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LECTURE NOTES ON INDEX NUMBERS

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1. Introduction:

The first application of index numbers took place in 1764 by Carli. He tried to compute the purchasing power of money. For that purpose he computed the relative price changes for three goods, grain, wine and oil, between 1500 and 1750. His index consisted of a simple average of these three. The first big push to the application of Index numbers was given by Edgeworth and Marshall. They studied the development of prices in the last part of the 19th century. In the beginning of the 20th centruy a great deepening in the theoretical knowledge of Index numbers took place. This led to a book by I. Fisher "The Making of Index Numbers". Basically an index number can be defined as a quantity which by reference to a base period shows by its variation the changes of a magnitude over a period of time[1]. They are usually expressed as a percentage of a base period. With the use of index numbers it is possible to compare through indirect measurement magnitudes that cannot be measured directly.

In the next sections we shall start from the easiest possible Index numbers and proceed to more advanced definitions. Further, in sections 5, we shall discuss the question, whether it is possible to say that one Index Number is better than the other. Finally, some examples on the application of Index numbers to economic phenomena, such as the Index of Industrial Production and the consumer Price Index, will be given to show the importance of Index Numbers in economic life nowadays.

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1) Sometimes we rather try to measure the difference existing between two geographical places.

2. Simple Index numbers.

To define the concept of a simple index number we look at the prices of one good on different points of time. If we look at the prices of sugar in Table I, we may notice that they have the character of an average. E.g. they show us the average price over the year '58. Also it is possible to have an average between two cities (say Cairo and Alexandria) on a certain day (e.g. 30th of June) In order to converse these price series into series of price indexes we choose a certain year as a base , in which the price of a good is equal to 100.
Now the following statement can be noticed.
Definition 1 : If we write p(0), p(1), p(2) ... for the sequence of prices of a certain good in t = 0,1,2, then the simple price index numbers for t = 0,1,2 can be defined as: 100, $\frac{p(1)}{p(0)}$ x 100, $\frac{p(2)}{p(0)}$ x 100, ...
This index fails to give us information about the real prices in Pt. per kilo because the index is a ratio between two prices. Whith the help of this price index it is now possible to compare the price movement of sugar and beer. A comparison of the absolute prices doesn't make sense because the units of measurement are different (kilo and liter respectively), while in other cases the absolute price levels of the two goods may differ considerably.

It is clear that the choice of the base year is one of the factors deciding the level of the index number. This is shown in table I where year 1952 is now taken as the base year, in which the index number for sugar is equal to 100. The index in year 1953 is computed as $\frac{7,58}{5,90}$ x 100 = 128.

Table I

Simple Price Index Numbers for Sugar and Beer,[1)]

Year	Price of sugar in PT/kilo	Price index for sugar '52 = 100	Price of beer in PT/litre	Price index for beer '52 = 100	Price index for sugar '54 = 100	Price index for beer '54 = 100	Price index for sugar '53 = 100	Price index for beer '53 = 100
'52	5,90	100	19,86	100	63	127	78	101
'53	7,58	128	19,83	99	81	125	100	100
'54	9,29	158	15,76	79	100	100	123	80
'55	9,29	158	16,50	83	100	105	123	105
'56	9,97	169	16,37	82	107	104	132	104

1) The data on quantities and prices in Section 1 and 2 are taken from "Annual Statistics 1959". U.A.R. Department of Statistics.

Now we see that it depends on the base year which impression they make on the reader. However, the information they give us does not change when we change the base. The series on the base year '52 can be transformed in the series on base year '54 by multiplying all indexes for beer by $\frac{100}{79}$. Some small differences may appear, due to rounding off errors.

So the price index for sugar in year 1956 (on basis'54) is equal to.

$$\frac{100}{158} \times 169 = 107$$

On the base '56 it would be equal to $\frac{100}{128} \times 169 = 132$. This conversion can only take place in case we have a simple index number; i.e. where each index number relates to one good only.

In the same way a simple quantity index number can be developed for which the definition is as follows:

Definition 2: If we write $q(0)$, $q(1)$, $q(2)$ .... for the sequence of quantities of a certain good in t= 0, 1, 2 .. then the simple quantity index numbers can be defined as : 100, $\frac{q(1)}{q(0)} \times 100$, $\frac{q(2)}{q(0)} \times 100$,.......

Similar operations as performed with the simple price index number are applicable in case of the simple quantity index number, so that no further illustration is needed.

Sometimes it us preferable to have an index number with the immediately preceding period as a base. This prevents is from making the common mistake of interpretating a decrease from 125 to 105 as a decrease of 20%. In reality this is a change of 20 points equal to 20% of the value in the base period (see the price index for beer on base 1954 = 100

Table II Chain index number for the price of the cotton export

Year	'55-'56	'56'-'57	'57'-'58	'58'-'59.
Price in LE/ kantar	17,47	21,82	10,70	16,11
Chain index (underlined)	100	125/100	85/100	86/100

The Chain index number for year '57-'58 is found by taking $\frac{18,70}{21,82}$ x 100 = 85.

Our simple index number (base '55'-'56= 100) can be computed by taking for year '58-'59.

$$P'58\text{-}59 = \frac{16,11}{17,47} \times 100 = \frac{21,82}{17,47} \times \frac{18,70}{21,82} \times \frac{16,11}{18,70} \times 100 = 91,5$$

From this we see that our simple index number can be expressed with the use of Chain index numbers as follows

$$P_{'58-'59} = \frac{125 x 85 x 86}{100^2} = 91,5$$

3. WEIGHTED INDEX NUMBERS.

So far we only looked at the development of one quantity or price at a time. In many occasions however, it is necessary to have a price index for agricultural products in general instead of having many indexes for each of the agricultural products seperately. This means that in a certain time period we have a whole sequence of price ratios $p_1(t) / p_1(0)$ ..., $p_n(t) / p_1(0)$, where n stands for the different goods and t relates to the time period. The problem now is to find from these proportions a suitable central value. Several proposals have been made to solve this problem. Some of them will be given here. They are known as the index number of Laspeyres, Paasche and Fisher.

3,1 <u>The Laspeyres index</u>

The most important index in practical work is the laspeyres index. It was formulated by Eteinne Laspeyres in 1871. In the second section we have seen that index numbers can be divided in price index numbers and quantity index numbers, The same distinction is made here, A price is denoted by p_{ti}, where t mean different time periods and i mean different goods. In the same way we can define a quantity q_{ti} where t mean different time period and i mean different goods. So, for example the value of good 3 in period 4 is shown by $p_{43} x\ q_{43}$. If we have four goods the total value of all goods in period 2, is shown by

$\sum_{i=1}^{4} p_{2i} q_{2i}$

Now the Laspegres price index number can be defined as:

<u>Definition 3:</u> If we have a sequence of n prices for n different goods in T different time periods, and a sequence of quantities for the above mentioned goods in the some time periods the <u>Laspeyres price index</u> number for period is defined as

$$p_L^k = \frac{\sum_{i=1}^{n} p_{ki} \quad q_{1i}}{p_{1i} \quad q_{1i}} \times 100$$

where t = 1 is the base year

When we try to translate this formula into words we see that the money value of the quantities in the base period computed for prices in the k-th.period is divided by the value of the same quantities in prices of the base period.

Table III The combined Lasperyres price and quantity indexes for sugar and beer. ('52 = 100)

Year	Sugar quantity	Sugar price	beer quantity	beer price	Laspeyres price index	Laspeyres quantity index
'52	210	59,0	14,5	19,9	100	100
'53	206	75,8	12,1	19,8	128	98
'54	262	92,9	13,2	15,8	156	124
'55	289	92,9	13,7	16,5	156	136
'56	289	99,7	14,6	16,4	167	137

In Table III we find quantities and prices for sugar and beer. To facilitate the computations we have only two goods. However this does not change the argument. The Laspeyres price index number for production in different periods is found by

$$P_L^{'53} = \frac{75,8 \times 210 + 19,8 \times 14,5}{59,0 \times 210 + 19,9 \times 14,5} \times 100 = 128$$

$$P_L^{'54} = \frac{92,9 \times 210 + 15,8 \times 14,5}{59,0 \times 210 + 19,9 \times 14,5} \times 100 = 156$$

Further results are shown in the same table. We may notice that the result thus not differ much from the result we achieved by computing the simple price index for sugar. This is due to the fact that total production of beer is relatively small compared with the total production of sugar. It is also possible to compute this index number as a weighted average of simple index numbers of the goods included in the index. So we may write:

$$P_L^{'53} = \frac{12390 \times 128 + 288 \times 99}{12390 + 288} \times 100 = 128$$

The weights with which the simple index number are weighted are the total values of the different goods in the base period. In general this second method is cumbersome compared with the first and moreover, gives more chances for making rounding -off errors.

We have seen that the problem of weighted price index numbers consists of weighting many prices to come to one expression, in which the general price movement is shown. A similar problem arises if we have a sequence of quantities for which we want to compose one quantity index numbers. It is impossible to add sugar to beer, so a suitabke factor is needed to bring these quantities under one denominator. Using the same terminology we come to the definition of the Laspeyres quantity index number.

Definition 4: If we have a sequence of n prices for n different goods in T different time periods and a sequence of quantities for the above mentioned goods in the same different time periods, the Laspeyres quantity index number for period K is defined as

$$Q_L^K = \frac{\sum_{i=1}^{n} p_{1i} \quad q_{ki}}{\sum_{i=1}^{n} p_{1i} \quad q_{1i}} \times 100$$

where t=1 in the base year.

In words: the money value of quantities in the K-th period computed for prices in the base period is divided by the value of the base packet (i.e. the quantities in the base period multiplied by their "own" prices). We see that, contrary to the Laspeyres price index number, in this case the quantities vary and the prices are kept constant.

As an example we can use the data for sugar and beer in table

III. Application of the definition gives us the following results:

$$Q_L^{53} = \frac{59{,}0 \times 206 + 19{,}9 \times 12{,}1}{59{,}0 \times 210 + 19{,}9 \times 14{,}5} \times 100 = 98$$

$$Q_L^{54} = \frac{59{,}0 \times 262 + 19{,}9 \times 13{,}2}{59{,}0 \times 210 + 19{,}9 \times 14{,}5} \times 100 = 124$$

For further results see Table III

3,2 The Paasche index:

The Paasche index number was named after the German statistician Paasche who first used his index in 1874. As with the Laspeyres indices two types can, be distinguished. If we denote a price by p_{ti}, where t refers to the time period and i denote different different goods and a quantity by q_{ti}, where t and i have the same meaning as with the price, then we define as follows.

Definitions 5: If we have a sequence of n prices for n different goods in T different time periods and a sequence of quantities for the above mentioned goods in the same time periods, the Paasche price index number for period K is defined as

$$P_P^K = \frac{\sum_{i=1}^{n} p_{Ki} \quad q_{Ki}}{\sum_{i=1}^{n} p_{1i} \quad q_{Ki}} \times 100$$

where t = 1 is the base year.

In words: to obtain the Paasche price index number the money value of quantities in the K-th period computed for prices in that period is divided by the money value of quantities in the K-th period computed for prices in the initial period.

Using the same data as in the examples given above the Paasche price index number appears as:

$$P_P^{'53} = \frac{75,8 \times 206 + 19,8 \times 12,1}{59,0 \times 206 + 19,9 \times 12,1} \times 100 = 128$$

$$P_P^{'54} = \frac{93,0 \times 262 + 15,8 \times 13,2}{59,0 \times 262 + 19,9 \times 13,2} \times 100 = 139$$

Table IV The weighted Paasche price and quantity indexes for sugar and beer ('52=100)

Year	Paasche price index	Paasche quantity index
'52	100	100
'53	128	98
'54	139	124
'55	156	137
'56	167	137

By using the definition table IV can be completed. The Paasche Quantity index number can be defined analogous to the definition of its price index number.

Definition 6: If we have a sequence of n prices for n different goods in T different time periods and a sequence of quantities for the above mentioned goods in the same time periods the Paasche quantity index number for period K is defined as:

$$Q_P^K = \frac{\sum_{i=1}^{n} p_{Ki} \quad q_{Ki}}{\sum_{i=1}^{n} p_{ki} \quad q_{1i}} \times 100$$

In words: to obtain the Paasche quantity index number the money value of quantities in the K- th period computed for prices in that period is divided by the money value of the quantities in the base period computed for prices in the K- th period:

For illustrational purposes two indexes will be computed. They are:

$$Q_p^{'53} = \frac{75,8 \times 206 + 19,8 \times 12,1}{75,8 \times 210 + 19,8 \times 14,5} \times 100 = 98$$

$$Q_p^{'54} = \frac{93,0 \times 26,2 + 15,8 \times 13,2}{93,0 \times 210 + 15,8 \times 14,5} \times 100 = 124$$

Along the same line the remaining computation can be performed. The results are shown in table IV.

3.3 The Fisher Index

When we compute the Laspeyres and Paasch index numbers Some errors are made[1]. Therefore, Irving Fisher developed his Fisher index number. This index number is sometimes called the "ideal index", which however doesn't mean that it needs preference in all cases. It can be defined with the help of the Paasche and Laspeyres index numbers.

Definition 7: If we have a sequence of T Laspeyres price index numbers and a sequence of T Paasche price index numbers for the same period, the Fisher price index number for period can be defined as:

$$P_F^K = \sqrt{P_L^K \times P_P^K}$$

In words: the Fisher price index number can be interpreted as the geometric mean of the corresponding Laspeyres and Paasche index numbers.

For our case the results can be formulated as:

$$P_F^{'53} = \sqrt{128 \times 128} = 128 \qquad P_F^{'54} = \sqrt{156 \times 139} = 147$$

To complete our system we come to the last definition.

Definition 8: If we have a sequence of T Laspeyres quantity index number and a sequence of T Paasche quantity index numbers for the same period, the Fisher quantity index numbers of period K can be defined as:

$$Q_F^K = \sqrt{Q_L^K \times Q_P^K}$$

1. See chapter 5 for an explanation.

In words: the Fisher quantity index number can be computed by taking the geometric mean of corresponding Laspeyres and Paasche index number.

One of the drawbacks will be clear from the beginning: the computation is exessively laborious. Furthe rmore, there may be questions with regard to the meaning of this index and what it measures actually. Therefore, the statisticians will in practice continue to rely on the less exact Laspeyres and Paasche index number.

3.4 Shifting the base of an index number:

In section 2 we have seen how we can transform an index number series into another series having a different base year. We found that this doesn't decrease our information.

Specially when we work with weighted index number this procedure is applied frequently. This springs from the necessity to bring the base down to a more recent period or to make it possible to compare our data with other data, which were computed on another base. In the last case we transfer our series to a series having the same base year as the other series with which it has to be compared. Only then valid comparisons can be made. The technique of shifting the base is illustrated by the following data:

Table V The whole sale price index in UAR

Year.	1957	1958	1959	1960	1961
The index on base '39=100	422	417	417	418	425
The index on base '58=100	101	100	100	100	102.

Source: Dept of Statistics and Census, UAR

This technique we is also used to join overlapping index number series. Long historical series (e.g. consumer price index) require this type of procedure because the number of commodities and the importance of each specific commodity changes over time.

4. A comparison of the Laspeyres and Paasche index number.

In this section we shall limit our arguments to the case of Laspeyres and Paasche price indexes. The same reasoning as shown below can be applied to quantity indexes. As this does not open new aspects it is omitted here.

4.1. Fixed weight versus changing weight.

To compute a Laspeyres index we need base year commodity prices (p_{1i}), given year commodity prices (p_{Ki}) and base year commodity weights (q_{1i}). These quantity weights are fixed. As a result the denominator is fixed over time. So the Laspeyres index is called a fixed weight index. The data needed to compute a Paasche index number are base year commodity prices (p_{1i}), given year commodity prices (p_{ki}) and given year commodity weights (q_{ki}). It is now necessary to have a new set of weights for each year we prepare an index number. For this reason the Paasche index is called a changing weight index.

In practice the computation of the Paasche index is often impossible because we have to find a new set of weights for every year we want to compute an index. Moreover, the amount of computation is almost twice that of the Lapeyres method. For these reasons in statistical work the Lspeyres index has a preference above the Paasche index.

4.2. The difference between the Laspeyres and Paasche index

We may wonder whether it is possible to say something about the difference in absolute height of the Laspeyres and Paasche indexes. To discover this we compute the Laspeyres index for period 2 and express our formula in the price ratio p_{2i}/p_{1i} of the separate goods. This gives us.

$$P_L^2 = \frac{\sum p_{2i} \quad q_{1i}}{\sum p_{1i} \quad q_{1i}} = \frac{\sum p_{1i} \cdot q_{1i} \cdot \frac{p_{2i}}{p_{1i}}}{\sum p_{1i} \cdot q_{1i}}$$

This implies that P_L^2 is the weighted arithmetic mean of the individual price ratios, with the expenses on each good in the base period ($P_{1i} \; q_{1i}$) as weights.

The same technique can be applied to the Paasche index. For that purpose we consider the reciprocal of P_P^2

$$\frac{1}{P_P^2} = \frac{\sum p_{1i} \cdot q_{2i}}{\sum p_{2i} \cdot q_{2i}} = \frac{\sum p_{2i} \cdot q_{2i} \Big/ \frac{p_{2i}}{p_{1i}}}{\sum p_{2i} \cdot q_{2i}}$$

This means that P_P^2 is the weighted harmonic mean[1)] of the individual price ratios $P_{2i} \;/\; P_{1i}$ with the expenses on the relating goods in the second period (P_{2i}/q_{2i}) as weights.

Next we consider the difference between the Laspeyres and Paasche indexes.

$$P_L^2 - P_P^2 = \frac{(\sum p_{2i} \; q_{1i})(\sum p_{1i} \; q_{2i}) - (\sum p_{1i} \; q_{1i})(\sum p_{2i} \; q_{2i})}{(\sum p_{1i} \quad q_{1i}) \; (\sum p_{1i} \quad q_{2i})}$$

1) Z is the weighted harmonic mean of $Z_1, Z_2 \ldots Z_n$ with weights $w_1, w_2 \ldots w_n$ if the reciprocal of Z is equal to the weighted arethmatic mean of the reciprocals of the values for Z with these weights: $1/_Z = (1/\sum w_i) \sum (w_i/Z_i)$.

The individual relative price and quantity changes from the second period compared with our base period can be expressed by:

$$p_{2i} = p_{1i} \; (1 + d_i)$$

$$q_{2i} = q_{i1} \; (1 + e_i)$$

Substituting these two expressions in our formula for $P_L^2 - P_P^2$ and multiplying both sides in this formula by $(\sum p_{1i} \cdot q_{1i})$ $(\sum p_{1i} \; q_{2i})$ gives us the following result:

$$(P_L^2 - P_P^2)\,(\sum p_{1i} \; q_i)\,(\sum p_{1i} \; q_{2i}) = (\sum p_{1i} \; q_{1i}(1+d_i)) \times (\sum p_{1i} \; q_{1i})(1+e_i)\,) - \left[\sum p_{1i} \; q_{1i}\right] \left[\sum p_{1i} \; q_{1i} \times (1+d_i)(1+e_i)\right]$$

$$= (\sum p_{1i} \; q_{1i} \; d_i)(\sum p_{1i} \; q_{1i} \; e_i) - (\sum p_{1i} \; q_{1i}) \times (\sum p_{1i} \; q_{1i} \; d_i e_i)$$

Now we can write $r_{1i} = p_{1i} \; q_{1i}$ for the expenses on the i-th good in the base period. Both sides of our formula are divided by $(\sum p_{1i} \; q_{1i})^2 = (\sum r_{1i})^2$. At the same time we substitute our definition for the Laspeyres quantity index number in the left hand side. This gives us:

$$(P_L^2 - P_P^2)\; Q_L^2 = \frac{\sum r_{1i} \; d_i}{\sum r_{1i}} \cdot \frac{\sum r_{1i} \; e_i}{\sum r_{1i}} - \frac{\sum r_{1i} \; d_i e_i}{\sum r_{1i}}$$

For the share w_i of the i-th good in the total value of the expenses in the base year we may write.

$$w_i = \frac{r_{1i}}{\Sigma r_{1i}} = \frac{p_{1i} \quad q_{1i}}{\Sigma p_{1i} \quad q_{1i}}$$

The principal property of this definition is, that $w_i = 1$ we can now simplify our formula to

$$(P_L^2 - P_P^2)\, Q_L^2 = (\Sigma w_i \quad d_i)(\Sigma w_i \quad e_i) - \Sigma w_i \quad d_i \quad e_i$$

$$= -\Sigma w_i \; (d_i - \bar{d}) \; (e_i - \bar{e}),$$

where $\bar{d}$ and $\bar{e}$ are the weighted arithmetic means of price and quantity changes respectively with the values of w_i as weights. The right hand side of the last expression can now be considered as the numerator of a correlation coefficient. It relates to the weighted correlation between two sets of figures d_i and e_i respectively ($i = 1, \ldots n$) where a weight w_i is given to the i-th observation (= i-th good). In regression analysis this means, that the i-th observation appears with a frequency w_i and the averages used in the computations are computed as weighted averages ($\bar{d}$ and $\bar{e}$) with the values of w_i as weights.

From our formula it appears that the difference between the laspeyres and Paasche price index numbers, multiplied by the laspeyres quantity index numbers is contrary equal to the numerator of the correlation coefficient between the relative price changes and the relative quantity changes. Now Q_L^2 is always positive. It follows that $P_L^2 > P_P^2$ if this correlation is negative and $P_L^2 < P_P^2$ if this correlation is positive.

A general conclusion can now be obtained for the cost of living index. Here it is plausible on economic grounds that the correlation will be negative. On account of the substitution effect we may on an average expect that the use on goods which have relatively risen in price most or have diminished in price least of all, will rise in use least or will deminish in use most of all goods. So in most cases we find here $P_L > P_P$. Our argument is illustrated in the subjoined table:

Table VI Cost of living index for the Netherlands (1938=100)

	1948				1949			
	Jan.	April	July	Oct.	Jan.	April	July	Oct.
"Laspeyres index"	202	204	207	205	215	218	219	217
"Paasche index"	186	188	190	187	195	196	199	197

Source : "Statistische en econometrische onderzoekingen", New Series, Volume 7 (1952) P. 105

It is useful to stress here that the extent of the correlation and its sign both are predominantly decided by the goods with the greatest share in the base period.

5. Which is the best index number?

A common mistake made by wage earners is, that they consider the cost of living index to be more favourable to them if it only contains more goods. They believe, that with the number of goods in the starting period as large as possible, the price rises will be expressed stronger. However it is a tale that the index would increase sharper by including rather fixed rents and transport expenses in it. In that case we can only say that the index is more representative for the actual situation. Another problem arises when new goods are introduced in the market. E.g. should we include TV.sets in the computations for the next period? Sometimes goods are very heterogeneous, giving us dificulties in fixing a price for the good. E.g. which price should we take for dresses?

Methods to search the merits of different index numbers were developed by Irwing Fisher in: "The Making of Index Numbers" He developed a number of criteria which have to be fulfilled by the indexes. Some of them will be mentioned below.

1. The proportionality test: if the prices in the current period all increase by a factor k, then th index number should also increase by a factor k.

It can easily be verified that the definition for the Laspeyres, Paasche and Fisher price indices meet this requirement. By formulating this test in quantities instead of prices, the Laspeyres, Paasche and Fisher quantity index also fulfil its condition.

2. The symmetry test: the dependency of the price index on the seperate prices and quantities should be such, that by interchange of the two last categories the corresponding quantity index occurs. We give an illustration by applying this interchange on the Paasche price index number:

$$\frac{\sum p_{2i} \; q_{2i}}{\sum p_{1i} \; q_{2i}} \text{ is changed in } \frac{\sum q_{2i} \; p_{2i}}{\sum q_{1i} \; p_{2i}}$$

As a result the Paasche quantity index appears. So the Paasche definitions fulfil the requirements of the symmetry test. In the same way it can be shown that the Laspeyres and Fisher index number fulfil its condition.

3. The factor reversal test: This is the best known test to stress the importance of Fisher's ideal index. It requires the price index multiplied by corresponding quantity index to be identically equal[1] to the relative total value.

The only index which satisfies this condition is the Fisher index. We see:

$$Q_F^2 \cdot P_F^2 = \sqrt{Q_P^2 \cdot \; Q_L^2 \cdot \; P_L^2 \cdot \; P_P^2}$$

$$= \sqrt{\frac{\sum p_{2i} \; q_{2i}}{\sum p_{2i} \; q_{1i}} \; \frac{\sum p_{1i} \; q_{2i}}{\sum p_{1i} \; q_{1i}} \cdot \frac{\sum p_{2i} \; q_{1i}}{\sum p_{1i} \; q_{1i}} \cdot \frac{\sum p_{2i} \; q_{2i}}{\sum p_{1i} \; q_{2i}}} = \frac{\sum p_{2i} q_{2i}}{\sum p_{1i} q_{1i}}$$

It is left to the ingenuity of the reader to verify that by multiplication of either the Laspeyres price index and the Paasche quantity index or the Laspeyres quantity index and the Paasche price index, the same condition is fulfilled.

4. The time reversal test: If for an index number we interchange the indexes 1 and 2 of the base period and the current period, the result should be equal to the reciprocal of the original index number. Consider the case for the Laspeyres price index number. By P_L^{12} we mean an index number with 1 as base period and 2 as current period and by P_L^{21} an index number with 2 as base period and 1 as current period. Now we have:

1) i.e. for all conceivable values for seperate prices and quantities in both the current and the base period.

$$P_L^{12} \cdot P_L^{21} = \frac{\sum p_{2i} \; q_{1i}}{\sum p_{1i} \; q_{1i}} \cdot \frac{\sum p_{1i} \; q_{2i}}{\sum p_{2i} \; q_{2i}}$$

$$= \frac{Q_L^2}{Q_P^2}$$

The result is not equal to 1, so that the time reversal test is not met by the Laspeyres price index number. Unfortunately the same conclusion goes for the Laspeyres quantity index and the Paasche indexes. The same exercise as above can be performed for the Fisher price index. This gives us.

$$P_F^{12} \cdot P_F^{21} = \sqrt{P_L^{12} \cdot P_P^{12} \cdot P_L^{21} \cdot P_P^{21}}$$

$$= \sqrt{\frac{Q_L^2 \cdot Q_P^2}{Q_P^2 \quad Q_L^2}} = 1$$

So the Fisher index meets the requirements of the time reversal test.

5. <u>The circular test</u>. Assume we have two periods 1, 2 and 3 and also three price indexes P^{12}, P^{13} and P^{23}. It is obvious to require that $P^{13} = P^{12} \cdot P^{23}$. For, as the price level in 2 surpasses the price level in 1 by a factor 1,2 and the price level in 43 surpasses the price level in 2 by a factor 1,4, it is reasonable to demand, that the price level in 3 surpasses the price level in 1 by a factor 1,2, x 1,4 = 1,68. We shall give the Lapeyres price index as an example.

$$P_L^{12} \cdot P_L^{23} = \frac{\sum p_{2i} \; q_{1i}}{\sum p_{1i} \; q_{1i}} \cdot \frac{\sum p_{3i} \; q_{2i}}{\sum p_{2i} \; q_{2i}} \neq P_L^{13}$$

In general, the Laspeyres, Paasche and Fisher definitions do not fulfil this condition. It can be verified, that the chain index number is the only index number given here fulfilling this requirement.

6. The Index of Industrial Production[1)]

6.1. Object and use of the Index

The aim of this index is to combine series representing changes in the volume of work done in various sectors of industry, limited to the production of commodities excluding agriculture and services. It compresses many facts into a few simple figures and, in conjunction with other data, their use in economic analysis is in summarizing past developments and to help people in forecasting future trends. Since industrial production is one of the more dynamic and fluctuating elements in the economy this index can be of great help in making decisions on economic policy. This index of production is subject to the familiar limitation of all index numbers. The series available are generally an imperfect and incomplete representation of the whole field. Further, these series can be combined according to different formulae[2)] which give different results. Finally, its use is valid only for relatively short-run comparisions. It should be reviewed at intervals, the weights being changed.

National product is the aggregation of product in the sense of value added, or value of work done, in the various sectors of the economy. It covers industries producing raw materials and semi-finished goods as well as final goods. The

1) For the greater part this chapter is an summary from "Index Numbers of Industrial Production, Studies in Methods no 1, statistical office of the United Nations.

2) See section 3.

index of production can then be regarded as the extension of the value of national product over time in volume terms, i.e., as a valuation of national product at constant prices. In weighted average form, the index would combine series of the volume of work done in the various industries with weights set by their net outputs contributions to national product. The index would relate to the output of establishments and would be grouped according to an industrial classification of establishments. Because of the practical difficulties we meet in allowing for deprecation, the concept of gross national product is generally preferred with weights proportional to net output before deduction of depreciation allowances.

6.2. Scope and gronping

The index number of industrial production should cover the industries and trades making up the major groups 11 to 51 inclusive of the International Standard. Industrial Classification of all Economic Activities,[3] i.e., mining and quarrying manufacturing, construction, and electricity and gas. The index should embrace factory production, workshops and handicraft, but should exclude work in the home or farm and repair work generally performed in the connexion with a service trade. In most cases it is very difficult to obtain data from small establishments. The compilation of an index covering large establishments and an index covering the whole field can be considered. In the last case production by small establishments can be shown seperately. "Small" establishments are defined such, that they cover not more than 10% of the industrial production. It is also possible to compile seperate indexes for one or more of the above mentioned major groups for certain industries.

3) See Appendix A

6.3. Formula and Base

The index is of a fixed weight type. (i.e. the Laspeyres definion is used here). Here the quantity should be a series representing the volume of work done in a particular industry. The price should be the margin added by the industry and expressed per unit of work done. Then the product of quantity and price represents some valuation of net output. Or else, in this index quantum relatives are averaged with values of net output in the base period it single year is selected as the weight base of the index. This base weights need to be changed once in five years to keep the index up to data. In the weight base year the index is equal to 100.

6.4. Weights and Series

So far, we have not considered the determination of the weights to be attached to the various industries in the base period and the selection of particular series of quantum data to measure current changes in production. This problem will be tackled below. First we have to fix the main industry groups within the general industrial classification and the weights (value of net out put) appropriate to them. Second we have to determine the particular series to represent each main industry group and their internal weighting.

The list of industries selected should be as detailed as possible. The weight for each industry is its net output in the sense of its contribution to gross national product in the base period. Net output is regarded as "census value added" (the selling value of gross output less the cost of materials and fuel used. Certain other deduction can be made[4]. The census value added includes the value of any change in work in progress over the census period, i.e. it should

4) See further Appendix B.

be a valuation of changes in work in progress. In fact only relative values of net output are needed. The weights should be proportional to values of net output; otherwise the actual values of net output do not matter.
Another problem is, to obtain quantum series to represent work done in each census industry i.e. to extend "the net output weight assigned to the industry. These series should take into account:

a) Variations in types and quantities of products made, and of materials[5] used, in the industry.

b) Changes in work in progress, including stocks of intermediate goods.

c) Changes in the amount of processing applied to materials (i.e. in the technical input-output relations).

The measurement of work done in the industry in one period as compared withe another can be approximated statistically by an index derived from valuations in each period of net output at constant prices of products and materials. The measure proposed by Geary :

$$q_{01} = \frac{\sum P_0 Q_1 - \sum \pi_0 \mu_1}{\sum P_0 Q_0 - \sum \pi_0 \mu_0}$$

where P and Q denote price and output of a product, π and μ price and consumption of a material, suffixes 0 and 1 the periods compared and $\sum$ a summation over all products or all materials in the industry. This formula approaches most nearly to the concept of changes in the volume of work done. Variations in types and quantities of products and materials are allowed for because each variety can be included separately in the computation of the index. The index allows for changes in work in progress by means of the inclusion of intermediate goods. Finally the index allows, at least in part for changes in the amount of processing

5) materials always include fuels, electric power, packaging and business services supplied by other industries.

applied to materials. The data required for compiling q_{01} are unlikely to be available except from a census of production or an extensive sample inquiry. For this reason in most years the computation of an index in accordance with the Geary formula is hardly possible and any series representing changes in work done in an industry should be regarded as a substitute.

In practice two types of series can be constructed from the less complete data available.

1. <u>Output Series:</u> They should represent current production of completed items at the end of a stage of production. They should not be equal to deliveries to the next stage of production. These series can be obtained directly in physical units or by deflating value of output with the aid of a suitable price index. The basic difficulty here is, that the output of an establishment is homogeneous very seldom. This is solved by devising some quantity index to cover the varying qualities. And alternative way is to take the value of output of various types and qualities and to deflate with an index representing changes in the level of output, for example, a series of value of output of clothing can be deflated with an index of clothing prices. One should realise however, that output series only approximately represent work done and are valid only as long as changes in work in progress and in the amount of processing applied to materials are known to be small.

2. <u>Input Series:</u> They are less homogeneous than output series, and the different types are in differing relation ship to work done. The main series in use refer to the input of labour, materials or energy. Labour series are most generally present as man-hours worked and give a fairly direct representation of

work done. However, they do not take account of changes in labour productivity and therefore their use is limited to industries where changes in labour productivity are known to be small. They should be confined to short period movements. To use series of inputs of materials involves the assumption that net output is constant per unit of materials used[5]. They should be considered only if a single homogeneous material is the main material used in production and if other series are not available. A last possibility is the use of series on the consumption of energy. They can be useful over short periods, in industries where it is known that no important technological changes affecting energy consumption have taken place.

In fixing internal weights within an industry the measure most commonly used at present is apparently relative value of gross output. The use of man-hours worked is preferable, when gross output reflects mainly differences in costs of raw material. For the choice of series within industries no rule-of-thumb method for selecting is available. Although this is difficult to realise it is preferable to select a single series to represent the movement of work done in the whole industry.

Frequently data are almost completely lacking for some sectors. The gap will have to be filled by some sort of imputation. This method should be used only where there is real reason to expect parallel movements. It may be assumed that variations in output price of product or productivity of labour in the sector in which data are lacking follow the variations in the sector which is covered. No rule can be laid down on the choice to be made in particular cases.

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5) e.g. the consumption of flax in the linen industry.

A special problem in the choice of series arises in an industry with a long period of production e.g. construction and shipbuilding. To solve the problem of intermediate products here, it is necessary to break up the industry into stages and to devise quantum series for each stage. The most important method used is, to define certain arbitrary stages in the continuous production process and to estimate the average proportion of the total work which is performed during each stage.

6.5. The Compilation of the Index

Several factors should be considered in assessing the accuracy and availability of the total index of production. The individual series used should be neither too few (say less than 100) nor too many (say over 500) in number. Whatever the number no single series should bear a weight direct or imputed even as large as 5 per cent of the total unless it has been carefully examined for suitability and accuracy. Each series used should be assessed as regards its approximation to a true work done series and each should be graded as to the accuracy with which it measures what it is intended to measure. The grading need be no more precise than some simple qualitative grade (A,B,C...) of descending accuracy. Each series or group of series should be shown with its weight and with an indication of how much of the weight is direct and how much is imputed. When the index is revised, efforts should be made to improve low graded series and particularly those having large weights. The compilation should not be held up in any month because of delay in obtaining a few series, but the index should be shown in provisional form. It is desirable to limit the amount of revision needed.

If a completely new industry arises e.g. synthetic rubber or television sets, it is very difficult to adjust an existing index satisfactorily. Only a complete revision and reweighting of the index will meet the case. In some cases where new products are arising and replacing old products within an industry, it is possible to adjust the index without comlete overhaul. Within an industry, the series used and their internal weights can be adjusted provided that a link is made at the month of change. For example, the fixed weight of a chemical industry may be extended by a group of 5 series combined with approximate weights. In a selected month, one series may be dropped, two new one added and a revised set of internal weights fixed for six series. This new combined series can be linked to the old series and used to extend the same industry weight as before. Such substitution cannot be adopted too frequently. One of the reasons for periodic revision of the index is to avoid a series of adjustments having a cummulative biased effect.

The general index and the group indexes should all be published to one decimal place to facilitate the employment of the indexes for purpose of analysis. The publication of all national indexes to one decimal place would make possible more accurate conversions from one base period to another for purpose of international comparisons.

There are difficulties in compiling and using index numbers of production on a monthly basis, since calendar months are of unequal length and contain varying numbers of week-ends. Therefore, the primary index of production should be constructed on the basis of production per working week, i.e. that series representing product per week be applied to weights which are relative values of net outputs (equally per week or per year).

The index can be shown for "months" that is computed twelve times per year for periods which can be labelled January, February and so on.

The adjustments of basic data given for series is intended to reduce production to the primary weekly basis to eliminate the vagaries of the calendar. Another point is that, when the basic data are for calendar months, the adjustment will differ as between countries, and, within one country, as between different industries, because of variations in the normal work week. Application of these adjustments to the main components of the index can be adopted if most establishments in each industry have the same normal workweek and if industries with the same normal work-week are grouped together.

The movements over time in the primary index of production (per working week) are affected by seasonal factors. These include the incidence of public and annual holidays as well as such factors as the affect of weather. Hence, for comparisons involving the trend of production from month to month, it is important to have a secondary index number from which seasonal influences have been removed. The primary index may be first adjusted for holidays, and then a further secondary index obtained by eliminating other seasonal influences. This second adjustment can be made according to one of the standard statistical techniques for isolating seasonal variations. This secondary index computed by elimination of the effect of public and annual holidays, and allowing for change in the length of the working week may be described as an index of production per working day.

6.6 The Index of Industrial Production in UAR.

The Index is compiled by the Research Department of the National Bank of Egypt in Cairo and published in their Economic Bulletin. The annual index is published for industries based on agricultural products, mining and quarrying, manufacturing, and electric light and power. The Index represents approximately 87 percent of the total net value of industrial production in 1954 by private establishments employing 10 or more persons. Also a quarterly index is published, which covers only mining and manufacturing. The index computed on the original base 1954 = 100 is shown in Table VII...

Table VII. Index Numbers of Industrial Production in UAR
Base 1954 = 100.

Industries	1955	1956	1957	1958	1959
1. Industries based on Agr. products	134,8	105,6	132,1	154,5	-
2. Mining and Quarrying	103,6	95,8	117,6	153,3	153,5
3. Manufacturing	109,4	117,2	123,4	134,1	139,6
a) Spinning and Weaving	106,0	115,3	122,8	134,4	140,9
b) Other.	112,2	118,8	123,9	133,9	137,6
4. Electricity	113,8	124,6	136,5	153,6	-
5. General Index	109,0	115,1	123,7	137,3	-

Source: Mational Bank of Egypt.

The Index is computed as a base weighted arithmetic average. This takes the form.

$$Q = \frac{\sum q_n p_o}{\sum q_o p_o}$$

Where q_n is the quantity produced in the current perid, q_o is the quantity produced in the base period and p_o is census value added per unit in the base period. The index does not give an adjustment for seasonal variations or for differences in the numbers of working days. Changes in the volume of output of industrial products and industries are mainly represented by quantities produced.

The "International Standard Industrial Classification" is used to combine forty-two series into group indexes. The Industry groups are shown below.

Table VIII Series and Weighs in the Index of Industrial Production in UAR.

Industry groups	Numbers of Series		Weighting Coefficient	
Mining	3		9,4	
Crude Petroleum		1		8,1
Salt and Phosphates		2		1,3
Manufacturing	38		87,2	
Food		8		15,5
Beverages		3		2,2
Tobacco		1		6,8
Textiles		5		40,1
Footwear (other than rubber)		1		0,8
Furniture		1		0,8
Paper and Paper Products		3		1,3
Printing and Publishing		1		2,3
Leather (Tanning)		1		0,5
Chemicals and Chemical Products		5		6,2
Petroleum refining		1		1,5
Non Metallic mineral products		5		5,1
Basic Metals		2		2,3
Motor-Car repairing		1		1,8
Electricity	1		3,4	
Total		42		100,0

As mentioned above the individual series and industry groups are weighted by census value added in 1954.

1) Unfortunatily no data were available for industries based on agricultural products.

7. The Consumer Price Index.

7.1. General Scope of the Index.

As the price of loaf of bread increases, the number of loaves that can be purchased for one Egyptian Pound decreases or one might say, the purchasing power of the bread Pound decreases. Similarly, as the level of prices paid for housing, food, and other basic consumer goods and services changes, so also does the purchasing power of the consumer pound change. In general, if the Consumer Price Index doubles over a period of time, the purchasing power of the consumer Pound is cut in half.

The primary purpose of the Consumer Price Index is to provide the means for measuring changes in the purchasing power of wages. The goods and services priced and the weights given each, are based upon the average expenditure pattern characteristics of families of wage earners in urban centers.

This Index is not an over-all measure of prices paid by consumers. In most cases the term "consumer" is restricted to mean wage earners in urban places, and the goods and services priced are those entering into the level of living characteristic of such consumers. The effect of changes in prices on farmers will not be measured by the Consumer Price Index, unless, accidentally, this group has the same pattern of expenditures as that applied in the Index calculation. Further, the Index does not reflect differences in the cost of living between one area and another, whether between cities in the same countries or between one country and another.

Many titles can be adopted for this Index. If the Index is applicable to total consumption in a country and when it reflects an index of prices paid by the Consumer, the term "Consumer Price Index" is used. The traditional title for the index is "Cost of Living Index". In that case it only refers to wage earners. Sometimes an index of retail prices for consumer goods is used, "Retail Price Index" In general; this last index does not include rent and services.

In most countries the index is computed as a base weighted arithmetic average of relative prices. This Laspeyres formula takes the form:

$$P_T = \frac{\sum_{i=1}^{n} p_{1i} \quad q_{1i} \frac{p_{Ti}}{p_{1i}}}{\sum_{i=1} p_{1i} \quad q_{1i}}$$

In some cases the weighted aggregative method is used, i.e. the standard Laspeyres formula. As before p_{1i} and q_{1i} are prices and quantities respectively of individual items in the base period and p_{Ti} are prices in the given period.

The indexes used have in general three base pe iods which do not necessarily coincide.

They are
a the period to which the prices p_{1i} relate.
b the period to which the weights p_{1i} q_{1i} in ou formula relate.
c the base period in which the figures are published.
The Index is usually published by the various countries for the following subdivisions of the total cost of living index.
1. food
2. clothing
3. rent

4. heat and light
5. miscellaneous.

So far, no general recommendations for the composition of the consumer Price Index have been given. The "Supplement to the Monthly Bulletin of Statistics", United Nations, 1959 gives a short description of the methods used in several countries. Two examples will be taken from this publication, UAR and USA. The description of the last examples will be completed with data from other sources.

7.2 <u>The Cost of Living Index in the United Arab Republic</u>:

As the title of this section shows us, the Index in this country should be seen as a cost of living Index.

The Index is published in the "General Statistical Bulletin"[1) published by the Department of statistics and census.
As the original base of the Index June, August 1939 = 100 is used. This explains the high absolute amounts used in the Index and is shown in Table IX

Table IX The cost of Living Index in UAR (1939=100)

	1955	1956	1957	1958	1959	1960	1961
Index for all items	283	290	302	302	303	304	306

Source: Dept of Statistics and Census, UAR.

1) in arabic.

The Index is computed as a base weighted arithmetic average, according to the definition given in Section 7.1

The weights are based on an expenditure study of lower middle income families (in Cairo only) with monthly incomes of LE 12-LE 18. The average number of persons in a family was found to be 6.3. at that time. The number of items in each group and corresponding percentage weights are given in Table X.

Table X Goods and their relative weight in the Index

	Number of Items	Percentage weight
Food	28	41,5
Fuel and Soap	4	3,5
Rent	1	16,0
Clothing	33	16,7
Miscellaneous	....	22,3

Rent in this table includes the general and direct taxes on the premises. Clothing items are mainly piece goods before being prepared by outfitters. The choice of commodities in the foodstuffs and clothing groups has been made so as to ensure uniformity all the year round.

7.3. The Consumer Price Index in the United States.

The American Consumer Price Index is published in the "Monthly Labor Review". This index measures only changes in prices. It tells nothing about changes in the kinds and amounts of goods and services families buy, the total amounts families spent for living or the differences in living costs in different places. As shown in table XII. ..., the Index uses 1947-1949 as the original base year equal to 100. This latter period is called the "reference" base period. The Index is a chain computation using a formula equivalent to the base weighted arethmetic average of price relatives. Thus gives us:

$$p = \frac{\sum \frac{p_n}{p_{n-1}} \cdot p_{n-1}\, q_1}{\sum p_1\, q_1} \times 100$$

Here p_1 and p_n mean series of prices in the base period and current period respectively. In the same way q_1 means a series of quantities in the base period. We see, that the price of each item in the current period is divided by its price in the previous period. These ratios are then combined in a weighted average of relative calculations to obtain the current month index. This index is adjusted for seasonal variations.

The weights used for computation of group indexes and the general index for each city were obtained from a 1950 survey of expenditures of 17000 families of wage earners and clerical workers in 97 cities with incomes less than $ 10,000 after taxes. Families were defined as units of two or more persons. In this way the "market basket" of goods and services as priced for the index is representative of the goods and services bought by urban families with moderate income. Since urban wage-earner families buy many thousands of types of goods and services in thousands of stores and establishments, this sample was made. From this sample the kinds, qualities and amounts of goods and services purchased and the amount spent for each item were determined. This "market basket" was revised in 1952 to include new items such as TV sets and frozen foods. The average size in 1952 of the families in the index was estimated to be about 3,3 persons and their average family income was estimated at $ 4160 after taxes. These families represented in 1952, about 64 percent of all people living in urbon areas and about 40 percent of the total population of the United States. The country-wide group indexes and the general index are computed by weighting each city index by the population of families of wage earners and salaried clerical workers in the city metropolitan area and of other cities in the same region and size class. The relative importance of the components in the index in 1952 was as follows:

Table.XI... Composition of the weights in the price Index

	Number of items	Percentage weights
Food	87	29,8
Housing	70	32,2
Clothing	78	9,4
Transportation	18	11,3
Medical care	18	4,9
Personal care	13	2,1
Reading and Recreation	8	5,3
Other goods and services	4	5,0

Source: Bureau of Labor Statistics.

The total number of items priced is 310.[1] Each of the items shown in Table XI..... is subdivided again, e.g. Medical Care in Drugs and Prescription, Hospital Services, and Professional Services.

The Prices are collected from 46 cities consisting of : 20 large cities (population over 250,000) for which separate indexes are computed, 10 cities with population ranging from 30,000 to 250,000 and 16 small cities with a population under 30,000. The prices are collected monthly for most items in the five largest cities, and for food, fuel, cigarettes and rent in all cities. Other items are priced quarterly in the remaining large and medium-sized cities and once every four months in small cities. Rents are obtained yearly from the full sample of approximately 32,000 tenants of dwelling units in each city and monthly from sub-samples. Food prices are collected from about 1425 retail stores. Newspapers, fuels, public utility costs, local transportation, postal rates, home financing costs, taxes and insurance rates are obtained by mail. Other goods and services are priced by agents in about 4.000 retail stores and service establishments. Special discounts or sale prices in force more than one week for food and more than two weeks for other items are included. Each time prices are gathered in a city, they are compared with prices obtained during the preceding period, and the percentage change in the price of each item is calculated.
The Consumer Price Index is published for the twelve cities over one million in population and for eight of the ten cities between 250.000 and one million. The United States total is also published.

1) An increase in the number of items priced was foreseen in 1959.

Table XII... The Consumer Price Index for some Selected Groups in USA. (1947-'49 = 100)

Year	all items	Food	Rent[1)]	clothing
1953	114,4	112,8	124,1	104,8
1954	114,8	112,6	128,5	104,3
1955	114,5	110,9	130,3	103,7
1956	116,2	111,7	132,7	105,5
1957	120,2	115,4	135,2	106,9
1958	123,2	120,3	137,7	107,0
1959	124,6	118,3	139,7	107,9

Source: Bureau of Labor Statistics.

The Consumer Price Index is also published in the United Nations Monthly Bulletin of Statistics, where the indexes are shown in tables based on 1953 =100 They are available monthly for the general index and the food group index only.

1)The Subgroup Rent accounts for only 5,6 percent of the total, while the importance of the group Housing is equal to 32,2 percent.

APPENDIX A

The International Standard Industrial Classification of all Economic Activities

List of divisions and major groups to be covered by the index of industrial production

Division	Major Group
1. Mining and quarrying	11. Coal mining
	12. Metal mining
	13. Crude petroleum and natural gas
	14. Stone quarrying, clay and sand pits
	19. Non-metallic mining and quarrying not elsewhere classified
2-3. Manufacturing	20. Food manufacturing industries, except beverage industries
	21. Beverage industries
	22. Tobacco manufactures
	23. Manufacture of textiles
	24. Manufacture of footwear, other wearing apparel and made-up textile goods
	25. Manufacture of wood and cork, except manufacture of furniture
	26. Manufacture of furniture and fixtures
	27. Manufacture of paper and paper products
	28. Printing, publishing and allied industries
	29. Manufacture of leather and leather products, except footwear
	30. Manufacture of rubber products
	31. Manufacture of chemicals and chemical products
	32. Manufacture of products of petroleum and coal
	33. Manufacture of non-metallic mineral products, except products of petroleum and coal
	34. Basic metal industries
	35. Manufacture of metal products, except machinery and transport equipment.

	36. Manufacture of machinery, except electrical machinery
	37. Manufacture of electrical machinery, apparatus, appliances and supplies
	38. Manufacture of transport equipment
	39. Miscellaneous manufacturing industries
4. Construction	40. Construction
5. Electricity, gas, water and sanitary services	51. Electricity, gas and steam

APPENDIX B

Note on the measurement of an industry's contribution to gross national product

The contribution of an industry to gross national product (at factor cost) may be defined as the unduplicated aggregate value of the goods and services produced in the industry during a period of time including the increase in the value of work in progress, less the value of the goods and services bought from other industries and used up in the process of production. The contribution to net national product is obtained if the cost of maintaining capital intact is also subtracted. The following list of items that must be deducted from the gross value of output of an industry to obtain its contribution to gross national product (at factor cost) may be considered sufficient for practical purposes.

1. Costs of materials and fuels used and of work given out.
2. Printing stationery and other office supplies.
3. Advertising and other selling expenses.
4. Business insurance premiums.
5. Postage, telegraph and telephone payments.
6. Expenses for banking, legal, accounting, auditing, and similar business services.
7. Cost of small repairs, maintenance and servicing carried out by outside contractors.
8. Cost of materials and parts required for small repairs and maintenance work carried out by the establishment's employees.
9. Property taxes and water rates.
10. Excise and sales taxes if these have not already been deducted from the gross value of production (less subsidies, if any, received from the Government).

Items 3 and 6 refer only to outlay on services bought from other industries. If the contribution to gross national product is to be expressed in current prices, both the gross value of output and the cost of raw materials and services should be valued on that bases to avoid inclusion of capital gains or losses due to price fluctuations.

With respect to certain items of expenditure it is necessary to decide whether they should be considered as current business costs or as outlay on capital goods. In the former case, the item should be included in the above list of deductions.

The contribution of an industry to gross national product is equal to the aggregate income paid to the factors of production which contributed to the productive process plus the cost of capital used up. Therefore, it is equal to the total of the following items:

1. (a) Wages and salaries
   (b) Other labour income
2. Income of unincorporated enterprises
3. Corporate profits before taxes
4. Net interest
5. Net rents on lands and buildings, including royalties
6. Allowances for depreciation and obsolescence.

Wages and salaries include employees' contributions to social insurance and pension funds, income in kind, commissions, tips, bonuses, etc. Other labour income includes employers' contributions to social insurance and pension funds, pensions and compensations paid if no social security and pension funds exist, etc. Income of unincorporated enterprises and corporate profits exclude capital gains and losses, and dividends received from other enterprises. Corporate profits, allowances for depreciation and obsolescence and other items will have to be defined in accordance with the detailed treatment of various items in the first list.

Rent paid on lands and buildings might be considered as part of income originating in the real estate industry. If this treatment is adopted, the item should be included in the list of items to be deducted from the gross value of output.

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