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Mathematical Programming Models for Planning on Firm and Sectoral Levels.

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FOREWARD

This memo. treats the problems of building mathematical programming models for economic planning. It discusses the very relying of the medium- and long-term planning on these models. The application of mathematical programming models to the long-term planning in Egypt is still in the stage of a laboratory experiment inspite of the applying some I/O models to the national planning during the last 20 years. However, a real full-scale application, the every day mass use of the mathematical programming models, has not yet taken place.

It is not the aim of this memo. to enter into a detailed analysis of these numerous models. Nevertheless, it will, occassionally be necessary to refer to the problems of building a mathematical programming models in general. Since there are no cut and dried methods for medium - and long-term planning, the procedure is changed from one plan to another and it is not fully identical for the various sectors of the economy and of the different industries. Anyhow, there are some outstanding features which may, at least in general terms, be regarded as constants.

Five chapters are presented in this memo.:

- Chapter one, where the problems of building a mathematical programming models on the firm - and sectoral-level will be mentioned.

- Chapter two will deal with the basic ideas of the mathematical programming models for the long-term sectoral models. In this chapter, a demonstrative sectoral programming model due to S.A. Manne - a LP model for the crude oil refining industry in USA - is illustrated.
- In chapter three, the problems of the Egyptian cotton industry, in view of applying the mathematical programming models in economic planning, will be dealt with.
- In chapter four, the discussion is narrowed down and the cotton-Weaving industry model has been presented. In the light of this model, several other large models are modified in various respects and could be easily made. The man-made fibers industry, the aluminum industry, ... etc. could be analyzed in the same manner.

Uptill now, we built a mathematical programming models for one-industry in one company (project) considering the output obligations put to these models and the resource quotas as constants. Moreover, chapters three and four just answer the question of how one-industry's programming can be used for the purpose of criticizing the sectoral economic plan from below, and how the analysis of a part (one-industry or two industries compared with one another) may provide basis for proposals aimed at the improvement of the whole sector.

We shall now proceed further and draw the mathematical programming model of the whole cotton industry's sector into the arbit of our investigation. In chapter five, we endeavoured to suggest a large mathematical model for the whole sector of the cotton industry in Egypt assuming that we have 33 companies related to the public sector under the uspises of, for example, EGOCI. We treated the problem in the light of the model building for the national economy wide of investigating the organic relationship between the sectoral and the economy-wide planning.

The relationship will first be presented on a lower-level. It will be shown how programming for one-company may provide a base for the structural-decisions of the higher economy administration, how it may reveal possible contradictions and disequilibria in the whole sectoral plan and how it may contribute to improve the targets of the latter.

Then, we will deal with a higher level relationship describing the model of"projects - sector coordination" in economic planning which unites the mathematical programming models of the central (EGOCI) and the company long-term plans in a single comperhensive system.

Ultimately, since the sectoral programming models represent individual sectors of the national economy as a whole, it is believed that the mathematical programming model of an entire economy could be treated in a similar way.

CONTENTS

CHAPTER ONE

The Problems of Building Mathematical Programming Models on the Firm- and Sectoral-Levels

1.1 Introduction

Although the early applications of the mathematical programming models for sectoral and national economic planning were mainly introduced in developed countries, they are being excercized in some developing countries. For example, in Egypt some mathematical economic models for some sectors of the national economy have been used as early as 1955. Applications were on both macro- and micro-levels, we will refer to the problems and the obstacles they faced in Egypt and in the similar developing countries and the recommendations to overcome these obstacles at the end of this chapter.

1.2 The Need For a Model:

In constructing a model, it is necessary to draw a set of measures to realize a set of policy targets for the country. It is necessary to consider the domestic inflationary preasures, the shortage of some commodities, and the overall shortage of foreign exchange. This will be better than to let policy work blindly. The better the description of the economy these countries have, the better the economic policy can be designed.

1.3 Model Construction

In constructing a model, we have to consider the following three main stages:

1- The model description; this stage starts with a study of the technology of the industry and ends with a numerical model for the industry or the industrial-complex.

2- The model's algorithm; this provides a computing procedures to trace through the implications of the model quickly and economically.

3- The model's economic policy; by which we use the model to thraw light on practical problems of public policy.

Each of these stages is closely related to each other and not isolated in a time. For example, while the analyst is studying the technology of the industry, he is mulling over the alternatives with respect to selection; when he decides on aggregation, he must warry about data as well as computation. At each stage, he must return to find new informations about technology. The model builder has to ask: Is he going to build a whole series of models on micro-levels which will be linked together, is he attempting to build a model on macro-level. Anyhow, in many cases, the models on micro-levels are easier to work at and there are a good attraction to think about. Some of model constructors, like A.S. Noble (23), think that it is better to work on both levels at once. Whatever, we must be ware of getting down in a morass of details which prevents us from identifying the major facts influencing the industry(s) performance.

1.4. <u>The Tyes of a Model:</u>

There are many models for one industry as there are major activity areas. Also, there are quite different models of the same industry for different purposes. We have to ask: What sort of models for the industry we attempting to construct?; is it to be for operational, tactical, or strategic planning?; is it to be static or dynamic, deterministic or stocastic?. Is it to be some sort of optimization models or simply discriptive or exploratory models?.

There are three distinct types of models which are more widely used. These are:

I/O Models,

Mathematical Programming Models, and

Simulation Models.

These models are, in sequence, descriptive, optimizing, and exploratory models. The first and second types have tended to be deterministic and static, while simulation one has offered the only real possibility of examining dynamic and stochastic effects.

Mathematical programming models have been used largely in the operational and tactical planning applications, and in some strategic planning. I/O models can be regarded as means of increasing the understanding of the structure of an industry and its place in the sector and the economy as a whole. Simulation models have a wide scope, but the design of proper experiments with the model is difficult.

1.5 Areas of Application:

For physical resources of an industry, I/O model is a useful analytical tool as a preliminary to mathematical programming model applicable to tactical planning; For stratigic market and capital investment planning, a mathematical programming model is probably the best we do; For financial planning, simulation models are appropriate for studying the financial behaviour of the industry in relation to its shareholders, customers, and suppliers. Such models be linked in some way with stratigic planning models.

One of applying the mathematical programming models which particulary deserves attention is that of investment planning for the developing countries. Such planning, aiming at rapid economic growth, is subject to a group of constraints in the form of a bottle-neck resources such as the foreign exchange, the raw-materials, the skilled labour-force the building, etc.

Another one of applying the mathematical programming models is for military economics since these models may exist whenever part of a practical problem concerns either the feasibility or the cost of achieving a desired output in the presence of restrictions on some resources.

There are some differences in the range of applying such models, but each of them addresses itself to the following types of questions: is a particular national objective function economically feasible? are there constraints on some resources which make the economy in capable of achieving its objectives? what are the alternatives economic targets that are feasible?.

The greatest problems arise in constructing these models lie in obtaining suitable data. For such models, we require data for the structure of the industry - the plants existence and their characteristics, raw-materials and intermidiates used in manufactures, methods of transportation, costomers location and requirements. For large industry, this represents many hundreds of thousands of items of data. Moreover, the extra ordinary difficult is to ensure that the data is regularly kept up-to-date. The model constructor wants data which will help him to calculate the true relationship between output and costs, between sales and revenues.

Moreover, for the successful application of the models, education is necessary to use these models and interprete the output results. The model constructor must be able to comunicate his ideas effectively. He must simplify whenever possible without seriously weaking the value of the model in dealing with the real problems and he must do everything he can to ease the task of interpretation. Here, Dr. KENDALL (23) said: if even the system being modelled becomes more complex, the results obtained from that complex model are not necessarly themselves complex.

The applications of I/O models in Egypt was very early. Two I/O models were constructed in 1954 and in 1959 for the purpose of the Egyptian economy. One of the major difficulties arised in these times, perhaps, the calculation of the technical-coefficients due to the absence of some statistics of the inter-sectorial relationship.

We left out the applications of the decision models in Egypt which can be considered as an important class of models. There is a rapid growth, in recent years, in the use of these decision models together with a further developed theory of games.

1.6 Conclusion:

A number of difficulties and objections have been appeared in constructing these models in Egypt, and, likely, in the similar developing countries. We can state some of these examples and suggest some of basic requirements for the success of constructing these models in the developing countries:

1) Inability to identify the real objectives of the study.

Usually, the model constructor is called to conduct the study and is given only broad-out-lines of the problem. Then he has to identify the problem, find the causes, set up an appropriate target.

2) Lack of adequate and accurate data. The essence of and industry model depends in its processing of a large amounts of data so that it is now an absolute necessity to think of the large models as a part of a comperhensive Data Pracessing System, its powerful system Analysts, and in its computing facilities. One of the keys to the successful application of large models is the speed of response when a question is posed. This will be achieved only if the data are on-hand and up-to-date so that they do not take a time to collect, and if the output of results is immediately comperhensible.

3) Inadequency of the model to make a reasonably accurate description of the study which it is supposed to simulate. A greater weight of mathematical and economic talent together with expertise in other disciplines will have to study the fundamental problems. This will imply greater cooperation between factories, universities, and responsible institutions.

4) Inaccuracy of simplifying assumptions because of which the models are presented in their simplest form only. Assumptions such as the constancy of certain factors and the

linearity of the relationship among the variables do not necessarily agree with reality. It is sometimes assumed that all figures are precisely known. These assumptions are gradually resolved and the possible uncertainties of the figures are treated. Besides, simplifying assumptions can be viewed as first approximations which can be modified at later stages if required. Anyhow, in constructing a good model, it is necessary to continue to fail to recognize.

5) Lack of adequate and powerful computing facilities. It is necessary for the existence of a sufficient data processing systems which includes powerful electronic computers, system analysts, and programmers.

6) At last, the government and factories will soon have to start thinking much more positively about modelling and be organizing to do it and to use its results.

CHAPTER TWO

One Industry Experiment in Relying Upon the Applications of the Mathematical Programming Models for the Economic Planning on the Firm- and Sectoral-level

2.1 Introduction

The general purpose of this chapter is to show the main depending of one industry, e.g. PETROLEUM REFINING, upon the mathematical programming models rather than the traditional methods of planning alone for such economic planning on the firm- and sectoral-level.

The use of L_p, combined with the sources of information, hold out the promise of greater forecasting reliability than is otherwise attainable.

In this chapter, a L_p model for the USA economy is considered (27). This model will consider the possiblity of substitution between alternative production processes. The model provides a more satisfactory allowance for both the substitutability and the complementarity effects between the alternatives. The success here, as we mentioned in the first chapter, depends upon the progress in the formulation of the problem, the collection of the suitable data, the computation facilities for a large-scale problem, and the testing of the output results.

One of the basic questions that can be answered with the help of this model is: Using the refining equipment and the raw--and intermediate-materials available in the USA on Jan. 1st, 1953 - What is the product-mix alternatives were possible as between the output of JP-Jet fuel and the output of other refinery products, and how would these alternatives be affected by a reduction in the available capacity of the refining equipment?.

2.2 The Model's Assumptions:

1- The model neglects the fact that the transportation resources may be limited, and it neglects the implications of new investments of the inventory accumulation (12).

2- It was assumed that the reference date for all the calculations is Jan. 1st, 1953.

3- All end-items specifications are set at the average levels prevailing on this date.

4- Of the petroleum refinery inputs other than the crude oil, most of items are taken to be available during the investigation of the model. Also, all other inputs (labor, electric power, and sulpharic-acid) are considered to be non-limitational and are excluded from the investigation.

5- Only 25 categories of crude oil are considered.

2.3 The Model's Statement:

How to produce a specified quantities of JP-4Jet fuel which maximize f(x), the product-mix i.e. the non-jet-fuel (product-mix).

The prevailing constraints with this model were:

1- The equipment capacity restrections which ensure that no program will call more refining equipment of any type than is available within the USA economy.

2- The raw- and intermediate-materials which ensure that no program will exceed the net initial availabilities of crude oil and other intermediate materials.

3- The refinery gases and straight-rum streams.

4- The converted streams,

5- The end-items requirements,
These three groups of constraints constitute the material balance equations on the various intermediate and end-items.

6- The gasoline and Jet-fuel specifications which constraints all the gasoline and Jet-fuel blends to fall within acceptable specification limits.

2.4 The Model's Formulation:

Max. f(x), the standard product-mix:

$$f(x) = \sum_{j} \alpha_{j} x_{j} \qquad (1)$$

S. to:

$$\sum a_{ij} x_{j} \leq q_{i} \quad \forall i \qquad (2)$$

&

$$x_{j} \geq 0 \quad \forall j \qquad (3)$$

where

x_j : represents the product-mix variables. There are 205 different variables in this model.

α_j : The coefficients of the objective function. They represent the level of the output of one unit of the product-mix items which, except for the Jet-fuel, are proportional to the actual 1952-53 output of these items.

a_{ij} : the main matrix in this model. It consiste of 105X205 elements.

q_i : the constants representing the net initial availabilities at each process of the crude oil refining industry.

Within this model, there are numerous possibilities for varying the product-mix. The key-points of choices are to be made according to the three main stages in the refining process:

1- The primary crude distalation,

2- The conversion of straight-run streams into blending stocks,

3- The blending of the end-items.

The production requirement for Jet-fuel is varied by means of the corresponding parameter q_i. Since this model deals with a special case of maximizing the level of the non-jet-fuel product-mix subject to the production of stipulated amounts of Jet-fuel, the model will trace out two substitution curves. One in case of normal utilization and the other for the case of reduced capacities of some industry processes. If there be interest in any product-mix more, it will be easy to obtain by a slight revision in the coefficients of the objective function.

2.5. Conclusion :

It is clear, as mentioned in chapter one, that is no easy task to construct a mathematical programming model of one industry, a single sector, let alone an entire economy. The formulation of the problem, the collection of data, and the numerical analysis are examples of combersome.

L_p, as one of the mathematical programming models, provide a tool for dealing with such problems. L_p allows both the possibility that a single item may be produced by more than one process, and the possibility that a single process may produce more than one item. But this is not to say that L_p represents the best of all possible methods for computing numerical answers to optimization problems.

Anyhow, the implementation of the mathematical programming models in general is not a routine mechanical procedure. Without intimate collaboration between the model constructor, the data collector, and the numerical analyst, there is little hope for success.

The model here represents no more than a modest attempt to demonstrate the feasibility of constructing a mathematical programming model for one industry or a single sector of the national economy.

CHAPTER THREE

The Problems of the Egyptian Cotton Industry in View of Applying the Mathematical Programming Models in Economic Planning

3.1 Introduction:

The cotton industry in Egypt was known thousands of years ago. This industry expanded and increased remarkably. The increased production was not based on a plan set to realize any clear or studed objectives. Such had been the case till 1952, when the government exerted true efforts for organizing and orienting this industry on a basis (even relatively) of planning and supervision to realize its stability in the local market and ability to withstand competition in foreign markets.

Anyhow, we have to know that the cotton industry represents one of the main targets in the Egyptian economic structure. It was considered as one of the broadest sectors of the light industry, and the most significant for the purposes of the home consumption and for exports. Moreover, it is one of the basic industries whose development establishes the surprising growth of the Egyptian economy as a whole.

3.2 The Problems of the Egyptian Cotton Industry:

The main problems of this industry are:

1. The production planning problems,
2. The raw- and intermediate-materials planning problems,

3. The labor force planning problems,

4. The marketing planning problems,

5. The allocation planning problems,

6. The cotton spinning industry problems, and

7. The cotton weaving industry problems.

3.3 Proposed Mathematical Programming Models for Some of the Cotton Industry Problems:

3.3.1 The Cotton Spinning Industry Model(s):

a) Notations:

n : Numer of spinning factories (halls) in the firm (company) as a whole.

N : Required number of the different yarn brands.

A_{jk}: Quantities demanded from each brand of yarns for the needs of the weaving industry in the same firm (being considered as constants or plan directives).

ν_j : External delivary obligations from the different brands of yarns.

M : Number of operations necessary to produce a specified brand of yarns.

t_{ijk} : Time in operation i necessary to produce one unit (ton) of yarn i in spinning factory k;

$i = 1, 2, \ldots, M,$

$j = 1, 2, \ldots, N,$ and

$k = 1, 2, \ldots, n.$

C_{jk} : Cost function of the j^{th} product in the R^{th} factory.

$$C_{jk} = \sum_{\mu} c_{jk} \quad \forall \; j \; \& \; k$$

; μ refer to the cost elements.

P_{ik} : Available operating time for operation i in hall k.

X_{jk} : The output target of product j produced in hall k.

r_{jkl} : Profit/unit of output j produced in hall k and sold to customer ℓ.

b) <u>The Model's Objective Function(s):</u>

To determine the production-programme which fulfiles the demands from all products given as constants at minimum costs or maximum profits according to the circumstances of the problem, we have to solve one of the following objective functions:

$$\left.\begin{array}{l} \text{Min.} \quad \sum_{j} \sum_{k} C_{jk} \; X_{jk} \\ \underline{\underline{\text{OR}}} \\ \text{Max.} \quad \sum_{j} \sum_{k} r_{jkl} \; X_{jk} \end{array}\right\} \qquad (1)$$