWMUM=TWMAX(K)
      GOTO70
65  TWMUM=TWMAX(I)
      K=I
70  CONTINUE
      PUNCH6,TWMUM
1  FORMAT(2I3,F4.2)
2  FORMAT(13,5X,6(F8.4,4X))
3  FORMAT(17X,F9.2,36X,2F9.2)
4  FORMAT(22X,AVERAGE WAITING TIME =,F10.4)
5  FORMAT(22X-MAXIMUM WAITING TIME =,F10.4)
      END

```


LIST OF ABBREVIATIONS IN PROGRAM 2

NH	: Total No. of Arrivals
L	: Length of Each Sector
CMEW	: Service Rate
N	: No. of Sectors
TWO	: No. of Orders With Zero Waiting Time (For each sector)
TWM	: Accumulative Waiting Time (For each sector), then averaged
TSM	: " Service " (" " " "), " "
TSSM	: " Time spent in the system (For each sector)
TWMAX	: Max. Waiting Time (for each sector)
TWMM	: Accumulative Waiting Time (for all sectors), then averaged for all arrivals
TSSME	: Estimated total Time Spent in the System (for each sector)
TWMUM	: Max. Waiting Time (global)

CALCULATION OF STANDARD ERROR




```

C      PROGRAM 3
C      A PROGRAM TO ESTIMATE STANDARD ERROR OF FINAL PEAK QUEUING TIME
C      USES RESULTS OF PROGRAM 2 AS DATA
      DIMENSION TMM(20),DEV(20),DEV2(20)
5  READ1,N
1  FORMAT(12)
   DO7J=1,N
7  READ2,TMM(J)
2  FORMAT(20X,F8.4)
   READ3,TMM
3  FORMAT(22X,F10.4)
   SUM=0
   SUM2=0
   DO10J=1,N
   DEV(J)=TMM(J)-TMM
   DEV2(J)=DEV(J)*DEV(J)
   SUM=SUM+DEV(J)
10  SUM2=SUM2+DEV2(J)
   AN=N
   VAR=(SUM2/AN)-((SUM/AN)**2)
   STERR=SQRT(VAR)
   PUNCH2, STERR
   GOTO5
   END

```


LIST OF ABBREVIATIONS IN PROGRAM 3

DEV(J) : The Deviation Between Mean Waiting Time, for sector J, And The
Global Mean Waiting Time.
DEV2(J) : The Square of DEV(J)
SUM : The Sum Of Deviations
SUM2 : The Sum of Squares of Deviations
VAR : The Variance
STERR : The Standard Error


```

*1505                                PROGRAM 4
C
C   A PROGRAM FOR LINEAR REGRESSION
C   PROGRAMMED BY,
C       MISS LOBNA ZAYTON
C       MR. WAFAYE ELRIDY
C       MR. AHMED HASSAN ABD EL RAZEK
C
C   DIMENSIONY(10),X(10)
5 READ1,N
1 FORMAT(I2)
DO10I=1,N
10 READ2,Y(I),X(I)
AN=N
L1=1
L2=N-1
L2=L2-L1
AL=L2
S1=0
S2=0
S3=0
S4=0
S5=0
DO15I=1,N
S1=S1+X(I)
S2=S2+X(I)*X(I)
S3=S3+Y(I)
S4=S4+Y(I)*Y(I)
15 S5=S5+Y(I)*X(I)
CY=S3*S3/AN
SST=S4-CY
CX=S1*S1/AN
E=S2-CX
ZED=S5-S1*S3/AN
B=ZED/E
SSREG=B*B*E
SSR=SS1-SSREG
SSRM=SSR/AL
SGMA=SQRT(SSRM)
FSEG=1.56*SGMA
R=SSREG/SSRM
A=(S3-B*S1)/AN
END OF CALCULATIONS
C
C
C   PUNCH3
3 FORMAT(2X,25HREGRESSION ANALYSIS TABLE)
PUNCH4
4 FORMAT(1X,45H-----)
X-----)
PUNCH6
6 FORMAT(6X,15HSOURCE OF VARIATION,8X,5HS.SG.,8X,4H.F.,8X,7H.S.SG.
X)
PUNCH4
PUNCH7,SSREG,L1,SSREG
7 FORMAT(6X,17HDOE TO REGRESSION,4X,E14.5,4X,15,4X,E14.5)
PUNCFE,SSR,L2,SSRM
8 FORMAT(6X,6HRESIDUAL,13X,E14.5,4X,15,4X,E14.5)
PUNCH4
PUNCH5,SST,L3
9 FORMAT(6X,5HTOTAL,16X,E14.5,4X,15)
PUNCH4
PUNCH11,A,B,R
11 FORMAT(1X,6X,6H A =E14.7,7X,6H B =E14.7,7X,4HF =E14.7)
PUNCF12,SGMA,FSEG
12 FORMAT(1X,6X,6HSGMA =E14.7,7X,6HFSEG =E14.7)
PUNCH13
13 FORMAT(1X,77H-----)

```


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```
X-----)
GOTO 5
2 FORMAT(2CX,2(FE.4,4X))
END
```


LIST OF ABBREVIATIONS:

S1	: Sum Of X's
S2	: Sum Of X Square
S3	: Sum Of Y's
S4	: Sum Of X square
S5	: Sum Of X.Y
A	: Intercept From Y Axis
B	: Slope Of Line

RESULTS FOR ARRIVAL TIME

MEAN ARRIVAL RATE = 1
STANDARD DEVIATION = .25

ARRIVAL	1	.54	2.22	2.99	3.69	5.06	5.81	6.81	7.92	9.08	9.60
ARRIVAL	2	11.00	12.33	12.96	13.51	15.01	16.24	16.48	17.47	19.08	19.76
ARRIVAL	3	21.06	22.30	23.12	24.13	25.12	26.32	27.22	27.75	29.29	29.86
ARRIVAL	4	31.72	31.78	33.54	33.90	35.28	36.06	36.86	37.65	38.79	39.86
ARRIVAL	5	41.21	42.05	42.92	44.17	44.52	46.10	46.75	48.32	49.12	49.96
ARRIVAL	6	51.22	51.55	52.77	53.93	54.81	55.64	56.60	57.65	59.14	59.56
ARRIVAL	7	60.74	61.54	62.58	64.06	64.84	65.88	66.66	68.23	69.07	70.15
ARRIVAL	8	70.97	72.23	73.37	74.04	74.87	75.84	76.84	78.08	79.03	79.88
ARRIVAL	9	81.18	81.50	83.10	83.87	84.90	86.06	87.04	87.99	89.21	90.58
ARRIVAL	10	90.65	92.16	92.64	94.43	94.90	95.76	96.58	97.90	98.67	100.08
ARRIVAL	11	101.02	101.78	102.77	103.86	104.53	105.89	106.74	108.38	109.16	109.98
ARRIVAL	12	110.66	112.08	112.75	114.10	115.30	115.90	116.84	117.92	119.22	120.05
ARRIVAL	13	121.06	121.87	122.79	123.73	124.54	125.83	126.66	127.88	129.17	130.11
ARRIVAL	14	130.75	132.24	132.77	134.34	135.32	135.80	136.80	137.68	138.24	139.52
ARRIVAL	15	141.11	141.89	143.20	144.16	144.77	146.42	147.14	148.30	149.59	149.99
ARRIVAL	16	151.26	152.08	152.93	153.93	155.22	156.00	156.76	157.98	159.29	159.92
ARRIVAL	17	161.10	162.15	163.01	163.82	164.89	166.24	167.19	168.45	169.94	169.97
ARRIVAL	18	171.29	172.20	173.29	174.32	175.03	176.00	176.93	177.89	178.82	179.67
ARRIVAL	19	181.03	181.81	182.65	184.01	185.07	186.02	187.02	188.05	189.96	190.02
ARRIVAL	20	191.21	191.67	193.29	193.94	194.74	195.80	196.30	198.07	199.96	200.16

RESULTS FOR SERVICE TIME

MEAN SERVICE RATE = 1.702

SERVICE 1	1.50	.21	.58	2.18	.51	4.78	.15	2.87	12.06	1.62
SERVICE 2	.13	1.90	.07	1.52	.34	3.19	.15	4.92	.01	.29
SERVICE 3	.17	1.51	1.18	.13	4.84	1.89	2.18	1.84	2.29	1.14
SERVICE 4	.01	.10	.40	1.15	.33	4.66	.36	.36	.49	2.62
SERVICE 5	.02	.21	.66	2.14	.26	.88	.90	6.28	11.77	.03
SERVICE 6	.84	.14	1.02	5.13	.15	.07	.12	.13	.12	.12
SERVICE 7	.01	1.18	.13	.66	5.29	1.76	3.56	.51	.12	.10
SERVICE 8	1.07	.74	1.78	2.25	.12	.27	.48	.48	.24	.27
SERVICE 9	.39	.05	.31	.66	4.65	2.75	1.01	.64	1.62	.03
SERVICE 10	.01	1.03	.16	.91	.52	2.35	.36	.64	4.64	.57
SERVICE 11	.41	2.56	.02	.21	.31	2.40	.45	.73	1.62	1.01
SERVICE 12	.62	1.65	1.57	.39	.12	1.56	.11	.12	4.50	.21
SERVICE 13	.66	2.40	.60	.30	1.52	1.14	.01	.21	.53	2.19
SERVICE 14	.18	1.59	.08	.45	.50	6.06	1.73	3.64	.09	.87
SERVICE 15	.07	.33	2.36	.90	.55	.27	.17	.22	.20	.10
SERVICE 16	2.78	.19	3.15	4.52	.82	1.47	.10	.39	.19	.72
SERVICE 17	2.43	5.01	.60	1.89	.22	.73	.57	.51	1.75	2.09
SERVICE 18	.09	.59	2.42	.13	6.81	.02	.10	.12	.40	.29
SERVICE 19	.20	.42	.59	.47	.11	.04	0.00	3.69	2.71	2.26
SERVICE 20	.28	1.42	1.86	.38	5.10	.86	2.21	.10	.80	.35

SIMULATION TABLE

I	TA	TS	TAD(I,1)	TAD(I,2)	TC(I,1)	TC(I,2)	TA	TS
1	.54	1.50	.94	9999.00	2.44	9999.00	0.00	1.50
2	2.22	.21	9999.00	2.22	9999.00	2.43	0.00	.21
3	2.99	.58	2.99	9999.00	3.57	9999.00	0.00	.58
4	3.65	2.18	3.69	9999.00	5.87	9999.00	0.00	2.18
5	5.06	.51	9999.00	5.06	9999.00	5.57	0.00	.51
6	5.81	4.78	9999.00	5.81	9999.00	10.59	0.00	4.78
7	6.81	.15	6.81	9999.00	6.96	9999.00	0.00	.15
8	7.52	2.87	7.52	9999.00	10.79	9999.00	0.00	2.87
9	9.08	12.08	9999.00	10.59	9999.00	22.67	1.51	12.59
10	9.60	1.62	10.79	9999.00	12.41	9999.00	1.19	2.81
11	11.00	.13	12.41	9999.00	12.54	9999.00	1.41	1.54
12	12.33	1.90	12.54	9999.00	14.44	9999.00	.21	2.11
13	12.96	.07	14.44	9999.00	14.51	9999.00	1.48	1.55
14	13.91	1.52	14.51	9999.00	16.03	9999.00	.40	2.12
15	15.01	.34	16.03	9999.00	16.37	9999.00	1.02	1.36
16	16.24	3.19	16.37	9999.00	19.56	9999.00	.12	2.32
17	16.48	.15	19.56	9999.00	19.71	9999.00	2.08	2.23
18	17.47	4.92	19.71	9999.00	24.63	9999.00	2.24	7.16
19	19.08	.01	9999.00	22.67	9999.00	22.68	1.55	2.60
20	19.76	.29	9999.00	22.67	9999.00	22.97	2.92	2.21
21	21.06	.17	9999.00	22.97	9999.00	23.14	1.91	2.08
22	22.30	1.91	9999.00	23.14	9999.00	25.05	.84	2.75
23	23.12	1.18	24.63	9999.00	25.81	9999.00	1.51	2.65
24	24.13	.13	9999.00	25.05	9999.00	25.18	.92	1.05
25	25.12	4.84	9999.00	25.18	9999.00	30.02	.06	4.90
26	26.32	1.89	26.32	9999.00	28.21	9999.00	0.00	1.89
27	27.22	2.18	28.21	9999.00	30.39	9999.00	.99	2.17
28	27.75	1.84	9999.00	30.02	9999.00	31.86	2.27	4.11
29	28.29	2.29	30.39	9999.00	32.68	9999.00	1.10	3.29
30	29.86	1.14	9999.00	31.86	9999.00	33.00	2.00	2.14
31	31.72	.01	32.68	9999.00	32.69	9999.00	.96	.97
32	31.78	.10	32.69	9999.00	32.79	9999.00	.91	1.01
33	33.54	.40	33.54	9999.00	33.94	9999.00	0.00	.40
34	33.90	1.15	9999.00	33.90	9999.00	35.05	0.00	1.15
35	35.28	.23	35.28	9999.00	35.61	9999.00	0.00	.23
36	36.06	4.66	36.06	9999.00	40.72	9999.00	0.00	4.66
37	36.86	.36	9999.00	36.86	9999.00	37.22	0.00	.36
38	37.65	.38	9999.00	37.65	9999.00	38.03	0.00	.38
39	38.79	.49	9999.00	38.79	9999.00	39.28	0.00	.49
40	39.86	2.62	9999.00	39.86	9999.00	42.48	0.00	2.62
41	41.21	.02	41.21	9999.00	41.23	9999.00	0.00	.02
42	42.05	.21	42.05	9999.00	42.26	9999.00	0.00	.21
43	42.92	.66	42.92	9999.00	43.58	9999.00	0.00	.66
44	44.17	2.14	44.17	9999.00	46.31	9999.00	0.00	2.14
45	44.52	.26	9999.00	44.52	9999.00	44.78	0.00	.26
46	46.10	.88	9999.00	46.10	9999.00	46.98	0.00	.88
47	46.75	.90	46.75	9999.00	47.65	9999.00	0.00	.90
48	48.32	6.28	48.32	9999.00	54.60	9999.00	0.00	6.28
49	49.12	11.77	9999.00	49.12	9999.00	60.89	0.00	11.77
50	49.96	.03	54.60	9999.00	54.63	9999.00	4.64	4.67
51	51.23	.84	54.63	9999.00	55.47	9999.00	2.92	4.14
52	51.55	.14	55.47	9999.00	55.61	9999.00	2.92	4.06
53	52.77	1.02	55.61	9999.00	56.63	9999.00	2.84	2.86
54	53.93	5.13	56.63	9999.00	61.76	9999.00	2.70	7.83
55	54.81	.15	9999.00	60.89	9999.00	61.04	6.08	6.22
56	55.64	.07	9999.00	61.04	9999.00	61.11	5.40	5.47
57	56.60	.12	9999.00	61.11	9999.00	61.22	4.51	4.63
58	57.65	.13	9999.00	61.23	9999.00	61.36	2.58	2.71
59	59.14	.13	9999.00	61.36	9999.00	61.49	2.22	2.35
60	59.56	.13	9999.00	61.49	9999.00	61.62	1.93	2.06
61	60.74	.61	9999.00	61.62	9999.00	62.23	.88	1.49
62	61.54	1.18	61.76	9999.00	62.94	9999.00	.22	1.40

63	62.56	.13	9999.00	62.58	9999.00	62.71	C.CC	.13
64	64.06	.66	64.06	9999.00	64.72	9999.00	C.CC	.66
65	64.64	5.29	64.64	9999.00	70.13	9999.00	C.CC	5.29
66	65.88	1.76	9999.00	65.83	9999.00	67.64	C.CC	1.76
67	66.66	3.56	9999.00	67.64	9999.00	71.20	.56	4.54
68	68.22	.51	70.13	9999.00	70.64	9999.00	1.90	2.41
69	69.07	.13	70.64	9999.00	70.77	9999.00	1.57	1.70
70	70.15	.10	70.77	9999.00	70.87	9999.00	.62	.72
71	70.97	1.07	70.97	9999.00	72.04	9999.00	C.CC	1.07
72	72.22	.74	72.23	9999.00	72.97	9999.00	C.CC	.74
73	73.37	1.76	73.37	9999.00	75.15	9999.00	C.CC	1.76
74	74.04	2.25	9999.00	74.04	9999.00	76.29	C.CC	2.25
75	74.67	.12	75.15	9999.00	75.27	9999.00	.28	.40
76	75.84	.27	75.84	9999.00	76.11	9999.00	C.CC	.27
77	76.64	.46	76.64	9999.00	77.32	9999.00	C.CC	.46
78	78.08	.45	78.08	9999.00	78.53	9999.00	C.CC	.45
79	79.03	.34	79.03	9999.00	79.37	9999.00	C.CC	.34
80	79.88	.37	79.88	9999.00	80.25	9999.00	C.CC	.37
81	81.18	.39	81.18	9999.00	81.57	9999.00	C.CC	.39
82	81.50	.05	9999.00	81.50	9999.00	81.55	C.CC	.05
83	83.10	.31	83.10	9999.00	83.41	9999.00	C.CC	.31
84	83.87	.66	83.87	9999.00	84.53	9999.00	C.CC	.66
85	84.90	4.65	84.90	9999.00	89.55	9999.00	C.CC	4.65
86	86.06	2.75	9999.00	86.06	9999.00	88.81	C.CC	2.75
87	87.04	1.01	9999.00	88.81	9999.00	89.82	1.77	2.76
88	87.99	.64	89.55	9999.00	90.19	9999.00	1.56	2.20
89	89.21	1.63	9999.00	89.82	9999.00	91.45	.61	2.24
90	90.58	.03	90.58	9999.00	90.61	9999.00	C.CC	.03
91	90.69	.01	90.69	9999.00	90.70	9999.00	C.CC	.01
92	92.16	1.03	92.16	9999.00	93.19	9999.00	C.CC	1.03
93	92.84	.16	9999.00	92.84	9999.00	93.00	C.CC	.16
94	94.43	.51	94.43	9999.00	95.34	9999.00	C.CC	.51
95	94.90	.52	9999.00	94.90	9999.00	95.42	C.CC	.52
96	95.76	2.25	95.76	9999.00	98.11	9999.00	C.CC	2.25
97	96.58	.36	9999.00	96.58	9999.00	96.94	C.CC	.36
98	97.90	.64	9999.00	97.90	9999.00	98.54	C.CC	.64
99	98.87	4.64	98.87	9999.00	103.51	9999.00	C.CC	4.64
100	100.08	.57	9999.00	100.08	9999.00	100.65	C.CC	.57
101	101.03	.41	9999.00	101.03	9999.00	101.44	C.CC	.41
102	101.78	2.56	9999.00	101.78	9999.00	104.34	C.CC	2.56
103	102.77	.02	103.51	9999.00	103.53	9999.00	.74	.76
104	103.86	.21	103.86	9999.00	104.07	9999.00	C.CC	.21
105	104.53	.31	104.53	9999.00	104.84	9999.00	C.CC	.31
106	105.89	2.40	105.89	9999.00	108.29	9999.00	C.CC	2.40
107	106.74	.45	9999.00	106.74	9999.00	107.19	C.CC	.45
108	108.38	.73	108.38	9999.00	109.11	9999.00	C.CC	.73
109	109.16	1.62	109.16	9999.00	110.78	9999.00	C.CC	1.62
110	109.98	1.01	9999.00	109.98	9999.00	110.99	C.CC	1.01
111	110.68	.62	110.78	9999.00	111.40	9999.00	.10	.72
112	112.08	1.65	112.08	9999.00	113.73	9999.00	C.CC	1.65
113	112.75	1.57	9999.00	112.75	9999.00	114.32	C.CC	1.57
114	114.10	.39	114.10	9999.00	114.49	9999.00	C.CC	.39
115	115.30	.12	115.30	9999.00	115.42	9999.00	C.CC	.12
116	115.90	1.56	115.90	9999.00	117.46	9999.00	C.CC	1.56
117	116.84	.11	9999.00	116.84	9999.00	116.95	C.CC	.11
118	117.92	.13	117.92	9999.00	118.05	9999.00	C.CC	.13
119	119.23	4.50	119.23	9999.00	123.73	9999.00	C.CC	4.50
120	120.05	.21	9999.00	120.05	9999.00	120.26	C.CC	.21
121	121.06	.66	9999.00	121.06	9999.00	121.72	C.CC	.66
122	121.87	2.40	9999.00	121.87	9999.00	124.27	C.CC	2.40
123	122.75	.60	123.73	9999.00	124.33	9999.00	.54	1.54
124	123.73	.30	9999.00	124.27	9999.00	124.57	.54	.64
125	124.54	1.52	124.54	9999.00	126.06	9999.00	C.CC	1.52
126	125.83	1.14	9999.00	125.83	9999.00	126.97	C.CC	1.14
127	126.66	.01	126.66	9999.00	126.67	9999.00	C.CC	.01
128	127.88	.21	127.88	9999.00	128.09	9999.00	C.CC	.21

129	129.17	.53	129.17	9999.00	129.70	9999.00	C.CC	.52
130	130.11	2.19	130.11	9999.00	132.30	9999.00	C.CC	2.19
131	130.79	.18	9999.00	130.79	9999.00	130.97	C.CC	.18
132	132.24	1.59	9999.00	132.24	9999.00	132.82	C.CC	1.59
133	132.77	.08	132.77	9999.00	132.85	9999.00	C.CC	.08
134	134.34	.45	134.34	9999.00	134.79	9999.00	C.CC	.45
135	135.32	.50	135.32	9999.00	135.82	9999.00	C.CC	.50
136	135.80	6.06	9999.00	135.80	9999.00	141.86	C.CC	6.06
137	136.80	1.73	136.80	9999.00	138.53	9999.00	C.CC	1.73
138	137.68	3.64	138.53	9999.00	142.17	9999.00	.85	4.49
139	138.24	.09	9999.00	141.86	9999.00	141.95	2.52	2.61
140	139.52	.87	9999.00	141.95	9999.00	142.82	2.42	2.29
141	141.11	.07	142.17	9999.00	142.24	9999.00	1.06	1.12
142	141.89	.33	142.24	9999.00	142.57	9999.00	.35	.66
143	142.20	2.36	143.20	9999.00	145.56	9999.00	C.CC	2.36
144	144.16	.90	9999.00	144.16	9999.00	145.06	C.CC	.90
145	144.77	.55	9999.00	145.06	9999.00	145.61	.29	.84
146	146.42	.27	146.42	9999.00	146.69	9999.00	C.CC	.27
147	147.14	.17	147.14	9999.00	147.31	9999.00	C.CC	.17
148	148.30	.22	148.30	9999.00	148.52	9999.00	C.CC	.22
149	148.99	.20	148.99	9999.00	149.19	9999.00	C.CC	.20
150	149.99	.10	149.99	9999.00	150.09	9999.00	C.CC	.10
151	151.36	2.78	151.36	9999.00	154.14	9999.00	C.CC	2.78
152	152.08	.19	9999.00	152.08	9999.00	152.27	C.CC	.19
153	152.93	3.15	9999.00	152.93	9999.00	156.08	C.CC	3.15
154	152.93	4.92	154.14	9999.00	159.06	9999.00	.21	5.12
155	155.22	.82	9999.00	156.08	9999.00	156.90	.86	1.68
156	156.00	1.47	9999.00	156.90	9999.00	158.27	.90	2.37
157	156.76	.10	9999.00	158.37	9999.00	158.47	1.61	1.71
158	157.98	.39	9999.00	158.47	9999.00	158.86	.45	.88
159	159.39	.19	159.39	9999.00	159.58	9999.00	C.CC	.19
160	159.95	.73	159.95	9999.00	160.68	9999.00	C.CC	.73
161	161.10	2.43	161.10	9999.00	163.53	9999.00	C.CC	2.43
162	162.15	5.01	9999.00	162.15	9999.00	167.16	C.CC	5.01
163	163.01	.60	163.01	9999.00	164.13	9999.00	.52	1.12
164	163.82	1.89	164.13	9999.00	166.02	9999.00	.31	2.20
165	164.89	.22	166.02	9999.00	166.24	9999.00	1.12	1.25
166	166.24	.73	166.24	9999.00	166.97	9999.00	C.CC	.73
167	167.19	.57	167.19	9999.00	167.76	9999.00	C.CC	.57
168	168.45	.51	168.45	9999.00	168.96	9999.00	C.CC	.51
169	168.94	1.75	9999.00	168.94	9999.00	170.69	C.CC	1.75
170	169.97	2.09	169.97	9999.00	172.06	9999.00	C.CC	2.09
171	171.29	.09	9999.00	171.29	9999.00	171.38	C.CC	.09
172	172.20	.59	172.20	9999.00	172.79	9999.00	C.CC	.59
173	173.29	3.42	173.29	9999.00	176.71	9999.00	C.CC	3.42
174	174.32	.12	9999.00	174.32	9999.00	174.45	C.CC	.12
175	175.03	6.81	9999.00	175.03	9999.00	181.84	C.CC	6.81
176	176.00	.02	176.71	9999.00	176.73	9999.00	.71	.73
177	176.93	.10	176.93	9999.00	177.03	9999.00	C.CC	.10
178	177.89	.13	177.89	9999.00	178.02	9999.00	C.CC	.13
179	178.82	.40	178.82	9999.00	179.22	9999.00	C.CC	.40
180	179.87	.39	179.87	9999.00	180.26	9999.00	C.CC	.39
181	181.03	.20	181.03	9999.00	181.23	9999.00	C.CC	.20
182	181.81	.42	181.81	9999.00	182.23	9999.00	C.CC	.42
183	182.85	.59	182.85	9999.00	183.24	9999.00	C.CC	.59
184	184.01	.47	184.01	9999.00	184.48	9999.00	C.CC	.47
185	185.07	.11	185.07	9999.00	185.18	9999.00	C.CC	.11
186	186.02	.04	186.02	9999.00	186.06	9999.00	C.CC	.04
187	187.02	C.CC	187.02	9999.00	187.02	9999.00	C.CC	C.CC
188	188.05	3.69	188.05	9999.00	191.74	9999.00	C.CC	3.69
189	188.96	2.71	9999.00	188.96	9999.00	191.67	C.CC	2.71
190	190.02	2.26	9999.00	191.67	9999.00	193.93	1.64	2.90
191	191.21	.28	191.21	9999.00	192.12	9999.00	.53	.91
192	192.12	1.42	192.12	9999.00	192.54	9999.00	.45	1.87
193	193.54	1.86	193.54	9999.00	195.40	9999.00	.25	2.11
194	193.94	.28	9999.00	193.94	9999.00	194.32	C.CC	.28

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195	194.74	5.10	9999.00	194.74	9999.00	199.84	0.00	5.10
196	195.80	.86	195.80	9999.00	196.66	9999.00	0.00	.86
197	196.20	2.21	196.66	9999.00	198.87	9999.00	.26	2.87
198	198.07	.10	198.87	9999.00	198.97	9999.00	.80	.90
199	198.96	.80	198.97	9999.00	199.77	9999.00	.01	.81
200	200.16	.25	200.16	9999.00	200.51	9999.00	0.00	.25

MEAN SERVICE RATE=1.717

 * ALL THROUGH THIS TABLE, 9999.00 IS PUT INSTEAD OF BLANKS

LIST OF NECESSARY STATISTICS

	TWC(J)	TWM(J)	TSM(J)	TSSM(J)	TSSME(J)	TWMAX(J)
1	.4000	.5690	1.9500	2.9190	4.6390	3.5900
2	.4500	.6735	1.4035	2.0770	3.7970	2.2700
3	.4500	2.0560	1.5505	3.6065	5.3265	6.0000
4	.6500	.3225	1.0900	1.4125	3.1325	1.9000
5	.8500	.1970	1.1655	1.3625	3.0825	1.7700
6	.9000	.0420	1.0290	1.0710	2.7910	3.7400
7	.7500	.4135	.2375	1.1510	3.3710	3.5200
8	.6000	.2005	.5955	1.2840	3.0040	1.6100
9	.8000	.1335	1.3940	1.5275	3.2475	1.1300
10	.6500	.2020	1.1975	1.3995	3.1195	1.6400

SOURCE OF VARIATION	S.S.	D.F.	M.S.S.
DUE TO REGRESSION	128.53853E-02	1	128.53853E-02
RESIDUAL	198.53832E-02	8	248.17290E-03
TOTAL	327.07686E-02	9	

A = -1.17662E-00 B = 1.3112872E-00 F = 5.1793944E-00
 SGMA = 4.98165E-01 FSEG = 9.7641232E-01

DISCUSSIONS OF RESULTS AND CONCLUSIONS:

- Global mean waiting time = 1.0569 hour
- Maximum waiting time = 6.08 hour
- Standard error for the average waiting time = 1.1183

Referring to tables of t test, we find that this error is insignificant. This result obviates the insignificant changes in results between different sectors of simulation table (which reflect simulated system).

It also interprets that the behaviour of the real system over a moderate time is not going to depart appreciably from that predicted using our simulation technique. So, we have succeeded to simulate the system to a good accuracy.

- As a result of the REGRESSION ANALYSIS, we repeat:
 - a = -1.176628
 - b = + 1.3112872
 - F = + 5.179394

From which we interpret that, since F is too large, a trend in the relation between the mean waiting time and the mean service time is insignificant. This is ideal in a queuing situation.

APPENDIX I


```

C      APROGRAM TO CALCULATE RANDOM ARRIVAL TIMES
C      ACCORDING TO STOCHASTIC NORMAL VARIATES
      DIMENSION TA(200),X(200),IR1(200),IR2(200),IR(200)
500 READ1,N,AMEH,PY,SEGMA
      1 FORMAT(I3,3F7.4)
      2 FORMAT(1CX,10(3X,14))
      K=1
      5 L=1
      M=10
      15 READ2, (IR(I),I=L,M)
      IF(M-N)20,25,25
      20 L=L+10
      M=M+10
      GOTO15
      25 GOTO(30,40),K
      30 CO35I=1,N
      IR1(I)=IR(I)
      35 IR(I)=C
      K=K+1
      GOTO5
      40 CO45I=1,N
      45 IR2(I)=IR(I)
      K=1
      55 L=1
      M=10
      65 PUNCH2, (IR1(I),I=L,M)
      IF(M-N)70,75,75
      70 L=L+10
      M=M+10
      GOTO65
      75 BMEW=AMEH
      CO10I=1,N
      R1=IR1(I)
      R1=R1*.CCCC1
      R2=IR2(I)
      R2=R2*.CCCC1
      S1=CO5F(2.*PY*R2)
      S2=SQRTF(-2.*LOGF(R1))
      X(I)=S1*S2
      TA(I)=BMEW+(X(I)*SEGMA)
      10 BMEW=BMEW+AMEH
      J=1
      L=1
      M=10
      175 PUNCH3,J, (TA(I),I=L,M)
      IF(M-N)180,185,185
      180 L=L+10
      M=M+10
      J=J+1
      GOTO175
      185 GOTO500
      3 FORMAT(17ARRIVAL,I3,10(F7.2))
      ENB

```


APPENDIX II


```

C      APROGRAM TO GENERATE RANDOM SERVICE TIME
C      ACCORDING TO HYPEREXPCENTIAL DISTRIBUTION
C      THROUGH APPLYING MARSAGLIA TECHNIQUE
C
      DIMENSION IRO(200),IR1(200),IR2(200),TS(200),AFAC(15),E(20),RN(15)
      DIMENSION IR(200),EXP(50),AXP(50)
500  READ 1,NH,P1,AMEW,BMEW,EX
      1 FORMAT(12,F6.4,2F4.2,F7.5)
      N=NH
      K=1
      5 L=1
      M=10
      115 READ2, (IR(I),I=L,M)
      IF(M-N)12C,125,125
      120 L=L+10
      M=M+10
      GOTO115
      125 GOTO(13C,14C,15C),K
      130 CO135I=1,N
      IRO(I)=IR(I)
      135 IR(I)=C
      K=K+1
      GOTO5
      140 CO145I=1,N
      IR1(I)=IR(I)
      145 IR(I)=C
      K=K+1
      GOTO5
      150 CO155I=1,N
      IR2(I)=IR(I)
      IR(I)=C
      155 CONTINUE
      N=0
      2 FORMAT(1CX,10(3X,14))
      READ3,NC,MC
      3 FORMAT(2I3)
      K=0
      A=0
      B1=0
      EX1=EX-1.
      EXP(1)=EX1/EX
      AXP(1)=EXP(1)
      CO215I=2,50
      EXP(I)=EXP(I-1)/EX
      215 AXP(I)=AXP(I-1)+EXP(I)
C
      COSOI=1,NH
      R=IRO(I)
      R=R*.CCC1
      IF(R-P1)1C,1C,15
      10 CMEW=AMEW
      A=A+1.
      GOTO20
      15 CMEW=BMEW
      B1=B1+1.
      CALCULATE M
      20 R1=IR1(I)
      R1=R1*.CCC1
      CUM=0
      M=0
      M1=M+1
      CUM=CUM+(R1-M1)
      IF(R1-CUM)25,25,30
      30 M1=M1+1
      M=M+1
      IF(M-45)31,34,34

```



```

23 GOTO25
24 M=50
C  CALCULATE N
25 R2=IR2(I)
   R2=R2*.CCC1
   AR2=R2*(EX-1.)
   AN=1
   AFAC(1)=1
   BFAC=1
40 AN=AN+1.
   N=AN
   AFAC(N)=1./(AN*AFAC(N-1))
   BFAC=BFAC+AFAC(N)
   IF(AR2-BFAC)45,45,4C
C  GENERATE N NUMBERS
45 IF(K-1)5C,65,65
50 CO 55II=1,NC,2
55 B(II)=1
   N1=NC-1
   CO60JJ=2,N1,2
60 B(JJ)=C
65 CALL RNGEN(B,RN,N,N1,NC,PC)
   K=K+1
   KL=1
   CO8CJM=2,N
   IF(RN(KL)-RN(JM))7C,75,75
70 RNMIN=RN(KL)
   GOTO8C
75 RNMIN=RN(JM)
   KL=JM
80 CONTINUE
   AM=M
50 TS(I)=(AM+RNMIN)*CMEW
   AN=NF
   CMEW=((A*AMEW)+(B1*BMEW))/AN
   PUNCH7,CMEW
   J=1
   L=1
   M=1C
175 PUNCH6,J,(TS(I),I=L,M)
   IF(M-NF)18C,185,185
180 L=L+1C
   M=M+1C
   J=J+1
   GOTO175
185 COTO5CC
4  FORMAT(1C,1CF7.4)
6  FORMAT(7H-SERVICE,I3,1C(F7.2))
7  FORMAT(F8.3)
   END

```