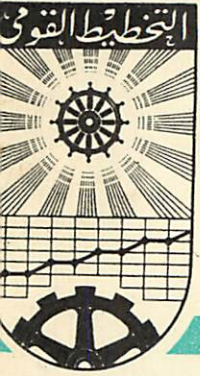


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SOME IDEAS ABOUT ECONOMIC PLANNING
IN DEVELOPING COUNTRIES

by

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I. Preliminary Remarks

The author intends to discuss in this paper some questions out of the wide scope of economic planning seeming to be of importance with respect to the subject of the First International Seminar of the Institute of National Planning at Cairo.

The first question is what objective conditions have to exist, or have to be created by the leading social forces of the nation, not only for the elaboration of a plan corresponding to the objective possibilities within the society but for a successful implementation of the targets of a plan, too. Only if this question is answered in the positive, it is useful and will lead to real success, to discuss methods for elaborating an overall economic plan and managing the economic activity of the different units in which the processes of production and consumption are going on. Even the methods for the elaboration of economic plans are not indifferent to the given social conditions. This, too, has to be recognized fully, whereas this coherence is usually rather accepted with respect to the methods of the State's management of the economic activity.

II. The Lawfulness of the Process of Economic Development and Economic Planning

It is almost always accepted that the economic activity of men is not a Robinsonade but a social process. However it is not always accepted, that this process is going on within the framework of objective laws, created by the social conditions within the society, and changing with changes in these conditions. The cognition of this total objective lawfulness is a precondition for successful measures in the field of economy.

Each society is characterized mainly in the following way:

- a) with respect to the state of development of productive forces, i.e., the level of technique, implemented in production, and the level of education and qualification of the people, as well as by the extension of the production facilities, generally speaking by the overall level of the productivity of labour;
- b) with respect to property of the means of production, i.e., either private, mainly capitalist, ownership-which excludes the working people from ownership, or ownership of the whole people, represented by the State, and of groups of producers in the form of cooperatives;
- c) with respect to the character of the State; i.e. whether it is the instrument of power of the classes of working people or of the bourgeois class; that is very closely linked with the prevailing form of ownership of the means of production.

Historical experience shows obviously that there exist a lawful coherence between the character of the productive forces and the property relations. The more advanced the technique of production the more comprehensive is the social scale of the process of production, the more is hampering the private ownership of the means of production the extension of economy and the increase of the productivity of labour; i.e., the more increases the objective necessity to remove private and to create social ownership. From the conformity of the character of productive forces with the property relations comes the prime motive for an accelerated economic development. This prime motive is, of course, an internal motive. It is, as was stated in the Background Paper for the Seminar of the 4th November, 1961, of social, and by that, of political character,

and today not to separate from the character and the activity of the State.

As for developing countries, they are usually coming in this state of history by overcoming colonialism, and by democratic revolution. In this process feudal chains were removed and the society is starting and going its way as a bourgeois one. A great number of these countries proclaimed economic planning as an instrument of economic, social and cultural development of the whole population. The results, however, are very different. This is due to the fact, that there was not recognized the above mentioned coherence. Capitalism was increasing very quickly in these countries, and along with that, firstly, the capitalist class on the one hand, and the workers' class on the other hand, were strengthening, i.e., class contradictions increased, and secondly the economic laws of surplus value, and of concurrence and anarchy of production made all endeavours to develop the country in the originally proclaimed manner, i.e., by planning, more and more, and at least completely insufficient. On the other hand, the economic development in developing countries, where there was created a public sector of sufficient and increasing extension, and where there was support of the development of cooperative production in agriculture and handicraft is successful, and leads to the increase of national income, by that of accumulation in production and in increase of the standard of living of the people. The public sector is an elementary form of social ownership of the means of production. If it is sufficiently extended, i.e., includes the main part of industry, the foreign trade, traffic, banking and wholesale enterprises, the process of reproduction within the society is a general social one. With respect to the public sector, the State is the representative of the collective

social owner. Within this sector exists objectively a common economic interest, no more the private profit interests of the different capitalists. If a State is going on to develop the national economy in this way, it will open the prime source for economic development to the advantage of the masses of population.

The above mentioned lawful social coherence exists objectively, i.e., outside of consciousness and independent of the will of man. If recognizing that and policy making on the basis of this cognition, economic policy on the basis of plans for economic development will be successful and lead to an accelerated development, for it removes the hampering of the utilization of the highest level of technique in the process of production step by step, and it will be based on the primary interests of the masses of population by which will mobilized their will to strengthen their efforts to produce more and to improve the production and their qualification, too.

Under the conditions of existence of a public sector of sufficient extension as mentioned above, it becomes possible to manage the whole economy on the basis of a plan successively. On the other hand requires the existence of such a public sector necessarily a centralized management on the basis of an elaborated plan, for reasons of the centralized and unified ownership of the means of production. Otherwise each state owned factory, band, etc., would act as a single capitalist; there would be concurrence and rivalries on the market with the same losses of social work, crises, very small economic development or even stagnation of the economy. etc., like in the capitalist countries of today.

Creating such social conditions the State is mobilizing the main forces, and opening the main source for the economic development of the country. In this way has - in the author's opinion - answered the question with respect to the prime motive of economic development, as was questioned in the "Background Paper".

Beyond that there is a secondary motive, viz., economic assistance from outside the country. However, as it is properly stressed in the "Background Paper", foreign economic assistance can be a double-edged sword. Such aid can be a real one only, when there are two conditions: a low rate of interest firstly, and advantageous conditions of repayment in time as well as in products with relative high internal productivity of labour on the one hand, and international prices which secure at least full compensation of the internal cost of production, i.e., avoiding any loss of national labour. This underlines expressively what has been said in the "Background Paper" (page 2) to this problem. Are the above mentioned conditions not fully given, credits from abroad will become necessarily serious obstacles far outweighing such foreign "aid". It is well known that credits from imperialist countries have all the mentioned negative characteristics whereas credits given by socialist countries fulfill the mentioned positive conditions for the credit taking developing countries.

Provided that there is a third motive of importance on that depends the growth rate of economic development. This is the ability to elaborate an optimal plan for economic development. The optimal plan is characterized in this manner:

1. Such a plan puts the State authorities in a position of management to achieve a steady and maximal increase of the overall productivity of labour and by that of national income and standard of living, too.
2. Such a plan is framed by the given social, natural (natural resources, population and disposable manpower), technical (and scientific) conditions, and economic starting conditions at the beginning of the plan-periods as well as by the political conditions and aims (national and international).

To be able to elaborate such a plan provides that there is full cognition of the lawful coherence of the objective economic laws and their manner of effect under the given social conditions, particularly the lawfulness of the process of expanded reproduction. The knowledge received by the study of this objective coherence shows to what extent, and in which degree of exactness an overall economic plan can be elaborated, and implemented, too. The better the conditions and their cognition, the more exact can be the methods of elaboration and implementation of a plan, and the greater will be the results.

It needs not proved again that real planning of economic development—which includes necessarily the implementation of the plan targets — is completely impossible in the developed capitalist countries like the more or less great imperialist states in Western Europe and America. What has been called planning there is nothing as prognosticizing some rough tendencies, and experience of history shows not only that all experiments of so-called "planning" in this area have failed, except some short periods of war — economy with absolute government control and compulsion. And, besides that, by no means was achieved a real and durable improvement of the

social and material conditions of the masses of the people. The interests of each capitalist and monopolist is contradictory to the objective social interest, and denies objectively and economic activity according to an overall plan in general, and to the interests of people, too. What has been called "planning" in these countries were state-monopolistic measures to sure the interests of the ruly grand-bourgeois class and sometimes feudal remainders to detain the decay of capitalism as a social system, and to oppress and exploit the working people and foreign nations.

In the young developing countries, having liberated themselves from colonialism and feudal restraints, are existing very different conditions that in the mentioned socalled old capitalist countries. As it has been mentioned above there are relatively good conditions to avoid the painful way of capitalism, which today usually leads to a state in which such countries remain in their role of poor raw-material producing appendages of the great and industrially high developed imperialist countries, which profit by the terms of trades (they dictate the prices) as well as by high interests for credits and by high profits from capital export. Some governments of developing countries did recognize that and try to complete their national sovereignty by getting to the political independence the economic independence. They create the main condition for an accelerated and planful economic development in the form of an extensive public sector in the sphere of production and circulation and support the unification of small producers in agriculture and handicraft into cooperatives in order to enable them to rise their productivity of labour to a great extent and rapidly.

Under such conditions overall state plans for economic development becomes possible to be elaborated more and more

exactly, and with an increasing success with respect to implementation. In this way a planned economic policy contributes to achieve the great aims of humanism.

III. The Elaboration of an Optimal Plan of the National Economy

Provided what has been said about possibility and necessity of economic planning in developing countries, it seems to be scientifically correct to discuss about the problems of elaboration and implementation of plans in such countries. Of course, it is difficult to a certain extent for the author to discuss about that, because he does not have sufficient experience about the economy of such countries, and there might be lack of knowledge with respect to the existing concrete conditions. For that it may be allowed to discuss only about the problem of optimalization of the plan of national economy (or of an overall economic plan as it is sometimes called) on the basis of the premises that the public sector is predominant in the economy and the Government tries to guide strongly the economic activity in the cooperative and private sectors.

Within the frame of a paper it is necessary to discuss this problem of optimization in using a lot of simplifications with respect to the high variety of practice, and it requires sometimes a very brief description of the economic background of the described methods.

1. The Characteristics of an Optimal Plan of the National Economy

An optimal plan of national economy has the following main features: It secures - under the above mentioned political aims and conditions as well as the natural, technical, and economic starting conditions - full proportionality with up-building of rational but sufficient reserves, and maximal effectivity of the total fund of productive labour spent by the society as a whole

(or with other words: a maximal increase of the productivity of labour within the whole society) over a certain period of time which is usually fixed by political aims or can be connected with economic projects of outstanding importance for the whole country (e.g., a huge valley project like the Assuan High Dam.)

Full proportionality is to be achieved by an elaborated and interconnected system of balances embracing the whole process of production, circulation, distribution, and consumption, foreign trade, reproduction of manpower, and creating-reserves. Such a system of balances has been developed in the socialist countries in the form of the Balance of National Economy.

Of course, the higher the degree of proportionality, the less will there be disproportions and frictions during the process of implementation of the plan, and the smaller will be the losses of productive labour. However, this is not the true problem of effectivity of productive labour. This consists rather in the fact, that - under conditions of full proportionality - with the same fund of productive labour today a different increase of the volume of the social product (in its structure according to needs) can be achieved tomorrow (that means: after the average time of construction of investment). Whereas proportionality is today possible to achieve with a sufficient elaborated system of overall balances, maximal effectivity in the abovementioned sense is still a problem. Some economists propose special methods to measure economic effectivity of single investment-objects compared with others; other economists propose special methods to measure the proportionality of foreign trade. However, all these proposals consider the economic effectivity of one investment-object or one commodity in foreign trade isolated from the whole process of national economy. This is the main defect of these methods. The task is to determine the overall economic effectivity of each combination of all possible variations (variants) of

1. development of technique, technology, and social organisation of labour in all (existing and newly rising) branches,
2. of foreign trade, and
3. of increase of the standard of living.

The elementary condition for each combination of variants is full proportionality, of course. Provided that the overall economic effectivity for each combination of variants becomes evident with the increase of the productivity of labour of the whole national economy. This can be measured by the coefficient of saving of productive labour for the disposable national gross product of the plan-year compared with the labour that would have been necessary for the same mass of products in the year before. I.e., the criterion of overall economic effectivity is "maximal saving of labour (-time)". This proves "That all economy is lastly 'economy of time'". It is evident that a plan, starting from the above mentioned frame of given conditions and securing full proportionality as well as maximal overall economic effectivity serves best to the great national aims. Recognizing that the methodology of the Optimal Plan has been developed.

2. The Mathematical Model of the Balance of National Economy for the Optimization of the National Plan

The model that has been described in the following sections has to describe the total causality of national economy by its main features by means of the quantities of the economic categories, and with the help of functional relations in the form of mathematical expressions (proportions and equations). This can be done with scientific exactness only in that way that every functional relation describes a direct causal relation. Otherwise the relation between two (or more) quantities and its change with the changing conditions cannot be calculated

exactly within the balance-system. This is the first elementary requirement for exact planning and utilization of mathematical models in economic planning. The second one is that the model must be formulated in such a manner that it allows calculation of combination of variants as said above. Thirdly it has to provide for full proportionality for each combination of variants, and for the calculation of the criterium of overall economic efficiency, in order to compare the different combinations and to find that combination what serves with highest effectivity best to the fixed national aims.

Full proportionality means - in short - that production and import is equal to internal demand and export, that capacity and manpower are in proportion to each other and with respect to production (in each branch), that demand and supply of manpower balances, that the balance of payment is balanced, and that national income balances with investment (accumulation) and consumption. On these requirements has to be based the model of the Balance of National Economy in order to find out the optimal plan for the national economy. Furthermore, this model has to be built up for a year's plan, and a long-range plan has necessary be a sequence of exactly interrelated year-steps. The most important link between the year-plans is long-range investment which began in the present or a previous year and becoming producing in one of the following years.

The model consists of three parts:

1. A system of balance equations providing for proportionality.
2. The equation for the criterion of overall economic effectivity.
3. A system of equations for the calculation of the national value of each product for which is in the first system a special balance.

The first one includes seven kinds of balances:

- a) balances of production and consumption of all material products (single products and groups of products),
- b) balances of the development of the fixed capital stocks of all branches,
- c) balances of the development of the circulating capital stocks of all branches,
- d) balances of manpower, total and of all professions,
- e) balance of payment,
- f) balance of consumption (especially standard of living),
- g) balance of national income.

The model shall be described with the following main simplifications:

- a) There are only five branches of production, each producing only one kind of product:

Branch No. Zp 1, producing product No. P 1 = investment goods

Branch No. Zp 2, producing product No. P 2 = raw material

Branch No. Zp 3, producing product No. P 3 = food

Branch No. Zp 4, producing product No. P 4 = other consumer goods

Branch No. Zp 5, producing product No. PG = gold.

- b) There are only two kinds of professions:

Profession No. E 1 = skilled workers

Profession No. E 0 = non-skilled workers

- c) Foreign trade occurs with only one country:

Country No. X 0 = the own country

Country No. X 1 = the foreign country.

- d) Non-productive consumption takes place only in one (public) branch and in two groups of families (with different income):

Branch No. Zk 1 = State apparatus

Family-Group No. F 1

Family-Group No. F 2.

The construction of symbols is as follows:

The basic symbol is characterizing the economic category; it is used a capital letter for so-called absolute figures like gross product, number of workers, etc.,; a small letter is used for a so-called relative figure (a quotient of two or more absolute figures).

As indices were used:

Left side above: the branch in which the described process takes place resp. the category is belonging to;

left side below: the year of the described process;

right side above: the unit of measurement;

right side below: the special quality of the described category.

3. The Balance of Manpower

By the balance of manpower has to be achieved equality between supply and demand with respect to the total number as well as to professions for each year on the one hand, and it has been shown the development of the number of manpower during the year. This is shown in the appendix, formulae No. 1 up to No. 6. The symbols mean:

E^M = Number of manpower, measured in persons (man = index M) in general

$Z_{t \overline{E}}^{pi} M_{Ei}$ = Number of workers of branch No. Z_{pi} with profession No. Ei in the average of the year "t".

Z_{pi}^1 = relative share of branch No. Z_{pi} of the total number of workers available for all branches of production.

$Z_{t \overline{e}}^{pi} E_{Ei}$ = relative share of workers with profession No. Ei of the total number of workers of branch No. Z_{pi} , in the average of the year "t".

$Z_{t \overline{y}}^{pi} (E..)$ = relative share of the "time of functioning of the whole year, concerning the retiring or new beginning workers of branch No. A_{pi} ."

The right hand below indices mean:

- (Ean) = retiring of natural causes,
- (Eas) = retiring of other causes,
- (Ezb) = new beginning after having finished education or vocational training,
- (Ezs) = new beginning of other causes.

4. The Balance of Fixed Capital Stocks

By the balance of fixed capital stocks is to be shown the development of capacity and value of these kind of stocks and the demand for investment goods as well as the requirements of allocation of the accumulation-fund as a part of the consumed national income. This is shown in formulae No. 7 up to 14. (The process of reproduction of fixed capital stocks is very much simplified).

The symbols mean:

$Z_{pi}^G Kn(M-P_1)$ = Capacity of the fixed capital stock of branch No. Z_{pi} , measured in numbers of produced products of the kind "Pi" per hour.

$Z_{pi}^y (G..)$ = Cf. the same symbol used in the balance of manpower,

The right hand below indices mean: the parts of eliminated or new capacities and the corresponding parts of the fixed capital stock:

(Gap) = eliminated capacities because of physical wear and tear,

(Gam) = eliminated because of modernization,

(Gze) = new capacity for "Gap"

(Gzm) = new capacity created by modernization investment,

(Gzn) = new capacity created by new investment.

$gk \frac{M/k}{Pi}$ = Number of products No. Pi , necessary to construct one unit of capacity in branch No. Z_{pi} .

$Pi \frac{H}{t}$ = value of one unit of product No. Pi , in year t , measured in "hours of simple labour" (see later)

G^H = Total value of the fixed capital stock, measured in hours of simple labour.

$Z_{pi} \frac{Ca}{t}^H$ = Depreciation (Amortization) of the Fixed capital stock of branch No. Z_{pi} , in year t , in terms of hours of simple labour.

$Z_{pi} \frac{\bar{d}}{t}$ = Rate of depreciation in the average of year t .

$Z_{pi} \frac{Ag}{t}^H$ = Absolute share of the branch No. Z_{pi} in the fund of accumulation (of the whole national economy),

$Z_{pi} \frac{Ai}{t-b}^H(t)$ = Absolute share of the branch No. Z_{pi} in that part of the fund of accumulation, which is determined for the increase of non-finished investment, in the year " $t-b$ ", which shall become investment in the year " t " (b = period of construction, in years).

5. The Balance of Culculating Capital Stocks

By means of this balance has to be calculated the volume of this kind of stocks in the branches of production (the State's reserve is excluded here for the sake of simplicity), and the participation of each branch in the fund of accumulation. See formulae No. 15 up to 17.

The symbols mean:

$V \frac{M}{P_i}$ = Number of products No. P_i belonging to the circulating capital stock.

$Z_{t \frac{M}{P_i}}^{Av}$ = Increase of that stock in branch No. Z_{P_i} in the year t .

$Z_{t \frac{H}{P_i}}^{Av}$ = Total value of the increase of the circulating capital stock of branch No. Z_{P_i} , in terms of hours of simple labour.

6. The Analysis of the Process of Production

The process of production is going on in the different branches. The elements of production-manpower, instruments of work and objects of work are working together under the organizing will of men, in the form of workers, fixed capital stocks, circulating capital stocks and material input; the result of this process is its gross product. Between the elements and the result of this process exist certain proportions according to the level of technique, technology, organization of labour, ability, qualification and social consciousness of workers as well as to other social factors like ownership and ability to planning and State management of economy. These proportions appear in the form of so-called relative figures; and they are characterizing the economy of each branch (i.e., of each in this way aggregated process of production). These proportions are described by the formulae No. 18 up to 30.

The symbols mean:

- $Z_{t \text{ } h \text{ } jr}^{pi} (E)$ = Effective number of working-hours of the workers in branch No. Z_{pi} in year t .
- $t \text{ } tn(E)$ = Number of legal working-days in year t .
- $Z_{t \text{ } h \text{ } (E)}^{pi \text{ } tn}$ = Number of legal working hours a day per worker.
- $Z_{t \text{ } at \text{ } u, k, s}^{pi} (E)$ = Number of falling out working days because of legal days of leave, days of illness, and others.
- $Z_{t \text{ } p}^{pi} \bar{p} \text{ } M/h$ = Production per working hour, measured in natural units of products, in the average of year.
- $Z_{t \text{ } B \text{ } Pi}^{pi} \text{ } M$ = Gross product of branch No. Z_{pi} (and of products No. P_i) in year t , measured in natural units of products.
- $Z_{t \text{ } ap}^{pi} \text{ } M/h$ = Productivity of labour, measured in natural units of products per working hour, in the average of year t .
- $Z_{t \text{ } k}^{pi} \bar{k}^e (E)$ = Degree of average extensive utilization of one working hour ($k^e \leq 1$)
- $Z_{t \text{ } k}^{pi} \bar{k}^i (E)$ = Coefficient of average deviation from the normal degree of the intensity of labour ($k^i \geq 1$).
- $Z_{t \text{ } h \text{ } (G)}^{pi} \text{ } jr$ = Effective number of the hours of utilizing the capacity during the year t .
- $Z_{t \text{ } sf}^{pi}$ = The shift-factor (relation of the daily hours of utilizing the capacity to number of legal working hours per day per worker, in the average of the year ($sf = 1$))

$Z_{pi} \overline{bt}_t$ = Relation between the number of working days of the factory and the number of legal working days per worker ($bt = 1$)

$Z_{pi} \overline{at}_t (G)$ = Number of falling out working days of the capacity (for reasons like repairs etc.) in the average of the year.

$Z_{pi} \overline{g}_t M/H$ = Coefficient of "utilization of the fixed capital stock", in the average of the year.

$Z_{pi} \overline{k}^e_t (G)$ = Coefficient of average extensive utilization of capacity.

$Z_{pi} \overline{k}^i_t (G)$ = Coefficient of average intensive ($k^e \leq 1$) utilization of capacity ($k^i \geq 1$).

$Z_{pi} \overline{c}_{Pi}^M$ = Material input of branch No. Z_{pi} of products No. P_i in year t , measured in physical units.

$Z_{pi} \overline{c}_{Pi}^{M/M}$ = Effective input per unit of output, in the average of the year.

$Z_{pi} \overline{m}_{Pi}^{M/M}$ = Norm of input per unit of output.

$Z_{pi} \overline{k}_{(C-Pi)}^i$ = Coefficient of utilization of raw material

$Z_{pi} \overline{v}_{Pi}$ = Rate of turnover of that part of the culculating capital stock that consists of products No. P_i .

To analyse the future development of all these figures for each branch is a very important economic task and a precondition for the elaboration of a plan. From the point of view

of each branch of production it is not possible to fix for each future year, t , t plus 1, t plus 2 etc. only one set of figures, because of the fact that one cannot know how many workers will the branch get, and to what extent will it participate in the fund of accumulation. Therefore it is objectively necessary to elaborate for each branch a reasonable number of variants (as mentioned above, cf. III, 1).

7. The Analysis of the Process of Consumption

The process of consumption includes public and private (family household) consumption. The task of the analysis of these processes is to find out the requirements for the allocation and distribution of national income on the one hand, and to get knowledge of the demand for consumer goods on the other hand. The volume of public consumption has to be fixed by decision of State, whereas the volume of private consumption depends from the development of the productivity of labour, of the number of hours of labour per year, the distribution-system (here is - for the sake of simplicity - assumed that there are only workers earning wage, therefore: wage-system), the number and structure of earners within a family, and the consumption structure of the different groups of families. This coherence can be fixed in a mathematical form as is shown in formulae No. 31 up to 39.

The symbols mean:

${}_t K^H$ = Fund of consumption of the year t in terms of hours of simple labour.

${}_t K_i^H$ = Fund of private consumption.

${}_t Z_{K_i} K_g^H$ = Fund of public consumption (here assumed only one branch No. Z_k 1, see above).

${}_t E_{i_L}^H$ = Wage per hour for a worker with the skill No. E_i , in terms of hours of simple labour.

E_{tW} = Coefficient of the wage of a skilled worker to the unskilled one.

\bar{E}_t^M $E_i(F_i)$ = Number of workers with the skill No. E_i belonging to the family-group No. F_i .

F_{tF_i} = Relative share of the family group No. F_i of workers with the skill No. E_i .

F_t^F = Number of families of the group with the average yearly income U .

F_{tU} = Average incomes of the family-group No. F_i per year, measured in hours of simple labour.

$R_{tP_i}^M$ = (Private Resp. public) consumption of products No. P_i in year t (in family-group No. F_i or Branch No. Z_{ki}).

$\bar{r}_{tP_i}^{M/H}$ = Number of consumed products No. P_i per unit (hour of simple labour) of expenses (in family-group No. F_i resp. branch No. Z_{ki}).

With respect to public as well as to private consumption exists the possibility, and the necessity as well, to go with variants into the calculation of the optimal plan. We can put different volumes of public consumption (e.g., State apparatus, defence, culture, medical services, scientific research), and we can also make the calculation with different minimal-wages EO_L^H as well as with different sets of the wage-structure E_{tW} . These variants affect, of course, the fund of accumulation. That t can lead to different rates of growth in the future rate of growth must be excluded.

8. The Balance of Payments

Exportation and importation as well as so-called pure payments are a part of the national economic activity; therefore they have to be included properly into the balance of national economy. Pure payments are in their character export and import of the general equivalent, gold. For the sake of simplicity we do not consider any gap between delivery and payment.

In order not to become either a debtor nor a creditor (with exemption on the basis of international treaties) the balance of payments has to balance every year, and in terms of gold, i.e., lastly national labour. Imports are sometimes objectively determined by national limitations of natural resources. Beyond that, and with respect to export there are possibilities to utilize the advantages of the international division of labour; i.e., that there is the necessity to fix a number of variants with respect to import (which is always connected with internal production development-variants) as well as to the export-structure. The payments without connection to import and export of use-values have to be fixed by decision (on the basis of planned credits and their repayment, etc.).

The system of balance equations with respect to the foreign relations is to be seen in formulae 40 up to 42.

The symbols mean:

$t \text{ Ex}_t^M \text{ Pi}$ = Export of products No. Pi in the year t, in terms of natural units.

$t \text{ Im}_t^M \text{ Pi}$ = Import of products No. Pi.

$t \text{ Pi}_{Pwe}^{\text{Dev}}$ = World market price of product No. Pi in terms of units of foreign currency.

$t \text{ Dev}_t \text{ Yn}$ = Legal gold content of one unit of the foreign currency.

9. The Balance of National Income

National income in the Marxist sense is the new created value by human labour spent in the process of production, the net product, taking in consideration the gains and losses connected with the realisation of export goods and the prices for import goods. It appears in its volume of labour as well as in the mass of products disposable for accumulation and consumption. It is calculated from one year, conventionally. The balance equation is shown in formulae No. 43 up to 46. (The "foreign-trade value-" differences will be explained later on.)

$t N_d^H$ = National income in year t .

$t N^H$ = Net product in the year t , in terms of hours of simple labour.

E_i
 t^q = Factor of reduction of skilled to unskilled (=simple) labour.

E_i M
 t^Q P_i = (Absolute) share of the products No. P_i of the cost of reproduction of a worker with the skill No. E_i .

E_0 M
 t^Q P_i = Ditto, of the unskilled worker.

P_i H
 t^D = Foreign trade value difference per unit of product No. P_i , in terms of hours of simple labour.

t^A H = Fund of accumulation in year t , in terms of hours of simple labour.

$Z_{P_i A_i}$ H
 t = Part of the fund of accumulation used for the increase of non-finished investment for branch No. Z_{P_i} (=increase of non-finished products.)

That last mentioned part of the fund of accumulation is a very important position, for from its size depends the rate of increase of production, and of productivity of labour in the future years,

after the period of construction of the investment (i.e. in the year $t + b$). Only such combination of variants can be considered proper in which this fund is greater than in the year before; otherwise will be surely not achieved an accelerated growth of the national economy.

10. The Balance of Production and Consumption of Material Products.

By means of that balance system the proportionality with respect to production and consumption of all material products has to be achieved in the plan. We assume here that all branches are producing ready-made products only, except branch No. $Z_p 1$ which produces ready-made and non-finished investment goods, too. In its elaborated form this balance system is that one of which has been spoken in part III 2 of this paper. Formula No. 47 shows the elementary form of this balance equation. Having all the expressions which have been found in the foregoing sections of this IIIrd part of the paper we are able to formulate a system of P_i equation ($i = 1$ up to n) with the same number of unknowns. These unknowns are the " $Z_{pi_1, s}$ ". All the other figures are either discrete, however^t exchangeable, variable or figures known from the balance of the year before ($t-1$). That septem of equations is linear.

A presupposition for its solution is the knowledge of the figures $P_i \bar{H}$ showing the value of the unit of each product. This knowledge^t is necessary for the calculation of the criterion of economic effectivity for each combination of variants, too.

11. The Calculation of Value in Terms of Labor-Time (in Hours of Simple Labour.

The necessity of this calculation is given by the fact that prices do more or less diverge from value (in the sense of the Marxist-labour theory of value), therefore prices reflect by

no means the labour necessarily spent in the average of the national economy. From the economic standpoint of a single capitalist or even a monopoly not value but cost price and profits, i.e. price is of interest only. However, from the economic interest of the society, as owner of the means of production represented by the State, the question is only for the amount of labour that has to be spent for the production of each, and of all products, too. Overall economic effectivity can therefore be only measured in labour, i.e. labour time. But labour of one man is not the same than of another, because of the fact that skilled labour requests more cost of reproduction than less or unskilled labour; and there is an extensive graduation of skills. Skilled labour is complicated labour where as non-skilled one is simple labour. As Marx showed exactly, all complicated labour is nothing more than multiplied simple labour, which required to-day only primary school education.¹⁾ The reduction of complicated to simple labour is an objective process. The factor of reduction is given by the relation between the respective process. The factor of reduction is given by the relation between the respective cost of reproduction, as it was shown in III, 9, and ley formula No.44.

The value of each product is formed by two main parts;

- (a) the value of the consumed means of production - depreciation and material input, and
- (b) the value created by new labour.

The value created in the internal production is affected by imports and exports, viz., to the extent of the deviation of world market price from the value created in internal production. This is true for each real use-value as well as for that product that acts as general equivalent (gold). The national value of each real use-value is given by the cost of labour of

1) The character of simple is different from country to country and changes in time.

the total internal production minus the cost of labour for the exported products (which act as general equivalent plus the labour-equivalent) of import. The national value of gold is given by the labour cost of internal gold production plus the labour for the realized gold equivalent for the export goods. These relations are shown in formula No. 48 up to 51. With the knowledge of the national value of each product we are able to calculate the real gold content per unit of the national currency, and the real rate of exchange (if we know the real gold content per unit of foreign currency, too).

Cf. Formula No. 52 up to No. 54. The theoretical background of formula No. 53 is the equality of the sum of values and the sum of prices, which has been proved by Marx.

The knowledge of the value of each product is the basis for a planned development of prices of the internal market, for the cognition of the deviation of the prices from the values, and for the calculation of the absolute gains and losses of the country in the international trade (the foreign trade value differences see above) what is a much better method than the calculation of the terms of trade. This coherence is shown with formula No. 55 up to No. 56.

The formula No. 50 is in its elaborated form the basis for the calculation of the national value of each product (or group of products) which is balanced in the former mentioned balance system (Cf. III, 10). One gets a system of P_i equations with P_i unknowns, viz, the P_i H_i S_i . The system is not linear, and has been solved by iteration.

The symbols mean:

- P_{tH}^{Pi} = Value per unit of product No. Pi created in the internal production, in the year t, in terms of hours of simple labour.
- P_{tH}^{Pi} = (National) value per unit of product No. Pi, paying regard to the influences of foreign trade, in terms of hours of simple labour.
- P_{tH}^{G} = National value per unit (e.g. gram) of gold, in terms of hours of simple labour.
- P_{tPv}^{Pi} = National value per unit of product No. Pi in terms of grammes of gold.
- X_{tYr}^{Xi} = Real gold content per unit of currency of the country No. Xi.
- ${}_t W_r \left(\frac{X_2}{X_1} \right)$ = Real rate of exchange between the currencies of countries No. X2 and No. X1.
- ${}_t H^Z_{Pi}$ = Deviation of the internal price from the national value per unit of product No. Pi, in terms of hours of simple labour.

12. The Criterion of Overall Economic Effectivity.

According to what has been said already the overall economic effectivity is characterized by the total labour cost of the mass of material products disposable in the year "t" within the country, compared with the cost of the same mass of products in the year before. The greater the coefficient ${}_t E$ the higher the effectivity of the combination of variants.

For the calculation of the coefficient cf. formula No. 57.

13. Other Problems.

What has been said in the foregoing sections of the IIIrd part of this paper is obviously very much simplified, however it shows the principle of solution of the problem of optimizing the national plan. The limited space of the paper does not allow to discuss all problems connected with. Therefore only the following problems shall be mentioned: The figures of the branches are aggregated ones, from the point of view of the single units of production with different levels of technique etc., and belonging to different groups of ownership. With respect of the public sector one can have exact information, but usually not for factories, etc. of other kinds of ownership. Here one has to use general experience and statistical information of the past; estimates are mostly necessary. Accordingly the results of the calculation of plan-targets have a certain degree of unexactness. The second problem is connected with the branches of internal trade and transports. Additional information about the flow of products in that processes of circulation are necessary, particularly with respect to transport. Here we need information about the regional proportions of production and consumption, and about the location of factories. The third problem to be mentioned is the procedure of calculation. Finally we have to collect all figures for all possible variants of technical development, of consumption, and (of foreign trade). It is further possible to work with variants in total employment, if there is unemployment to-day which has been removed step by step.

Having collected these data for the year t , one has to calculate the national values for all products P_i for each combination of variants of technique etc. in the branches of production. After that the balance system for the elaboration

of the proportionality has to be calculated for each combination of variants of all kinds. Lastly the criterion of overall economic effectivity has to be calculated for that combination of variants which give a sufficient size of the increase of non-finished investment to secure an accelerated development in the future years. Then the whole process has to be repeated for the next year $t + 1$ and so on. Of course, there are also to consider several variants of the increase of non-finished investment from year to year by which is effected the rate of growth of national economy. It can easily be seen that such an exact and detailed calculation requires full mechanized and even electronic computers, for one has to take into consideration between 300 to 600 products and groups of products, and about 150 professions.

This seems to be very complicated, but the degree of enactness of the foundation of economic policy, and of the plan targets for the public sector as well, and the economic (and by that the political) results will surely far outweigh the cost of machinery as well as statistical information and projects of the technical development of branches according to variants.

14. Concluding Remarks.

The author feels the necessity to underline again the idea described in part III of this paper is a new one, and is developed hitherto only in a logical way, but is not based on practical verification. It has to be discussed therefore. However it has to be stressed too, that the planning process in a socialist country is principally going on in the same way, except the calculation of the overall economic effectivity (what is a completely new idea). The main difference between practice and the described procedure is,

firstly, the higher degree of exactness, and, secondly, the possibility to mechanize that part of work of the planning authorities which consists of pure calculating operations and of such logical operations which have to follow the objectively existing lawfulness of the economic process of the society. The economic, technical, and political character of the frame for each plan as well as the judgement of the results of the plan calculation which is of purely political character, can, of course, never become mechanized. The idea described in this paper may somewhat seem far from realization in practice because of the complicated and extended system of calculations. The author does not think so. Modern technique is conquering very quickly all fields of human activity. It will not stop before the doors of the planning authorities.

The Background Paper of the 4th November, 1961, asked the questions of the motives for an accelerated economic development, and of the methods to achieve that. The author tried to answer these questions with respect to two particular problems only.

The problems connected with the implementation of the plan targets is another complex question that required full knowledge of the concrete situation within the country. It can with respect to what has been said above not be discussed from the author in this paper.

Prof. Dr. Rudolph

Appendix: Proportions and Equations for the Optimization of the National Plan.

$$(1) \quad t^{\bar{E}M} = \sum_{Zpi=1}^5 Zpi \quad t^{\bar{E}M} + Zki \quad t^{\bar{E}M}$$

$$(2) \quad Zpi_{t1} = \frac{Zpi \quad t^{\bar{E}M}}{\sum_{Zpi=1}^5 Zpi \quad t^{\bar{E}M}}, \quad \sum_{Zpi=1}^5 Zpi = 1$$

$$(3) \quad t^{\bar{E}M}_{Ei} = \sum_{Zpi=1}^5 Zpi \quad t^{\bar{E}M}_{Ei} + Zki \quad t^{\bar{E}M}_{Ei}$$

$$(4) \quad Zpi_{t^e Ei} = \frac{Zpi \quad t^{\bar{E}M}_{Ei}}{\sum_{Ei=0}^1 Zpi \quad t^{\bar{E}M}_{Ei}}, \quad 1 \sum_{Ei=0}^1 Zpi \quad t^{\bar{E}M}_{Ei} = 1$$

$$(4a) \quad Zki_{t^e} = \text{like No. 4}$$

$$(5) \quad 1.1. t^{\bar{E}M} - t^{\bar{E}M}_{(An)} - t^{\bar{E}M}_{(As)} + t^{\bar{E}M}_{(Zb)} + t^{\bar{E}M}_{(Zs)} = 31.12. t^{\bar{E}M}$$

$$(6) \quad Zpi_{t^E} = 1.1. t^{\bar{E}M} - Zpi_{t^E_{(An)}} \left(1 - Zpi \quad t^{\bar{E}M}_{Y(An)} \right) - Zpi_{t^E_{(As)}} \rightarrow$$

$$- \left(1 - Zpi \quad t^{\bar{E}M}_{Y(As)} \right) + Zpi_{t^E_{(Zb)}} \quad Zpi \quad t^{\bar{E}M}_{Y(Zb)} + Zpi_{t^E_{(Zs)}} \quad Zpi \quad t^{\bar{E}M}_{Y(Zs)}$$

$$(5a) \quad 31.12 \, t_{Ei}^M, \quad Z_{Pi}^M \, t_{Ei}, \text{ and } Z_{Pi}^M \, t_{Ei} \text{ like No. 5}$$

$$(7) \quad \begin{aligned} & 1.1. \, t_{G}^{Kh(M-Pi)} - Z_{Pi}^{Kh(M-Pi)} \, t_{G(GAp)} - Z_{Pi}^{Kh(M-Pi)} \, t_{G(GAm)} + Z_{Pi}^{Kh(M-Pi)} \, t_{G(GZe)} \\ & + Z_{Pi}^{Kh(M-Pi)} \, t_{G(GZm)} + Z_{Pi}^{Kh(M-Pi)} \, t_{G(GZn)} = 31.12. \, t_{G}^{Kh(M-Pi)} \end{aligned}$$

$$(8) \quad \begin{aligned} & Z_{Pi}^{Kh(M-Pi)} \, t_{G} = 1.1. \, t_{G}^{Kh(M-Pi)} - Z_{Pi}^{Kh(M-Pi)} \, t_{G(GAp)} \left(1 - t_{Y(GAp)} \right) \\ & - Z_{Pi}^{Kh(M-Pi)} \, t_{G(GAm)} \left(1 - t_{Y(GAm)} \right) + Z_{Pi}^{Kh(M-Pi)} \, t_{G(GZe)} \, t_{Y(GZe)} \\ & + Z_{Pi}^{Kh(M-Pi)} \, t_{G(GZm)} \, t_{Y(GZm)} + Z_{Pi}^{Kh(M-Pi)} \, t_{G(GZn)} \, t_{Y(GZn)} \end{aligned}$$

$$(9) \quad Z_{Pi}^{M/K} \, t_{G}^{sk} = \frac{Z_{Pi}^M \, t_{G}^{Pi}}{Z_{Pi}^{Kh(M-Pi)} \, t_{G}}$$

$$(10) \quad Z_{Pi}^M \, t_{G}^{Pi} = Z_{Pi}^{Kh(M-Pi)} \, t_{G} \, Z_{Pi}^{M/K} \, t_{G}^{sk}$$

$$(11) \quad \begin{aligned} & Z_{Pi}^M \, t_{G}^{Pi} = Z_{Pi}^M \, t_{G}^{Pi(GZe)} + Z_{Pi}^M \, t_{G}^{Pi(GZm)} + Z_{Pi}^M \, t_{G}^{Pi(GZn)} \\ & = Z_{Pi}^{Kh(M-Pi)} \, t_{G(GZe)} \, Z_{Pi}^{M/K} \, t_{G}^{sk} + Z_{Pi}^{Kh(M-Pi)} \, t_{G(GZm)} \, Z_{Pi}^{M/K} \, t_{G}^{sk} \\ & + Z_{Pi}^{Kh(M-Pi)} \, t_{G(GZn)} \, Z_{Pi}^{M/K} \, t_{G}^{sk} \end{aligned}$$

$$(12) \quad Z_{Pi}^H \, t_{G} = Z_{Pi}^M \, t_{G}^{Pi} \, P_{1H}, \text{ generally } = \sum_{Pi=1}^n Z_{Pi}^M \, t_{G}^{Pi} \, P_{1H}$$

$$(13) \quad Z_{Pi}^H \, t_{G} = Z_{Pi}^H \, t_{G} \, Z_{Pi}^d = Z_{Pi}^{Kh(M-Pi)} \, t_{G} \, \sum_{Pi=1}^n Z_{Pi}^{M/K} \, t_{G}^{sk} \, P_{1H} \, Z_{Pi}^d$$

$$(14) \quad Z_{t^A_g}^{pi \ H} = \sum_{Pi=1}^n Z_{t^F_{Pi}}^{pi \ M} \frac{Pi}{t^H} - Z_{t^{Ca}}^{pi \ H} - Z_{t^{-b}^{Ai}(t)}^{pi \ H}$$

$$(15) \quad Z_{t^V_{Pi}}^{pi \ M} = \frac{Z_{t^{1.1} \cdot t^V_{Pi}}^{pi \ M} + Z_{t^{31.12} \cdot t^V_{Pi}}^{pi \ M}}{2}$$

$$(16) \quad Z_{t^{Av}_{Pi}}^{pi \ M} = \frac{Z_{t^{31.12} \cdot t^V_{Pi}}^{pi \ M}}{31.12 \cdot t^V_{Pi}} - \frac{Z_{t^{1.1} \cdot t^V_{Pi}}^{pi \ M}}{1.1 \cdot t^V_{Pi}}$$

$$(17) \quad Z_{t^{Av}}^{pi \ H} = \sum_{Pi=1}^n Z_{t^{Av}_{Pi}}^{pi \ M} \frac{Pi}{t^H}$$

$$(18) \quad Z_{t^h(E)}^{pi \ jr} = \left[t^{tn(E)} - \left(Z_{t^{at}(E)}^{pi \ U} + Z_{t^{at}(E)}^{pi \ k} + Z_{t^{at}(E)}^{pi \ s} \right) \right] Z_{t^h(E)}^{pi \ tn}$$

$$(19) \quad Z_{t^P}^{pi \ M/h} = \frac{Z_{t^B_{Pi}}^{pi \ M}}{Z_{t^E}^{pi \ M} Z_{t^h(E)}^{pi \ jr}}$$

$$(20) \quad Z_{t^P}^{pi \ M/h} = \frac{Z_{t^{ap}}^{pi \ M/h} Z_{t^k(E)}^{pi \ e} Z_{t^k(E)}^{pi \ i}}{t^k(E)}$$

$$(20a) \quad Z_{t^{ap}}^{pi \ M/h} \leq Z_{t^G}^{pi \ Kh(M-Pi)}$$

$$(21) \quad Z_{t^h(G)}^{pi \ jr} = \frac{Z_{t^h(E)}^{pi \ tn}}{t^h(E)} \frac{Z_{t^{sf}}^{pi \ M}}{t^{sf}} \frac{Z_{t^{bt}}^{pi \ M}}{t^{bt}} - \frac{Z_{t^{at}(G)}^{pi \ M}}{t^{at}(G)}$$

$$(22) \quad Z_{t^B_{Pi}}^{pi \ M} = \frac{Z_{t^G}^{pi \ Kh(M-Pi)}}{t^G} \frac{Z_{t^h(G)}^{pi \ jr}}{t^h(G)} \frac{Z_{t^k(G)}^{pi \ e}}{t^k(G)} \frac{Z_{t^k(G)}^{pi \ i}}{t^k(G)}$$

$$(23) \quad Z_{t^g}^{pi \ M/H} = \frac{Z_{t^B_{Pi}}^{pi \ M}}{Z_{t^G}^{pi \ H}} = \frac{Z_{t^G}^{pi \ Kh(M-Pi)}}{Z_{t^G}^{pi \ Kh(M-Pi)}} \frac{Z_{t^h(G)}^{pi \ jr}}{t^h(G)} \frac{Z_{t^k(G)}^{pi \ e}}{t^k(G)} \frac{Z_{t^k(G)}^{pi \ e}}{t^k(G)} \sum_{Pi=1}^n \frac{Z_{t^g_{Pi}}^{pi \ M/K}}{t^g_{Pi}} \frac{Pi}{t^H}$$

$$(24) \frac{Z_{pi} \frac{M}{M}}{t^{\bar{c}}_{Pi}} = \frac{Z_{pi} \frac{M}{t^{\bar{c}}_{Pi}}}{Z_{pi} \frac{M}{t^B_{Pi}}} = \frac{Z_{pi} \frac{M}{t^{\bar{c}}_{Pi}(\text{Norm})}}{Z_{pi} \frac{M}{t^B_{Pi}}} \frac{Z_{pi} \frac{M}{t^{k^i}_{(C-Pi)}}}{t^{k^i}_{(C-Pi)}}$$

$$(25) \frac{Z_{pi} \frac{M}{M}}{t^{\bar{c}}_{Pi}} = \frac{Z_{pi} \frac{M}{M}}{t^{\bar{m}}_{Pi}} \frac{Z_{pi} \frac{M}{M}}{t^{k^i}_{(C-Pi)}}$$

$$(26) \frac{Z_{pi} \frac{M}{M}}{t^{\bar{v}}_{Pi}} = \frac{Z_{pi} \frac{M}{t^{\bar{v}}_{Pi}}}{Z_{pi} \frac{M}{t^B_{Pi}}}$$

$$(27) \frac{Z_{pi} \frac{M}{M}}{t^B_{Pi}} = \frac{Z_{pi} \frac{M}{t^{\bar{E}}}}{t^{\bar{h}}_{(E)}} \frac{Z_{pi} \frac{M}{h}}{t^{\bar{a}p}} \frac{Z_{pi} \frac{M}{t^{k^e}_{(E)}}}{t^{k^e}_{(E)}} \frac{Z_{pi} \frac{M}{t^{k^i}_{(E)}}}{t^{k^i}_{(E)}} \\ = \frac{Z_{pi} \frac{M}{t^{\bar{E}}} - \frac{Z_{ki} \frac{M}{t^{\bar{E}}}}{t^{\bar{h}}_{(E)}} \frac{Z_{pi} \frac{M}{h}}{t^{\bar{a}p}} \frac{Z_{pi} \frac{M}{t^{k^e}_{(E)}}}{t^{k^e}_{(E)}} \frac{Z_{pi} \frac{M}{t^{k^i}_{(E)}}}{t^{k^i}_{(E)}}$$

$$(28) \frac{Z_{pi} \frac{H}{H}}{t^{\bar{G}}} = \frac{Z_{pi} \frac{M}{t^B_{Pi}}}{Z_{pi} \frac{M}{t^{\bar{G}}}}$$

$$= \frac{\left[\frac{M}{t^{\bar{E}}} - \frac{Z_{ki} \frac{M}{t^{\bar{E}}}}{t^{\bar{h}}_{(E)}} \right] \frac{Z_{pi} \frac{M}{h}}{t^{\bar{a}p}} \frac{Z_{pi} \frac{M}{t^{k^e}_{(E)}}}{t^{k^e}_{(E)}} \frac{Z_{pi} \frac{M}{t^{k^j}_{(E)}}}{t^{k^j}_{(E)}} \sum_{Pi=1}^n \frac{Z_{pi} \frac{M}{k} \frac{Pi}{t^{\bar{g}h}_{Pi}}}{t^{\bar{g}h}_{Pi}}}{\frac{Z_{pi} \frac{M}{t^{\bar{h}}_{(G)}}}{t^{\bar{h}}_{(G)}} \frac{Z_{pi} \frac{M}{t^{k^e}_{(G)}}}{t^{k^e}_{(G)}} \frac{Z_{pi} \frac{M}{t^{k^i}_{(G)}}}{t^{k^i}_{(G)}}}$$

$$(29) \frac{Z_{pi} \frac{M}{M}}{t^{\bar{v}}_{Pi}} = \frac{Z_{pi} \frac{M}{t^B_{Pi}}}{t^{\bar{v}}_{Pi}} \\ = \left[\frac{M}{t^{\bar{E}}} - \frac{Z_{pi} \frac{M}{t^{\bar{E}}}}{t^{\bar{h}}_{(E)}} \right] \frac{Z_{pi} \frac{M}{h}}{t^{\bar{a}p}} \frac{Z_{pi} \frac{M}{t^{k^e}_{(E)}}}{t^{k^e}_{(E)}} \frac{Z_{pi} \frac{M}{t^{k^i}_{(E)}}}{t^{k^i}_{(E)}} \frac{Z_{pi} \frac{M}{t^{\bar{v}}_{Pi}}}{t^{\bar{v}}_{Pi}}$$

$$(30) \quad \begin{matrix} Z_{pi} M \\ t^C_{Pi} \end{matrix} = \begin{matrix} Z_{pi} M \\ t^B_{Pi} \end{matrix} \begin{matrix} Z_{pi} M/M \\ t^C_{Pi} \end{matrix}$$

$$= \left[\begin{matrix} M \\ t^E \end{matrix} - \begin{matrix} Z_{pi} M \\ t^E \end{matrix} \right] \begin{matrix} Z_{pi} \\ t^L \end{matrix} \begin{matrix} Z_{pi} \\ t^{h(jr)}_{(E)} \end{matrix} \begin{matrix} Z_{pi} M/h \\ t^{ap} \end{matrix} \begin{matrix} Z_{pi} \\ t^{k(e)}_{(E)} \end{matrix} \begin{matrix} Z_{pi} \\ t^{k(i)}_{(E)} \end{matrix} \begin{matrix} Z_{pi} M/M \\ t^m_{Pi} \end{matrix} \begin{matrix} Z_{pi} \\ t^{k(i)}_{(c-Pi)} \end{matrix}$$

$$(31) \quad t^{KH} = t^{KiH} + \frac{Z_{ki} H}{t^K}$$

$$(32) \quad \frac{E_i}{t^W} = \frac{E_i H}{E_o H} \frac{L}{L M}$$

$$(33) \quad \frac{F_i}{t^f_{Ei}} = \frac{t^E_{Ei}(F_i)}{t^E M_{Ei}}$$

$$(34) \quad \frac{F_i}{t^F} \frac{F_i H}{t^U} = \sum_{Z_{pi}=1}^n \sum_{E_i=0}^n \left\{ \begin{matrix} M \\ t^E \end{matrix} - \begin{matrix} Z_{ki} M \\ t^E \end{matrix} \right\} \begin{matrix} Z_{pi} \\ t^L \end{matrix} \begin{matrix} Z_{pi} \\ t^e_{Ei} \end{matrix} \begin{matrix} Z_{pi} \\ t^{h(jr)}_{(E)} \end{matrix}$$

$$+ \begin{matrix} Z_{ki} M \\ t^E \end{matrix} \begin{matrix} Z_{pi} \\ t^e_{Ei} \end{matrix} \begin{matrix} Z_{ki} \\ t^{h(jr)}_{(E)} \end{matrix} \left\{ \begin{matrix} E_o H \\ t^L \end{matrix} \begin{matrix} E_i \\ t^W \end{matrix} \begin{matrix} F_i \\ t^f_{Ei} \end{matrix} \right\}$$

$$(35) \quad \frac{F_i M/H}{t^r_{Pi}} = \frac{\begin{matrix} F_i M \\ t^R_{Pi} \end{matrix} \begin{matrix} P_i \\ t^H \end{matrix}}{\begin{matrix} F_i \\ t^F \end{matrix} \begin{matrix} F_i H \\ t^U \end{matrix}}$$

$$(36) \quad \frac{F_i M}{t^R_{Pi}} = \frac{\begin{matrix} F_i \\ t^F \end{matrix} \begin{matrix} F_i H \\ t^U \end{matrix} \begin{matrix} F_i M/H \\ t^r_{Pi} \end{matrix}}{\begin{matrix} P_i \\ t^H \end{matrix}}$$

$$(37) \quad \frac{Z_{ki} M/H}{t^r_{Pi}} = \frac{\begin{matrix} Z_{ki} M \\ t^R_{Pi} \end{matrix} \begin{matrix} P_i \\ t^H \end{matrix}}{\begin{matrix} Z_{ki} H \\ t^K_g \end{matrix}}$$

$$(38) \quad \frac{Z_{ki} M}{t^R_{Pi}} = \frac{\begin{matrix} Z_{ki} H \\ t^K_g \end{matrix} \begin{matrix} Z_{ki} M/H \\ t^r_{Pi} \end{matrix}}{\begin{matrix} P_i \\ t^H \end{matrix}}$$

$$(39) \quad t_{iH}^K = \sum_{Fi} \frac{Fi}{t^F} \frac{Fi}{t^U} H$$

$$(40) \quad \sum_{Pi=1}^n \left[\frac{M}{t^{Ex}_{Pi}} \frac{Pi}{t^{P_{we}}} \frac{Dev}{t^{Y_n}} \frac{Dev}{t^{Y_n}} \right] + \frac{M}{t^{E}_{PG}} =$$

$$\sum_{Pi=1}^n \left[\frac{M}{t^{Im}_{Pi}} \frac{Pi}{t^{P_{we}}} \frac{Dev}{t^{Y_n}} \frac{Dev}{t^{Y_n}} \right] + \frac{M}{t^{Im}_{PG}}$$

$$(41) \quad t^{x}_{Pi} = \frac{\frac{M}{t^{Ex}_{Pi}} \frac{Pi}{t^{P_{we}}} \frac{Dev}{t^{Y_n}} \frac{Dev}{t^{Y_n}}}{\sum_{Pi=1}^n \frac{M}{t^{Ex}_{Pi}} \frac{Pi}{t^{P_{we}}} \frac{Dev}{t^{Y_n}} \frac{Dev}{t^{Y_n}}}$$

$$(41a) \quad \sum_{Pi=1}^n t^{x}_{Pi} = 1$$

$$(42) \quad t^{Ex}_{Pi} = \frac{t^{x}_{Pi}}{\frac{Pi}{t^{P_{we}}} \frac{Dev}{t^{Y_n}} \frac{Dev}{t^{Y_n}}} \left\{ \sum_{Pi=1}^n \left[\frac{M}{t^{Im}_{Pi}} \frac{Pi}{t^{P_{we}}} \frac{Dev}{t^{Y_n}} \frac{Dev}{t^{Y_n}} \right] + \frac{M}{t^{Im}_{PG}} - \frac{M}{t^{Ex}_{PG}} \right\}$$

$$(43) \quad t^N_H = \sum_{Zpi=1}^n \frac{Zpi}{t^N} H$$

$$\sum_{Zpi=1}^n \left[\frac{M}{t^E} - \frac{Zpi}{t^E} \right] \frac{Zpi}{t^1} \frac{Zpi}{t^{e_{Ei}}} \frac{Zpi}{t^{h(E)}} \sum_{Ei=0}^n \frac{Ei}{t^q}$$

$$(44) \frac{E_i}{t^q} = \frac{\sum_{P_i=1}^n \frac{E_i M P_i}{t^{Q_{P_i}} t^{\bar{H}}}}{\sum_{P_i=1}^n \frac{E_o M P_i}{t^{Q_{P_i}} t^{\bar{H}}}}$$

$$(44a) \frac{E_i}{t^q} > 1, \quad \frac{E_o}{t^q} = 1$$

$$(45) t_{Nd}^H = t^H + \sum_{P_i=1}^n \frac{P_i H}{t^D} + \left[\frac{M}{t^{Im_{PG}}} \frac{M}{t^{Ex_{PG}}} \right] \frac{PG}{t^{\bar{H}}}$$

$$(46) t_{Nd}^M = t^H + t^K = t^H + t^H + t^H + t^{Zki H} = t^H + t^H + t^H + t^H$$

$$= \sum_{Zpi=1}^n \frac{Zpi H}{t^{A_g}} + \sum_{Zpi=1}^n \frac{Zpi H}{t^{A_v}} + \sum_{Zpi=1}^n \frac{Zpi H}{t^{A_i}}$$

$$(47) \frac{Zpi H}{t^{B_{P_i}}} + \frac{M}{t^{Im_{P_i}}} = \sum_{Zpi} \frac{Zpi M}{t^{F_{P_i}}} + \sum_{Zpi} \frac{Zpi M}{t^{C_{P_i}}} + \sum_{Zpi} \frac{Zpi M}{t^{A_v_{P_i}}}$$

$$+ \sum_{F_i} \frac{F_i M}{t^{R_{P_i}}} + \sum_{Zki} \frac{Zki M}{t^{R_{P_i}}} + t^{Ex_{P_i}}$$

$$(48) \frac{Zpi H}{t^{B_{P_i}}} = \frac{Zpi H}{t^{Ca}} + \sum_{P_i} \frac{Zpi H}{t^{C_{P_i}}} + \frac{Zpi H}{t^N}$$

$$(49) \quad \frac{P_i}{t^H} = \frac{Z_{pi} \bar{t}^H}{t^B_{Pi}} \frac{Z_{pi} \bar{t}^d}{t^d} + \sum_{Pi} \frac{Z_{pi} M}{t^B_{Pi}} \frac{t^C_{Pi}}{t^H} +$$

$$+ \frac{Z_{pi} M}{t^B_{Pi}} \frac{t^E}{t^B_{Pi}} \frac{Z_{pi} \bar{t}^{jr}}{t^{h(E)}} \sum_{Pi} \frac{Z_{pi} E_i}{t^E_{Ei}} \frac{E_i}{t^q}$$

$$= \frac{Z_{pi} \bar{t}^d}{Z_{pi} \bar{t}^{jr} t^{h(G)} - Z_{pi} \bar{t}^k t^{k(G)}} \frac{Z_{pi} M/k}{t^{gh}_{Pi}} \frac{P_i}{t^H}$$

$$\sum_{Pi} \frac{Z_{pi} M/M}{t^m_{Pi}} \frac{Z_{pi} i}{t^k(C-Pi)} \frac{P_i}{t^H}$$

$$+ \frac{Z_{pi} \bar{t}^{jr}}{t^{ap}} \frac{Z_{pi} M/h}{t^{k(E)}} \frac{Z_{pi} \bar{t}^i}{t^{k(E)}} \left\{ \frac{Z_{pi} \bar{t}^e}{E_i} \sum_{Pi} \frac{E_i M P_i}{t^Q_{Pi} t^H} - \frac{Z_{pi} \bar{t}^e}{E_i} \sum_{Pi} \frac{E_o M P_i}{t^Q_{Pi} t^H} \right\}$$

$$(50) \quad \frac{P_i}{t^H} = \frac{\left[\frac{Z_{pi} M}{t^B_{Pi}} - \frac{M}{t^{Ex}_{Pi}} \right] \frac{P_i}{t^H} + \frac{M}{t^{Im}_{Pi}} \frac{P_i}{t^P} \frac{Dev}{t^{we}} \frac{Dev}{t^{Y_n}} \frac{PG}{t^H}}{Z_{pi} M - t^B_{Pi} - t^{Ex}_{Pi} + t^{Im}_{Pi}}$$

$$(51) \quad \frac{PG}{t^H} = \frac{Z_{pi} M}{t^B_{PG}} \frac{PG}{t^H} + \sum_{Pi} \frac{M}{t^{Ex}_{Pi}} \frac{P_i}{t^P} \frac{Dev}{t^{we}} \frac{Dev}{t^{Y_n}}$$

$$(53) \quad \frac{x_i}{t^{Yr}} = \frac{\sum_{Pi} \left[\frac{Z_{pi} M}{t^B_{Pi}} - \frac{M}{t^{Ex}_{Pi}} + \frac{M}{t^{Im}_{Pi}} \right] \frac{P_i}{t^H}}{\sum_{Pi} \left[\frac{Z_{pi} M}{t^B_{Pi}} - \frac{M}{t^{Ex}_{Pi}} + \frac{M}{t^{Im}_{Pi}} \right] \frac{P_i}{t^{Pe}}}$$

$$(54) \quad t_r^W \left(\frac{\bar{x}_2}{x_1} \right) = \frac{x_1}{x_2} \frac{t_r^Y}{t_r^Y}$$

$$(55) \quad \frac{P_i^H}{t^Z} = \frac{P_i^{\bar{x}_i}}{t^P_e} \frac{x_i}{t^Y_r} \frac{PG}{t^{\bar{H}}} - \frac{P_i}{t^{\bar{H}}}$$

$$(56) \quad \frac{P_i^H}{t^D} = \frac{P_i^{\text{Dev Dev}}}{t^P_{we}} \frac{PG}{t^Y_n} \frac{PG}{t^{\bar{H}}} - \frac{P_i}{t^{\bar{H}}}$$

$$(57) \quad t^{\epsilon} = \frac{\sum_{P_i} \frac{P_i}{t^{\bar{H}}} \left[\frac{Z_{P_i}^M}{t^B_{P_i}} - \frac{M}{t^{\text{Ex}}_{P_i}} + \frac{M}{t^{\text{Im}}_{P_i}} \right]}{\sum_{P_i} \frac{P_i}{t^{\bar{H}}} \left[\frac{Z_{P_i}^M}{t^B_{P_i}} - \frac{M}{t^{\text{Ex}}_{P_i}} + \frac{M}{t^{\text{Im}}_{P_i}} \right]}$$
