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FOREIGN LOANS AND ECONOMIC DEVELOPMENT

PART III

THE I.B.R.D. APPROACH

by

Dr. M.M. El-Imam

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CAIRO
3, MOHAMED MAZHAR - ZAMALEK

CONTENTS

I. Introduction:

In the first two parts of the present memo. we have discussed the conditions under which a single loan can have a given net effect on the size of the country's capital at a certain future date. Two variants of the annual annuity were considered; the decreasing and the fixed. We also discussed the effects of the existence of lags in the shape of gestation periods and grace periods. The guiding principle was to define the addition to the country's capital at a given future date, which was chosen initially as the date of full retirement of the debt. Later it was necessary to modify this rule since it did not fit in quite reasonably with the need to evaluate the actual capital resources required to finance a certain target of economic growth. It is one thing to say that a certain loan can add something to the country's resources at the end of its repayment period; and it is another to tell what it is going to add at a given future date irrespective of the length of that period.

As concluded in the previous parts, we still have to investigate the behaviour of the parameters of the economy in order to take full account of long term development aspects. Before doing that however, we have to discuss the implications of yet another approach, developed by the experts of I.B.R.D. In general, this approach is concerned with the effect of debt servicing on long-term growth. The starting point was to consider a given growth rate, g say, already determined by some model. If this rate requires more investments than could be financed by the self-generated savings of the country, the need for borrowing, and further, the size of foreign loans (assumed to be the only source of external finance) will be determined. The main question will, therefore, be: Under what conditions can the country maintain the given rate of growth and at the same time be able to service the ensuing debt?

In other words, the I.B.R.D. experts are interested in the capacity to service the debt under given development requirements. This is a point of view of a creditor rather than a debtor, which is natural if we look at the problem from the side of the Bank. However, it leaves much to be desired. In the first place, the approach does not tell the developing country how to fix its target growth rate, which will eventually be a function of loan conditions. The implicit advice given in the form of the necessity to stop borrowing if repayment is impossible within a reasonably long period in the future (e.g., over a generation) begs the problem. As was shown before, there are cases where a country could have ended with a larger capital stock without, rather than with, the loan. If this is so, we might face a situation in which the capacity to repay exists, but the net outcome is much below the level achievable in the absence of loans. Such situations do exist and they call for a reconsideration of the whole approach, in spite of its seemingly plausible foundations.(1)

Two variants of the I.B.R.D. model will be considered. The first was prepared about ten years ago by Gerald M. Alter for a conference organized at Rio de Janeiro, August 1957; by the International Economic Association. Alter's paper(2) is concerned with the determination of the critical level of the marginal propensity to save, required to ensure the realization of a certain rate of growth, and at the same time delimiting the cumulation of foreign debt within predetermined bounds. This model has been later simplified by Avramovic and his colleagues,(3) and used to study the so-called debt cycle. We shall give in the following sections a summary of these studies, then show how far the whole approach can be relied upon to deal with the real problems of economic development.

To the list of notations given in pages 4-5 of Part I of this Memo., we shall be in need of the following additional notation:

P = Population
p = rate of growth of population
g = rate of growth of G.D.P.
s_t= average propensity to save in year t
σ'= per capita marginal propensity to save
g'= rate of growth of per capita G.D.P.
L_t= new loans needed in year t.
D_t= Deficit of balance of payments in year t
Q_t= Cumulative debt at end of year t
n = target year for which the cumulative debt, or its ratio to G.D.P. reaches a certain (extreme) level.

(1) M.M.El-Imam: The Contribution of Foreign Loans to Economic Development. April, 1968 - T.U. 34, of specialized course on Financing of Development, organized jointly by I.N.P. and I.D.E.P., Cairo, 1968
(2) G.M. Alter: "The Servicing of Foreign Capital Inflow by Under developed Countries". in, H.S. Ellis and H.C.Wallich (eds): Economic Development for Latin America; Macmillan, N.Y. 1961.
(3) D. Avramovic, et al: Economic Growth and External Debt; I.B.R.D., Johns Hopkins, Baltimore, 1964.

Variables measured on a per capita basis will be denoted by the same symbols primed; e.g., Y', S', I'.

II. Alter's Model:

Alter prefers to work with per capita rather than aggregate variables. With constant rates of population growth, and of income growth, this implies a constant overall rate of growth of G.D.P.:

$$(1 + g) = (1 + g')(1 + p) \qquad (1)$$

However, this would generally lead to a variable overall marginal propensity to save, even though the per capita m.p.s. is assumed constant:

$$\sigma_t = \sigma' + \frac{p}{g} (s_{t-1} - \sigma') \qquad (2)$$

This means that if the m.p.s. is greater than the a.p.s. in the base year, the overall m.p.s. will be smaller than the per capita.

Starting from the definitions:

$$\sigma' = \frac{S'_t - S'_{t-1}}{Y'_t - Y'_{t-1}}, \qquad g' = \frac{Y'_t - Y'_{t-1}}{Y'_{t-1}} \qquad (3)$$

we obtain the difference equation:

$$S'_t = S'_{t-1} + \sigma' g' Y'_o (1 + g')^{t-1} \qquad (4)$$

The solution of this equation is:(4)

$$S'_t = S'_o + \sigma' Y'_o \left[(1 + g')^t - 1\right] \qquad (5)$$

Multiplying throughout by $P_t = P_o (1 + p)^t$, and denoting, as before, the a.p.s. in the base year by s, we obtain the formula for <u>overall savings</u>

$$S_t = \left[\sigma'(1 + g)^t + (s - \sigma') (1 + p)^t\right] Y_o \qquad (6)$$

(4) See Appendix A, equation (A/34)

On the other hand, overall investment required to realize income growth at the given rate can be determined by means of the capital coefficient:

$$I_t = \gamma g Y_o (1+g)^t \qquad (7)$$

Then the deficit of the current account of the balance of payments will be:

$$D_t = I_t - S_t = \left[(\gamma g - \sigma')(1+g)^t - (s - \sigma')(1+p)^t \right] Y_o \qquad (8)$$

If this quantity is positive, the country will be in no position to repay anything of the out-standing debt. In fact it has to borrow that amount together with interest charges on the out-standing debt. If according to the conditions of previous loans, a part of the principal is to be repaid, an equivalent amount should be borrowed.

Thus, the net flow of new loans in year t would be

$$L_t = r \cdot Q_{t-1} + D_t \qquad (9)$$

The cumulative debt outstanding at the end of year t will therefore be,

$$Q_t = Q_{t-1} + L_t$$

or

$$Q_t = (1+r)\, Q_{t-1} + D_t \qquad (10)$$

The solution of this difference equation for the case $g \neq r$ is:(5)

$$Q_t = \left[(\gamma g - \sigma') \left\{ \frac{(1+g)^{t+1} - (1+r)^{t+1}}{g - r} \right\} - (s - \sigma') \left\{ \frac{(1+p)^{t+1} - (1+r)^{t+1}}{p - r} \right\} \right] Y_o \qquad (11)$$

If $g = r$, we have:(6)

$$Q_t = \left[(\gamma g - \sigma')(t+1)(1+g)^t - (s - \sigma') \left\{ \frac{(1+p)^{t+1} - (1+g)^{t+1}}{p - g} \right\} \right] Y_o \qquad (12)$$

If $p = r$, a similar formula can be derived,

$$Q_t = \left[(\gamma g - \sigma') \left\{ \frac{(1+g)^{t+1} - (1+r)^{t+1}}{g - r} \right\} - (s - \sigma')(t+1)(1+g)^t \right] Y_o \qquad (13)$$

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(5) Appendix A, equation (A/40)
(6) Appendix A, equation (A/41)

The formula for cumulative debt can be used to answer the question: What are the conditions under which the country can ensure the service of debts, and at the same time go on with the implementation of its long term development activities?

If such conditions can be easily satisfied, there will be no problem in servicing the debt. On the contrary, the problem will be to absorb more loans, in the sense of finding more investment possibilities to be financed by means of these loans. On the other hand, if it is difficult to realize such conditions, it will be difficult, or even dangerous, to get envolved in any debt, however small.

Suppose that the values of the parameters, p, s, r, g and γ are given. Then knowing σ' we can calculate Q for any future date n considered by the planners. This will be a direct application of (11); but it does not help in formulating a decision. Therefore, Alter suggests that a certain restriction on the value of Q, then solve (11) to obtain the value of σ' necessary to realize that restriction.

III. Alternative Constraints on Debt:

The need for putting constraints on debt arises from the possibility that debt might grow on indefinitely, which is a situation impossible to sustain. Starting from a given base year deficit, debt will be growing so long as new deficits are either positive, or negative but smaller than what is necessary to provide for interest charges. At the beginning debt will be growing faster than income. Then there will be two possibilities. either it will remain to do so, or starts to grow at decreasing rates. We execlude the first alternative.

For the second, there must be a point at which the rate of growth of debt will fall down to that of income. In fact, if the rate of growth of debt at a point t is q_t, then it will be found that the condition that the ratio of debt to income is a maximum will be:

$$\frac{\partial}{\partial t}\left(\frac{Q_t}{Y_t}\right) = \frac{1}{Y_t^2}\left[Y_t \cdot q_t \cdot Q_t - Q_t \cdot g \cdot Y_t\right] = 0$$

which means that:

$$q_t = g \qquad (14)$$

If the rate of growth of debt diminishes after that, there will be a point at which that rate is equal to zero. This is, in fact the condition that debt will reach a certain maximum at this point.

$$q_t = 0, \qquad \text{or,} \qquad L_{t+1} = o \tag{15}$$

After this point the outstanding debt will start to decrease. There will arrive another point at which it will completely vanish:

$$Q_t = 0 \tag{16}$$

Afterwards the country has to worry no more about debts: It will become a net creditor rather than net debtor.

Thus, for a given point of time, n, we can find the value of σ' necessary to achieve any of the above three turning points. Let us therefore rewrite (11) as follows:

$$Q_n = \left[\sigma'\left(\frac{P-R}{p-r} - \frac{G-R}{g-r}\right) + \gamma g \left(\frac{G-R}{g-r}\right) - s\left(\frac{P-R}{p-r}\right)\right] Y_o \tag{17}$$

where,

$$P = (1+p)^{n+1}, \qquad R = (1+r)^{n+1}, \qquad G = (1+g)^{n+1} \tag{18}$$

The critical values of σ' can be obtained as follows:

1. The most restrictive condition is (16). Substituting (17) and solving for σ' we obtain:

$$\sigma' = \left[s\left(\frac{P-R}{p-r}\right) - \gamma g\left(\frac{G-R}{g-r}\right)\right] / \left[\left(\frac{P-R}{p-r}\right) - \left(\frac{G-R}{g-r}\right)\right] \tag{19}$$

2. A less restrictive condition is to assume that debt will reach absolute maximum in year n. we use (15), which means that:

$$L_{n+1} = r\,Q_n + D_{n+1} = 0$$

Hence,

$$\left[r\sigma'\left(\frac{P-R}{p-r} - \frac{G-R}{g-r}\right) + r\gamma g\left(\frac{G-R}{g-r}\right) - rs\left(\frac{P-R}{p-r}\right)\right] + \left[\sigma'(P-G) + \gamma g\, G - sP\right] = 0$$

This can be rearranged as follows:

$$\sigma'\left(\frac{pP-rR}{p-r} - \frac{gG-rR}{g-r}\right) + \gamma g\left(\frac{gG-rR}{g-r}\right) - s\left(\frac{pP-rR}{p-r}\right) = 0$$

Which can be solved to give:

$$\sigma' = \left[s\left(\frac{pP-rR}{p-r}\right) - \gamma g \left(\frac{gG-rR}{g-r}\right) \right] / \left[\left(\frac{pP-rR}{p-r} - \frac{gG-rR}{g-r}\right) \right] \qquad (20)$$

3. Finally, the mildest condition is to allow debt to reach a maximum rate of increase in year n, hence grow at the same rate as income, as in(14) This means that:

$$Q_{n+1} = (1+g)\ Q_{n+1}$$

or,

$$(1+r)\ Q_n + D_{n+1} = (1+g)\ Q_n$$

In other words:

$$(r-g)\ Q_n + D_{n+1} = 0$$

Substituting as before, we obtain:

$$\sigma' = \left[s \left\{ (p-g)\ P + (g-r)\ R \right\} - \gamma g(p-r)R \right] / (p-g)(P-R) \qquad (21)$$

Commenting on these formulae, Alter states that[7]

> "It is perhaps obvious that all other things being equal the target rate of increase of per capita income, compared with the rate that can be achieved in the absence of foreign capital inflow, may be put at a higher level and a larger volume of foreign capital inflow is permitted when:
> 1. the marginal savings ratio is higher;
> 2. the incremental capital-output ratio is lower,
> 3. the rate of population increase is lower,
> 4. the required rate of return on foreign capital inflow is lower,
> 5. the degree of independence of foreign capital that must be achieved within a given time period is lower;
> 6. the time period in which a given degree of independence must be achieved is longer".

No formal proof was given to verify the validity of these conclusions which were merely described as "obvious". What the above formulae indicate are certain critical values of σ', which could be compared with actual values in order to determine the possibility of achieving a certain degree of independence of foreign loans within a given period. However, some numerical examples can be calculated to illustrate the results, and the following is one given by Alter.

(7) op. Cit., p. 149

Example (1): Suppose that,

rate of growth of population, $p = 0.025$
rate of return on foreign capital, $r = 0.045$
initial average savings ratio, $s = 0.085$
capital/output ratio, $\gamma = 3.5$
target year, $n = 24$

It is required to investigate the marginal savings ratio σ' for three alternative targets of the rate of growth g' of per capita income, under the condition that external debt should reach a maximum in 24 years, (formula 20). Having done that, we can substitute in (17) to obtain Q_n, then relate it to Y_o or to aggregate investment. The results are summarized as follows:

Per capita Rate of Growth g'	Required marginal Savings Ratio σ'	Capital Inflow as Ratio of Initial National Income	Capital Inflow as Ratio of Aggregate Net Investment
0.005	0.23	0.23	0.07
0.01	0.28	0.61	0.14
0.02	0.31	1.42	0.22

Thus an increase in the marginal savings ratio from 0.23 to 0.31 (i.e., by about one-third), enables an increase in the rate of growth four times, and capital inflow could be increased over six times, while it remained possible to service it so that it reaches a maximum in 24 years. The rest of Alter's argument relates mainly to the risks befalling the creditor, which is natural for an expert on the supplying side.

IV. Implications of Alter's Model:

For any positive rate of growth of per capita income, we have $g > p$. Further, it is clear from (8), that for D_o to be positive, we should have, $\gamma g > s$. This means that $- sP > - \gamma gP$, where P is defined according to (18). Thus, if D is to become negative at some point of time, n+1 say, we should have:

$$0 > D_{n+1} > (\gamma g - \sigma')(G - P)$$

Which means that $\sigma' > \gamma g$. It follows that:

$$\frac{s}{\gamma} < g < \frac{\sigma'}{\gamma} \qquad (22)$$

Consider now equation (19) which is given in the same order suggested by Alter (Formula I, p. 159). First we notice that:

$$\frac{G-R}{g-r} - \frac{P-R}{p-r} = (1+r)^n \sum_{i=0}^{n} \left(\frac{1+p}{1+r}\right)^i \left[(1+g')^i - 1\right] > 0 \qquad (23)$$

Hence we multiply both numerator and denominator of (19) by - 1, to ensure that the latter is positive. This means that (19) can be rewritten as follows:

$$\sigma' = \gamma g + (\gamma g - s) \left(\frac{P-R}{p-r}\right) / \left[\frac{G-R}{g-r} - \frac{P-R}{p-r}\right] \qquad (24)$$

The factor multiplying (γg-s) is positive, which ensures that σ' satisfies (22).

Similar remarks can be made with respect to (20). It is evident that:

$$\frac{gG - rR}{g - r} = g \left(\frac{G-R}{g-r}\right) + R = \left[p+g'(1+p)\right]\left(\frac{G-R}{g-r}\right) + R$$

Further,

$$\frac{pP-rR}{p-r} = p \left(\frac{P-R}{p-r}\right) + R = p \left(\frac{P-R}{p-r}\right) + R$$

Hence

$$\frac{gG-rR}{g-r} - \frac{pP-rR}{p-r} = p\left[\frac{G-R}{g-r} - \frac{P-R}{p-r}\right] + g' (1+p) \cdot \left(\frac{G-R}{g-r}\right)$$

Again we have to multiply both numerator and denominator of (20) by (-1), to ensure that the latter is positive. Thus we can rewrite (20) as follows:

$$\sigma' = \gamma g + (\gamma g-s)\left(\frac{pP-rR}{p-r}\right) / \left(\frac{gG-rR}{g-r} - \frac{pP-rR}{p-r}\right) \qquad (25)$$

Finally, equation (21) can be rewritten as:

$$\sigma' = \gamma g + (\gamma g-s) \left[(g-r)R - (g-p)P\right] / (g-p)(P-R) \qquad (26)$$

Alternatively it can be rewritten as:

$$\sigma' = \gamma g + (\gamma g-s)\left[R - (g-p) \left(\frac{P-R}{p-r}\right)\right] / (g-p) \left(\frac{P-R}{p-r}\right) \qquad (27)$$

Comparing these forms, we find that they take the general formula:

$$\sigma' = \gamma g + (\gamma g - s) X_{jn} \qquad (j=1,2,3) \qquad (28)$$

Given that $\gamma g > s$, it follows that $\sigma' > \gamma g$ if X is positive. The factors X are;

$$X_{1n} = \frac{\frac{P-R}{p-r}}{\frac{G-R}{g-r} - \frac{P-R}{p-r}}$$

$$X_{2n} = \frac{\frac{pP-rR}{p-r}}{\frac{gG-rR}{g-r} - \frac{pP-rR}{p-r}} \qquad (29)$$

$$X_{3n} = \frac{R}{(g-p) \cdot (\frac{P-R}{p-r})} - 1$$

It is obvious that both

$$X_{1n} > 0 \quad \text{and} \quad X_{2n} > 0 \qquad (30)$$

Since X_{3n} is the difference between two positive quantities, its sign has to be further investigated . On the other hand two properties can be expected a priori:

1) The factor X is smaller for the milder conditions, which means that, for any $n > 0$:

$$X_{1n} > X_{2n} > X_{3n} \qquad (31)$$

2) The factor X is smaller for larger values of n; i.e., it is a decreasing function of n:

$$X_{j,n+1} > X_{jn} \qquad (j = 1,2,3) \qquad (32)$$

To prove these properties we introduce the following abbreviated notation:

$$a = \frac{P-R}{p-r}, \qquad b = \frac{G-R}{g-r}$$

$$A = \frac{P(1+p) - R(1+r)}{p - r}, \qquad B = \frac{G(1+g) - R(1+r)}{g - r}$$

$$c = A - a = \frac{pP-rR}{p-r}, \qquad d = B-b = \frac{gG-rR}{g-r}$$

$$\left.\begin{aligned} &\therefore\ c = ra + P, && d = rb + G \\ &C = \frac{P(1+p)p - R(1+r)r}{p - r}, && D = \frac{G(1+g)g - R(1+r)r}{g - r} \\ &\therefore\ C = (1+r)c + pP, && D = (1+r)d + gG \end{aligned}\right\} \quad (33)$$

It follows that:

$$X_{1n} = \frac{a}{a-b}, \qquad X_{2n} = \frac{c}{d-c}$$

For the first inequality in (31) to hold, we should have

$$(ad-ac) - (ca-cb) = a(B-b) - b(A-a) = aB - Ab > 0$$

But:

$$\begin{aligned} aB - Ab &= a\left[(1+r)b + G\right] - \left[(1+r)\ a + P\right] b \\ &= aG - bP \\ &= (1+g)^{n+1} \sum_{i=0}^{n} (1+r)^i (1+p)^{n-i} - (1+p)^{n+1} \sum_{i=0}^{n} (1+r)^i (1+g)^{n-i} \\ &= (1+g)^n (1+p)^{n+1} \left[\left(\frac{1+g}{1+p}\right) \sum_{i=0}^{n} \left(\frac{1+r}{1+p}\right)^i - \sum_{i=0}^{n} \left(\frac{1+r}{1+g}\right)^i\right] \\ &= (1+g)^n\ (1+p)^{n+1} \sum_{i=0}^{n} \left(\frac{1+r}{1+g}\right)^i \left[(1+g')^{i+1} - 1\right] \end{aligned}$$

For all $i \geqslant 0$, the last expression between square brackets is positive It follows that:

$$aB > Ab \qquad (34)$$

which ensures the first inequality in (31). In other words, the most stringent condition (1) requires a higher m.p.s. than the milder condition (2).

Further, if we subtract from both sides of (34) the quantity aA, we find that:

$$\frac{a}{b - a} > \frac{A}{B - A}$$

This means that property (2), stated by (32) in satisfied for X_{1n}, determining the m.p.s. for condition (1). To prove that this applies for condition (2), so that:

$$\frac{c}{d - c} > \frac{C}{D - C}$$

we have to prove the validity of a formula similar to (34), namely:

$$cD > Cd \qquad (35)$$

Now,

$$cD - Cd = c\left[(1+r)d + gG\right] - \left[(1+r)c + pP\right] d$$

$$= (ra + P)\ gG - pP(rb + G)$$

$$= rg\ (aG - bP) + (g-p)\ P\ (rb + G)$$

$$= rg\ (aB - Ab) + (g - p)\ Pd$$

All factors in both terms are psotive; hence, (35) is satisfied. Hence property (2) applies, and (32) is satisfied for condition (2).

To show that the second of inequalities (31) is true we evaluate X_{2n} - X_{3n}. For this purpose we rewrite them as follows:

$$X_{2n} = \frac{R + pa}{gb - pa}\ , \qquad X_{3n} = \frac{R - (g-p)\ a}{(g-p)\ a}$$

The sign of the difference depends on the sign of its numerator:

$$(g-p)\ a\ (R+pa) - (gb-pa)\left[(R - (g-p)\ a\right]$$

$$= (g-p)\ a\ (pa + gb - pa) + R\ (ga - pa + pa - gb)$$

$$= g\left[(g-b)\ ab + R(a-b)\right] = g\left[a\ (gb+R) - b\ (pa+R)\right]$$

$$= g\ (ad - bc) = g\ (aB - Ab)$$

This means that $X_{2n} > X_{3n}$.

Finally, for t = n + 1, we have:

$$X_{3,n+1} = \frac{(1+r)R - (g-p)A}{(g-p)\ A} = \frac{(1+r)R}{(g-p)\ (1+r)\ a + p} - 1$$

$$= \frac{R}{(g - p)\ a + \frac{P}{1+r}} - 1$$

It follows that (32) applies for case (3); i.e., it is true for all $j = 1,2,3$. The two properties are actually holding as expected.

However, as indicated before formula (30) need not apply in case (3). In fact if $r > g$ hence $r > p$ and $R > P$, we have:

$$X_{3n} = \frac{(r-p)R - (g-p)(R-P)}{(g-p)(R-P)} = \frac{(r-g)\ R + (g-p)P}{(g-p)(R-P)}$$

which is positive. But as soon as r falls below g (whether above or below p), the sign of X_{3n} is indeterminate. For $n = 0$, we have:

$$P = (1+p), \qquad P-R = p-r$$

hence,

$$X_{30} = \frac{(p-r)(1+p) + (r-g)(p-r)}{(g-p)(p-r)} = \frac{p + r + (1+g)}{g-p} > 0$$

On the other hand, as n approaches infinity X_{3n} becomes negative. Since X_{3n} is a monotonically decreasing function of n, there is a value n^* of n such that:

$$X_{3n} < 0 \qquad \text{for} \qquad n > n^*$$

In order that:

$$X_{3n} = \left(\frac{p-r}{g-p}\right)\left(\frac{R}{P-R}\right) - 1 \leqslant 0$$

we should have:

$$\frac{p-r}{g-p} \leqslant \frac{P-R}{R}, \qquad \text{or,} \qquad \frac{g-r}{g-p} \leqslant \frac{P}{R}$$

This means that:

$$n^* = \frac{\log\ (g-r) - \log\ (g-p)}{\log\ (1+p) - \log\ (1+r)} - 1 \qquad (36)$$

Example (2): Suppose: $p = 0.03$, $g = 0.07$. If $r = 0.015$, i.e., smaller than both, formula (36) gives $n^{x} = 20.6$. Thus if $n = 25$, $X_{3n} = -0.1925$, while for $n = 30$, $X_{3n} = -0.349$. Assuming that $s = 0.15$ and $\gamma = 3.5$, then $\gamma g = 0.245$. Substituting in (26), the value of σ' is found to be 0.227 and 0.212 respectively. If $r = 0.04$ (i.e., lying between p and g), then $n^{x} = 28.7$. For $n = 30$, $X_{3n} = - 0.034$, which means that $\sigma' = 0.242$.

To conclude; the three conditions suggested by Alter for maintaining a predetermined rate of growth while still generating enough resources to serve the debt, are found to satisfy the two properties defined by (31) and (32). The first two conditions satisfy also property (30). which ensures that (28) gives a value of σ' greater than γg, thus satisfying (22). This is not necessarily the case with the mildest condition (3). We have, therefore, to examine more closely the conditions given by Alter.

V. The Inadequacy of Alter's Conditions:

In commenting on the three conditions discussed above, Alter believes that:(8)

"To put it more exactly a country which is capable of repaying debt completely in twenty-five years will have reached a maximum volume of external indebtedness somewhat earlier and will have reduced the rate of net capital inflow even earlier. In other words, one can state a more liberal repayment requirement either in terms of complete independence to be achieved over a long time period or a less degree of independence to be achieved over a shorter time period."

The implications would, therefore, be that if a less degree of independence is accepted (as a shorter-term target), the full degree of independence will automatically follow:

If we accept this argument we might get some value σ' for a value $n < n^{x}$ given by (36) which ensures the weakest degree of independence without calling forth complete independence. The statement should be qualified by an explicit discussion of the necessary and sufficient conditions. We shall try to do that for the three degrees considered.

1) The first condition ensures full independence from debt. Hence we need not worry about it any further.

2) The second condition is satisfied with a maximal point on the curve. The first-order condition is stated, but we have to discuss the validity of the second-order condition to make sure that we have hit the maximum. Even if the latter is true the curve might completely fall above the horizontal axis, and no complete independence is acheived. We have to

(8) Alter; op.cit., p. 150

make sure that the solution of the first order condition for σ' is both necessary and sufficient for obtaining a maximum and for realizing full independence some time later.

Now the necessary (first-order) condition was stated to be:

$$Q_{n+1} - Q_n = I_{n+1} = rQ_n + D_{n+1} = 0$$

This means that:

$$D_{n+1} = - rQ_n < 0 \qquad (37)$$

which was found to lead to condition (22), namely

$$s < \gamma g < \sigma' \qquad (38)$$

The second-order condition requires that:

$$D_{n+2} - D_{n+1} < 0$$

But:

$$D_{n+2} - D_{n+1} = p\, D_{n+1} + (g-p) \ (\gamma g - \sigma')$$

Since $g > p$, it is clear from (37) and (38) that the second-order conditio[n] is satisfied, once the first-order condition holds.

However, even though the point achieved is that of a maximum, it does not necessarily follow that debt will vanish at some later date. To investigate this question, let us introduce the following notation for values at a point $t > n$:

$$\left.\begin{aligned} G' &= (1+g)^t = G(1+g)^{t-n} > G \\ P' &= (1+p)^t = P(1+p)^{t-n} > P \\ X_{1t} &= \left(\frac{P'-R'}{p-r}\right) / \left(\frac{G'-R'}{g-r} - \frac{P'-R'}{p-r}\right) \end{aligned}\right\} \qquad (39)$$

By (11) we have:

$$\begin{aligned} Q_t &= (\sigma' - s) \left(\frac{P'-R'}{p-r}\right) - (\sigma' - \gamma g) \left(\frac{G'-R'}{g-r}\right) \\ &= \left(\frac{P'-R'}{p-r}\right) \left[(\gamma g - s) - (\sigma' - \gamma g) \frac{1}{X_{1t}} \right] \end{aligned}$$

If this is to be non-positive we should have:

$$0 < (\gamma g - s)\, X_{1t} \leqslant (\sigma' - \gamma g) \qquad (40)$$

It is necessary for the realization of these inequalities that (30) and (38) should hold. We still need, however, a sufficient condition. It is clear from (40) that:

$$\sigma' \geqslant \gamma g + (\gamma g - s)\; X_{1t}$$

Comparing this with (28) it follows that we should have:

$$X_{2n} \geqslant X_{1t} \qquad (41)$$

To show that this is true we evaluate the limit of X_{1t} when t tends to infinty. It can be easily shown that if $r < g$, this limit is zero, which means that (40) can be satisfied for large enough values of t. But if $g < r$, the limit is $\frac{r-g}{g-p}$. Substituting this value we get:

$$\frac{1}{X_{1\infty}} - \frac{1}{X_{2n}} = \frac{g-p}{r-g} - \left[\left(\frac{rR-gG}{r-g}\right)/\left(\frac{rR-pP}{r-p}\right) - 1\right]$$

$$= \frac{r-p}{rR-pP}\left[\frac{r-p}{r-g}\left(\frac{rR-pP}{r-p}\right) - \left(\frac{rR-gG}{r-g}\right)\right]$$

$$= \left(\frac{r-p}{r-g}\right)\left(\frac{gG-pP}{rR-pP}\right) > 0$$

In this case also the debt will vanish at some large enough value of t. Hence Alter's condition, expressed by (25) is both a necessary and sufficient condition for reaching a maximum point and at the same time ensuring the possibility of completely paying off the debt at some later date.

3) The third condition (of a maximum growth rate of the debt ratio) has been already shown to allow cases of an ever increasing debt. First we notice that equation (21) can be rearranged as follows:

$$(\sigma' - s)\,(g-p)\,\left(\frac{P-R}{p-r}\right) = (\gamma g - s)R \qquad (42)$$

This means that:

$$\text{if} \quad \gamma g > s, \qquad \text{then,} \qquad \sigma' > s \qquad (43)$$

To ensure that g is a maximum rate of growth of Q/Y, we have to make sure that:

$$Q_{n+2} < (1+g)\, Q_{n+1}$$

or,

$$(g-r)\, Q_{n+1} > D_{n+2} \qquad (44)$$

Now,

$$(g-r)\, Q_{n+1} - D_{n+2} = (g-r)(\gamma g-s)\left(\frac{P-R}{p-r}\right)$$

$$- (g-r)(\sigma' - \gamma g)\left(\frac{G-r}{g-r} - \frac{P-R}{p-r} + \frac{gG-rR}{g-r} - \frac{pP-rR}{p-r}\right)$$

$$- (\gamma g-s)(P+pP) + (\sigma' - \gamma g)(G-P+gG-pP)$$

$$= (\gamma g-s)(1+r)\left[(g-p)\left(\frac{P-R}{p-r} + \frac{P}{1+r}\right) - R\right]$$

$$+ (\sigma' - \gamma g)(1+r)(g-p)\left(\frac{P-R}{p-r} + \frac{P}{1+r}\right)$$

$$= (1+r)(\sigma' - s)(g-p)\left(\frac{P-R}{p-r} + \frac{P}{1+r}\right) - (1+r)(\gamma g-s)\, R$$

Using (42) and (43), we find that:

$$(g-r)\, Q_{n+1} - D_{n+2} = (\sigma' - s)(g-p)\, P > 0$$

which means that the necessary condition is also sufficient.

Finally to investigate the possibility of debt repayment, we should have, as in (41)

$$X_{3n} > X_{1t} \qquad (45)$$

If $r < g$, the limit $X_{1\infty}$ is zero, which means that X_{3n} should be positive. As shown before, this need not be true for values of $n > n^{*}$, defined by (36). Debt will be increasing indefinitely. On the other hand, if $r > g$, the limit of X_{1t} is $\frac{r-g}{g-p}$. Hence:

$$X_{3n} - X_{1\infty} = \frac{R\,(r-p)}{(g-p)(R-P)} - \left(1 + \frac{r-g}{g-p}\right)$$

$$= \left(\frac{r-p}{g-p}\right)\left(\frac{R}{R-P} - 1\right) = \left(\frac{r-g}{g-p}\right)P > 0$$

In this case the condition for future repayment of debt is satisfied.

To conclude, conditions (1) and (2) ensure complete repayment of debt. In both cases σ' exceeds γg by a fraction sufficient enough for that purpose. This is not the case with condition (3) if the rate of interest falls below the rate of growth, and n is relatively large.

VI. The Shahid Husain Model:

A more recent formulation of Alter's model was prepared by S. Shahid Husain in the monograph produced lately by the I.B.R.D. group. The model remains essentially the same, apart from the use of overall rather than per capita parameters. Thus the m.p.s. which is assumed to remain constant is the overall propensity: This is more acceptable in cases where institutional, rather than personal, savings dominate the scene. Incidentally, this helps to simplify the formulae. In fact, the present case can be considered as a special case of Alter's model with $p = 0$ hence $P = 1$.

Given that the overall marginal propensity to save σ is constant, we can define it in the same manner as σ' in (3). The saving function is similar to (4) and (5)

$$S_t = S_{t+1} + \sigma g(1+g)^{t-1} Y_o$$

whose solution is:

$$S_t = S_o + \sigma \left[(1+g)^t - 1 \right] Y_o \qquad (46)$$

The investment equation remains as in (7)

$$I_t = \gamma g(1+g)^t \; Y_o$$

Hence the deficit of the current account is:

$$D_t = I_t - S_t = \left[(\gamma g - \sigma)(1 + g)^t + (\sigma - s) \right] Y_o \qquad (47)$$

which is similar to (8). Using (9) and (10)the formula replacing (11) would be:

(9) D. Avramovic, et al; op. cit; Mathematical Appendix, pp. 188-192.

$$Q_t = \left[(\gamma g - \sigma)\left\{\frac{(1+g)^{t+1} - (1+r)^{t+1}}{g - r}\right\} + (\sigma - s)\left\{\frac{(1+g)^{t+1} - 1}{r_g}\right\}\right]Y_o \quad (48)$$

If g = r, equation (12) becomes:[10]

$$Q_t = \left[(\gamma g - \sigma)(t + 1)(1 + g)^t + (\sigma - s)\left\{\frac{(1 + g)^{t+1} - 1}{g}\right\}\right]Y_o \quad (49)$$

The emphasis in this study was put on the determination of the critical rate of interest which would help to ensure an equiproportionate growth in both debt and income. This is Alter's mildest condition which had been shown before to be rather deficient. To derive the necessary formula, we use the condition:

$$Q_{n+1} = (1 + g) \; Q_n$$

or

$$(r - g) \; Q_n + D_{n+1} = 0$$

In other words:

$$(r - g)\left[(\gamma g - \sigma)\frac{R - G}{r - g} + (\sigma - s)\;\frac{R - 1}{r}\right]$$

$$+ (\gamma g - \sigma) \; G + (\sigma - s)$$

$$= (\gamma g - \sigma) \; R + (\sigma - s)\left[R - \frac{g}{r}(R - 1)\right] = 0$$

Solving for r, we find that:

$$r = \left(\frac{\sigma - s}{\gamma g - s}\right) g \left(\frac{R - 1}{R}\right) \quad (50)$$

Since the R.H.S. involves r, we can be satisfied with an approximation which holds true for infinitely large values of n, namely

$$\frac{R - 1}{R} = 1 - \frac{1}{R} = 1 \qquad \text{app.}$$

Hence, the upper bound on r is:

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(10) The formula given by Husain, (Ibid., p. 190). seems to be derived from Alter's formula which was based on Y_1 rather than Y_o. Hence n has to be replaced by (n + 1).

$$\hat{r} = \left(\frac{\sigma - s}{\gamma g - s}\right) g \tag{51}$$

Comparing this bound with what we have already obtained in Part I of the present memo.,

$$r^{*} = \frac{\sigma}{\gamma} \tag{52}$$

we notice that both rates belong to a more general formula:

$$r = \left(\frac{\sigma - c}{\gamma g - c}\right) g \tag{53}$$

where c is some non-negative constant. In the case of (51), c = s; while is (52), c = 0. It follows that:

$$\frac{\partial r}{\partial g} = \frac{c\,(c - \sigma)}{(\gamma g - c)^2} \tag{54}$$

If $c < \sigma$, this expression will be negative, Since it is usually assumed that $s < \sigma$, it can be seen that the higher the rate of growth g, the lower will be critical rate. In our case, where c = 0, the rate of change is equal to zero.

Further, comparing (51) and (52) we notice that:

$$\frac{\hat{r}}{r^{*}} = 1 + \frac{s(\sigma - \gamma g)}{\sigma(\gamma g - s)} \tag{55}$$

Now, if there will be any need for debt at all we should have $\gamma g > s$. On the other hand, if the deficit is to vanish at some future date (which is a necessary condition for the possibility of ending the debt cycle), we should have $\sigma > \gamma g$. We thus obtain a condition similar to (22) or (38), expressed in terms of σ rather than σ':

$$s < \gamma g < \sigma \tag{56}$$

This means that the R.H.S. of (55) is greater than unity, or:

$$\hat{r} > r^{*} \tag{57}$$

It should be remembered, however, that this is an asymptotic relationship, which might be violated at lower values of n. But, since the I.B.R.D. group had been satisfied with (51) rather than (50), one would expect policy recommendations which can lead to an ever increasing debt, and at the same time leading to a capital smaller than that which could have been achieved in the absence of the loan.

It can be also expected that the present model suffers from the same defect noticed before with respect to Alter's case (3). Solving (51) for σ, we obtain the critical (minimal) value:

$$\sigma = s + \frac{r}{g} (\gamma g - s) \tag{58}$$

Subtracting γg from both sides we can rewrite this as follows:

$$\sigma = \gamma g + (\gamma g - s) \left(\frac{r - g}{g}\right) \tag{59}$$

The second condition of (56) holds only if:

$$r > g \tag{60}$$

exactly as in Alter's model. In the opposite case, the debt can grow on indefinitely and the concept of the debt cycle vanishes. If we prefer to use the exact formula (50) instead of (51), we would obtain in place of (59):

$$\sigma = \gamma g + (\gamma g - s) \left[\left(\frac{r}{g} \left(\frac{R}{R - 1}\right) - 1 \right] \tag{61}$$

which means that we should have:

$$g < \frac{rR}{R - 1} \tag{62}$$

This defines the upper bound of g once r and n are given.

The factor multiplying $(\gamma g - s)$ in (61) is the value of X_{3n} for the special case $p = 0$. Condition (62) can be solved for n as before, for given values of $g > r$. The formula corresponding to (36) would be:

$$n^{*} = \frac{\log g - \log (g - r)}{\log (1 + r)} - 1 \tag{63}$$

This illustrates the contradiction of the I.B.R.D. model. The critical rate (51) gives a value of r greater than g, which satisfies (60). But it calls for accepting values of $r < \hat{r}$, consequently values of $r < g$. But in such cases any value of $n > n^{*}$ defined by (63) would lead to an ever increasing debt. Since (51) was actually derived on the basis of an infinite value of n, we might be easily led to fix the values of the parameters (e.g., σ) in such a way that debt grows indefinitely. The whole concept of closing the debt cycle will fall down.

VII. The Debt Cycle:

Let us investigate more closely the concept of debt cycle as envisaged by Avramovic and Hayes. The whole concept concentrates on the idea that there should be a future (probably finite) point of time at which debt would be completely repaid while a given rate of growth is maintained throughout. This means that the debt curve takes a parabolic form, with a point of inflexion and a point of absolute maximum. This behaviour has been summarized by Avramovic[11] in a thre-stages process, the so-called "The growth-cum-debt" process:

1) During the first stage there exists a deficit in the balance of current account due to excess of investment over savings. Hence loans are required to finance the investments necessary to maintain the growth rate. Foreign indebtedness increases more rapidly than these needs owing to interest charges on prior debt. But it is assumed that the investment - savings gap narrows gradually.

2) When the latter gap closes a second stage begins. Debt is not required to finance investment while the surpluses generated help to cover a part though not all of interest obligations. Hence the debt still grows to meet the remaining interest obligations, and its rate of growth will go down.

3) The second stage ends when the surplus is just enough to cover interest charges. From that point onwards a third stage begins during which the increasing surplus covers, besides interest charges, a part of the principal itself. Debt falls gradually until it completely vanishes, and the cycle is closed.

If the country cannot escape stage one, it will be inadvisable to attempt maintaining the given growth rate through foreign finance. The investments required to maintain that rate will always exceed savings, and the deficit increases indefinitely. For this reason Hayes calls for considering cases where the initial savings rate(s) is lower than the required investment rate (γg), but the marginal savings rate(σ) is higher than the required invest-

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(11) Avramovic, op. cit, pp. 53-55

(12) Ibid., pp. 168 - 171

ment rate. This means that the country should pass through at least three of the following phases

1) As long as deficit exists, debt grows at a rate faster than the rate of interest.

2) When deficit disappears debt grows at a rate equal to the rate of interest. If this latter is greater than g, debt will still grow faster than income.

3) The same will still occur even when some surplus exists, if such a surplus covers a part of interest charges, leaving a remainder greater in its ratio to debt than the growth rate itself.

4) Then follows a point at which the rate of growth of debt will be just equal the rate of growth of income.

5) Debt continues to grow but at a rate lower than that of income. Hence the ratio of debt to income grows smaller.

6) The surplus exceeds interest obligations, and debt starts to decrease both alsolutely and relatively.

Hayes' phases (2)-(5) are, in fact, phases of Avramovic's second stage. The least that the country would aim at is stage (4). Formula (51) was obtained on the assumption that such a stage would be obtained at least during an infinitely large value of n.

The six-stages process is based on the assumption that $r > g$. It can be easily seen that a similar process can be obtained forthe case $r < g$. The following table summarizes the six phases and their conditions in both cases:

	Phase	Condition on D_t
I.	Case $r > g$	
(1)	$Q_t > (1+r)Q_{t-1} > (1+g)Q_{t-1}$	$D_t > 0$
(2)	$(1+r)Q_{t-1} = Q_t > (1+g)Q_{t-1}$	$D_t = 0$
(3)	$(1+r)Q_{t-1} > Q_t > (1+g)Q_{t-1}$	$0 > D_t > -(r-g)Q_{t-1}$
(4)	$(1+r)Q_{t-1} > Q_t = (1+g)Q_{t-1}$	$0 > D_t = -(r-g)Q_{t-1}$
(5)	$(1+r)Q_{t-1} > (1+g)Q_{t-1} > Q_t > Q_{t-1}$	$0 > -(r-g)\ Q_{t-1} > D_t$
(6)	$Q_{t-1} > Q_t \geqslant 0$	$0 > -rQ_{t-1} > D_t$
II.	Case $r < g$	
(1)	$Q_t > (1+g)Q_{t-1} > (1+r)Q_{t-1}$	$D_t > (g-r)Q_{t-1} > 0$
(2)	$Q_t = (1+g)Q_{t-1} > (1+r)Q_{t-1}$	$(g-r)Q_{t-1} = D_t > 0$
(3)	$(1+g)Q_{t-1} > Q_t > (1+r)Q_{t-1}$	$(g-r)Q_{t-1} > D_t > 0$
(4)	$(1+g)Q_{t-1} > Q_t = (1+r)Q_{t-1}$	$D_t = 0$
(5)	$(1+g)Q_{t-1} > (1+r)Q_{t-1} > Q_t > Q_{t-1}$	$-r\ Q_{t-1} < D_t < 0$
(6)	$Q_{t-1} > Q_t \geqslant 0$	$D_t < -r\ Q_{t-1} < 0$

In case (I), Avramovic's first stage corresponds to phase (1), while his second stage corresponds to phases (2)-(5). It is clear that it is not sufficient to require that the country should fall in the second stage; it should also escape phases (2) and (3) belonging to it. Hence the minimum requirement that at least phase (4) should be achieved.

In case (II), phase (4) becomes phase (2) and falls within stage one. Therefore, to require that debt grows at a rate equal to the growth rate of income does not ensure that deficit will vanish. It follows that the country might achieve phase (2) but never move into phase (4). This explains the findings of the previous section about the insufficiency of the condition of equiproportionate growth whenever $g > r$.

Even if we assume that $r > g$, the condition of equiproportionate growth is not a realistic assumption. Since this condition is assumed to hold exactly at n approaching infinity, we have to replace it by the more workable assumption that at $n = \infty$, debt should completely vanish. Putting $Q_{\infty} = 0$, and solving for σ as before, we have to evaluate the limiting value of X_{1n} which was found before to be $(\frac{r-g}{g})$, since $p = 0$. It follows that

$$\sigma \geqslant \gamma g + (\gamma g - s)\left(\frac{r-g}{g}\right)$$

This can be solved to give the upper bound of r ($> g$),

$$r \leqslant \left(\frac{\sigma - s}{\gamma g - s}\right) g$$

which is equal to $\hat{r}$ given in (51). In other words if $r = \hat{r}$, debt will vanish at $n = \infty$, which means that it would have achieved a maximum and a growth rate equal to g at some large, but finite value of n. This being true at the equality sign, it will remain also true for the inequality, which subsumes cases of $r < g$.

VIII. The I.B.R.D. Numerical Example:

To illustrate the working of the above model, Avramovic has suggested a set of values for the parameters envolved which would be considered representative of conditions prevailing in a large number of developing countries.(13) These values are:

$s = 0.10$ $\sigma = 0.20$ $\gamma = 3.0$
$g = 0.05$ $r = 0.06$

Assuming that $Y_0 = 1000$ (e.g., million dollars), we can calculate the main aggregates as shown in Tables (B/1) - (B/2), of appendix (B). The three stages of Averamovic's debt cycle are:

(1) The first stage covers the years 0-14 during which debt increases to meet the (decreasing) deficit needed to finance investment as well as interest charges on previous debt. This represents phase (1) in Hayes' process.

(2) The second stage covers the period 15-23. In year 16, the surplus (9.1) is greater than $(r-g)\ Q_{t-1} = 0.01 \times 779.9 = 7.8$. Phase (5) begins and ends in year 23 when debt reaches the maximum of 962.4.

(3) The third stage (or phase 6) goes on until year 36, during which the surplus generated helps to pay the remainder of the debt and leave a net credit of 46.6

The fact that the present case leads to maximum indebtedness in 25 years has led Avramovic to call it, the "Single generation" variant. To warn against expecting that such a variant, based on average parameter values, would apply mechanically to actual situations, Avramovic has drawn attention to some qualifications:(14)

(13) Avramovic op. cit., pp. 57-61

(14) Ibid., pp. 61-64.

(1) Reaching a maximum in 25 years, the country need not go on repaying debts. It can go on borrowing and raising its growth rate.

(2) If there are other debts at the beginning of the base year, the cycle will become longer.

(3) The actual cycle will depend on the true values of the parameters, which calls for a study of the statistics of the given country. Further, the values of the parameters might change through the development process. For example the capital-output ratio might go down, thus improving the situation, and attracting more foreign capital.

(4) No discussion was made of the foreign trade gap. But it was mentioned that if exports are to grow at 4% only, a lot of import-saving has to take place. We shall deal with this problem later.

While such qualifications can be easily justified, they need not be the most important. We can enumerate a number of other considerations which have to be carefully studied before accepting the above model as a model of rational economic decisions. In what follows we refer to numerical results given in Appendix B.

(1) Planners are tempted to consider the finance of investment as divorced from the ensuing debt problem. Thus judging a 15-year plan, they would be happy to substitute an average growth rate of 5% for a lower one of 4.1% obtained without recourse to loans. This raises income in year 15 from 1816.4 to 2078.9, by 262.5 or 14.5%. A tendency to accept that rate might be fortified by the fact that in year 15 no further loans are required to finance investment; in fact a series of surpluses begins. The present model helps to show that the process of repayment goes on for more than another 20 years. But it is possible since the critical rate of interest is:

$$\hat{r} = \frac{0.20 - 0.10}{0.15 - 0.10} \times 0.05 = 0.10$$

which is much higher than the actual rate.

(2) Further, if income in year 15 is higher by 262.5, this means that the country was able to obtain an extra capital amounting to 3.0 x 262.5 = 787.5. The last column of table (B/1), shows that foreign capital actually obtained for finance of investment is 421.1, or nearly 50% only of that amount. The remainder was generated

domestically by the increased power of the economy itself. Judging by these results, the planner will be quite happy by setting the 5% target and declaring independence from foreign finance in 15 years, and at the same time meeting only about 50% of extra capital needs from abroad.

(3) Such conclusions are in fact unwarranted. But neither is the Avramovic conclusion that the year 36 is to be considered as "a bridge between poverty and self-sustained growth".(15) As can be seen from table (B/3), the increase in income in year 37 is 136.3 only or 2.3% above the level achieved with-out loans. In fact, if the country maintains the 5% rate three years more, its income will be lower by 1%. (But it would possess some capital abroad which could be used to make good the difference). The gain in year 37 is a function of our formula based on the critical rate of interest:

$$r^{*} = \frac{\sigma}{\gamma} = \frac{0.20}{3.0} = 0.0667$$

The smallness of the difference $(r^{*} - r)$ is responsible for the rather negligible benefit; a fact completely ignored by the I.B.R.D. group.

(4) A more startling fact is the amount of capital outflow, especially in connection with interest charges, required to pay back the extra 421.1 of capital needed from abroad. In year 14 the debt outstanding is 739.4. which means that a difference of 318.3 was to be borrowed in order to finance interest charges. Even, after exporting some surpluses, the debt grows to a maximum of 962.4 in year 23, or more than double the capital requirements. If we add up the column of interest charges in table (B/1), its total is found to be the incredible amount of 1334.8 or more than three times the actual capital needs.(16) Even if we try to account for differences in dates by calculating the present values in the base year of the two series (discounted at the rate 6%) we find that interest is 444.5 while capital needs are 327.0 only. Interest will be 135% of the principal. This indicates clearly the impact of the compound-interest nature of the process.

(15) Ibid, p. 61

(16) This can be also obtained as follows. From the last column of Table (B/1), we find that the cumulative deficit reaches - 1381.4 in year 36. Subtracting from it the export of 46.6, we obtain 1344.8.

(5) It follows that the country in attempting to procure 421.1 extra capital, has to pay back 421.1 + 1334.8 = 1755.9, or more than four times its needs. We then face the paradox that a country in need of capital has to become a very big exporter of capital in order to be able to borrow safely. If it so does, it will be sure that its income (hence capital) will exceed what it could obtain without importing or exporting capital, by2.3%. This aspect of the I.B.R.D. model is quite serious, but it has not been mentioned at all. Its neglection is probably the reason for many of the difficulties faced by countries embarking on foreign debts then discovering that they cannot get rid of them, with little done on fostering development.

(6) The next question is to see how the country would be able to export that amount of capital. It has been assumed by Avramovic that a reasonable rate of growth of exports would be 4% per annum. If exports are thus determined and if the deficit is already given, imports have to satisfy the identity:

$$M = D + X$$

Table (B/2) gives the estimates thus obtained, as X_1 and M_1. Considering this latter, we immediately observe the need for the import-saving program suggested by Avramovic and mentioned before. In fact, the marginal propensity to import over the whole period is as low as:

$$\frac{222.7 - 150.0}{6081.4 - 1000.0} = \frac{72.7}{5081.4} = 0.014$$

The average propensity to import has to drop sharply from 15% in the base year to 3.66% in year 37. This can be hardly defended especailly in view of the increasing importation needs required for accelerated development.

Let us therefore take another alternative assuming that total (direct and indirect) needs for consumption and investment are 0.10 and 0.40 respectively. This gives estimates M_2 of imports, from which exports X_2 could be derived as in Table (B/2). The average propensity to import would fall slightly to 14.2%, while the marginal remains constant at 14%. These estimates seem more reasonable, but they would imply that exports should grow at an average rate of 6.6%, to reach a level about 11 times its base year level, during 37 years. Another undesirable feature would be that the rate of growth of exports should be as high as 9.5% at the beginning of the period, then fall gradually to 5.4% One cannot, therefore, divorce the savings - investment gap from the foreign trade gap. Not only a high marginal propensity to save should be aimed at, but the feasibility of transfering such an increase into export channels has to be considered. It might be found necessary that a gradually increasing marginal propensity to save should be aimed at. However, this would complicate the model and elongate the debt cycle.

(7) Leaving aside this transfer aspect, we have to question the actual benefit from the debt. To do that we have to compare the outcome with the situation in the absence of loans. This is done in Table (B/3), which indicates that the immediate effect of debt would be to increase income by increasing percentages up to year 18, in spite of the fact that the rate of growth of income in the no-loan case would be increasing. From year 19 onwards this latter rate exceeds 5%, and goes on increasing to nearly double its initial level, and the ratio of excess of income falls down until it vanishes in year 39. What would have been actually happening is some income redistribution over time, with rather negligible impact on future incomes If this is so, we would expect that the actual benefit which the country can draw out of loans is a quick rise in its final consumption. This is in fact the case as shown by the percentages in the last column but one of Table (B/3). Debt would be financing consumption, and for this purpose a big capital exportation has to take place.

Not only that. The last column of table (B/3) shows that up to year 25, the absolute size of investment will be larger as a result of debt. But from year 26 onwards, the capacity of the economy to finance investments will be greater in the absence of loans. In spite of the fact that the absolute level of savings would be lower, it is not forced to divert any part of it for redemption purposes.

The no-loan case has more desirable features in the longer-run. The main question would be: whether it is possible (not to say desirable) to let the economy be satisfied with the lower levels of consumption in the earlier years. If this is not possible, the acceptance of loans should be made on the understanding that they are meant to finance immediate consumption rather than long run capital formation. The price in terms of excessive capital exportation should be also appreciated. It should be noticed that if exports are to increase by 4%, the average propensity to import would fall from 10% in the base year to 7.2% in year 37, which is more acceptable than in the case of a series of loans.

Thus we can conclude that the model suggested by the I.B.R.D. provides insufficient bases for decision on the advisability of borrowing The transfer problem has to be considered seriously, and an evaluation of the ultimate capital benefit from excessive borrowing should be explicitly made.

IX - The Compound Interest Mechanism:

In spite of the fact that simple interest is assumed, the process suggested by the I.B.R.D. has represented two interesting features:

(1) Interest has been actually working on a compound rather than simple basis.

(2) No explicit account was made of the terms of the loans. Even though it was assumed by Avramovic that the average maturity is 15 years, this did not affect the calculations in any way. The parameter e (maturity period) was not considered in the model itself.

The basic idea is that whatever repayments are made, there will be need - during the first 15 years - for new loans to finance the required investments. To this we should add provisions for interest charges. It follows that the sum of D and $r.Q_{-1}$ would represent the net flow of new loans. The gross flow would exceed the net by any principal repayments due. Thus if we assume e = 15, the amount of loans needed in year 15 would be $\frac{1}{15}$ of all previous loans or (739.4/15) = 49.3, plus the net flow 44.4 - 3.9 = 40.5. Hence the gross flow will be 89.8. If e were equal to 10, the flow will be 114.4; and so on. While the working of the model requires knowledge of the net flow, the actual application of the loan procedure needs an evaluation of the gross flow, which is the one to be actually contracted.

The implication of this approach is that investment requirements receive the first claim on the country's resources. If these resources are not sufficient, external finance is saught. After that allowance should be made for servicing previous debt. This is how the problem of long-term growth is envisaged by the I.B.R.D. in relation to external debt.

It is clear that this approach is different from our own approach, where first claim was given to servicing a given debt and investment was considered as a residual. To reconciliate the two approaches, one should reconsider this residual investment to see whether it will be sufficient to maintain a given rate of growth. If not, a new loan has to be considered in the same manner.

However, it follows from this order of priorities that whenever financial resources are not sufficient to meet debt service, an equivalent amount has to be borrowed. As a result, new interest claims will be charged, thus leading to compound interest even though each loan is treated on a simple interest basis. It might be argued, therefore, that a comparison of this approach to ours cannot be made unless we reconcile the orders of priorities.

Such a reconciliation is not difficult to make. In fact, every thing would depend on whether we consider a single loan or a series of loans. Let us therefore determine the base year loan on the assumption that it will be that required to let $Y_1 = (1 + g)Y_o$. Thus :

$$L_o = D_o = (\gamma g - s)\, Y_o$$

During the following e years, there will be claims on the country's resource equal to L_o/e plus interest on the remainder. Thus in year 1, provision has to be made for the annuity A_{o1} (of loan contracted in year o, to be paid in year 1):

$$A_{o1} = rL_o + L_o/e$$

To this we have to provide for investments sufficient to raise income by $gY_1 = g(1+g)Y_o$. Leaving aside the annuity, there will be a deficit

$$D_1 = \left[(\gamma g - s) - g(\sigma - \gamma g)\right] Y_o$$

To make sure that this amount will be forth-coming after allowing for the annuity, a new loan has to be obtained, amounting to:

$$L_1 = A_{o1} + D_1 = rL_o + \frac{L_o}{e} + D_1$$

Hence total debt outstanding at the end of year 1 will be:

$$Q_1 = L_o + L_1 - L_o/e = (1 + r)L_o + D_1$$

This illustrates the source of the compound interest nature of the I.B.R.D. process. On the other hand considering each loan separately, according to our model, we obtain the following results: Suppose that the rate of interest is just equal to the critical rate σ/γ. If no other loan is contracted up to year e, except L_o, income Y_{e+1} will be just equal to income obtained with-out any loan. If the rate exceeds the critical value, income will be smaller, and vice versa. Now, to judge the behaviour of income in any year $n > e+1$ we can use the series with no loans. We begin by year 1 and judge L_1 determined as before. If $r = r^*$, income in year $e + 2$ will be again equal to income generated without loans. On the other hand if $r > r^*$, we have two reductions in Y_{e+2}: The first is due to the fact that Y_{e+1} is less than Y'_{e+1} as a result of the effect of L_o, which means that in the absence of any further loans, Y_{e+2} will also be less than Y'_{e+2}. The second, is due to the further loss caused by contracting L_1 at the same conditions. Similar results can be obtained, even when we allow for a variable e.

Thus our critical rate will be still effective, and it implies the working of the compound interest rule as in the I.B.R.D. case. However, one might raise the question that the I.B.R.D. process abruptly ends the process of repayment without giving attention to the maturity period. Further it does not allow for using up all available resources, whenever they exceed what is required to maintain the given rate g. Is it possible that the capital generated after full repayment will be different from what would be obtained if we strictly adopt our process?

Let us assume again that $r = r^*$, which meets the requirement of being less than the I.B.R.D. critical rate. Since the model allows for net flows, assuming that whatever extra resources are available can be lent abroad at the same rate r, it follows that the country can lend any extra capital for one year and get a return equal to $r = \sigma/\gamma$, on it. The following year it will generate another surplus which together with the preceding one and the return on it can be lent and a similar return on it obtained. Alternatively, the country could invest such surpluses, increasing its income by $1/\gamma$ of their values, hence generating additional capital (through extra savings) by an amount equal to σ/γ, which is the same as the returns obtained from investment abroad. Income in the former case will be smaller, but national capital will be the same.

However, if $r < r^*$, income will be smaller in year n. But if instead of investing the surpluses at home, the country decides to lend them abroad, the cumulative returns will grow at the rate r which is smaller than σ/γ. It would be more beneficial to adopt our process rather than the I.B.R.D. process.

It follows that if $r > r^*$, it will be to the benefit of the country to invest abroad, thus making good some of the previous losses envolved. However, if this is true, one could argue that whenever the rate of interest exceeds our critical rate, it will be to the advantage of the country to act as a lender rather than a borrower, whenever this is possible.

X. Conclusions:

In the present part of this memo. we have discussed the model developed by the I.B.R.D. experts. The first variant was originally formulated by Alter; the second by Shahid Husain in collaboration with Avramovic and others. The basic structure of the model, especially in the latter variant, is the same used by Qayum and ourselves, in the first two parts. The main difference lies in the fact that here a series of loans is considered, and the capacity to repay the debt is chosen a criterion. Some of the main findings are:

(1) While the idea of considering the process of indebtedness as acceptable if it is possible to repay the debt without hampering a predetermined growth rate, seems reasonable, it is not clear how it is possible to determine the growth rate itself taking indebtedness into consideration.

(2) There had been suggested 3 degrees of independence from foreign debts to be considered as objectives. The final choice is left in the hands of the planner, without any qualification except that the m.p.s. is to exceed a certain limit. This gives more room for flexibility, but it leaves the final decision rather arlitrary.

(3) The conditions given by the I.B.R.D. group for the realization of the given degrees of independence are the necessary conditions. When we investigated the sufficient conditions for ensuring that debt will be eventually repaid we found that they necessarily hold for the condition of debt attaining a maximum. But this was not true for the mild condition that the ratio of debt to income will be a maximum, if the growth rate was higher than the rate of interest.

(4) The I.B.R.D. critical rate of interest was derived on the basis of an approximate relationship ensuring that debt will asymptotically grow at a rate equal to the rate of growth of income. We have shown that such a rate would in fact ensure that debt will be fully repaid at an infinitely large period. This gives more power to that rate.

(5) On the other hand, that critical rate is much higher than the critical rate which we have shown to define an upper bound beyond which income and capital will fall below the level attainable without loans. It follows that a country might accept loans according to the I.B.R.D. principle which would eventually empoverich it.

(6) In that sense it seems that the main contribution of such loans would be the immediate rise in income and consumption, rather than strengthening long-term development. Unless this is made clear, wrong decisions might be recommended.

(7) In spite of the fact that the numerical example discussed by Avramovic is based on acceptable values of the parameters, the computations have displayed certain undesirable features. In particular, the pattern of foreign trade was found to be difficult to realize. There is need for an explicit treatment of the foreign trade gap besides the savings investment gap.

(8) The same numerical example has shown that a large flow of capital exportation (amounting to more than four times the original needs) has to take place, with a final meager benefit of 2.3% increase in income over what could have been obtained without loans. The computations show that a country getting herself envolved in foreign debts, as a result of shortage of domestic capital, should be ready to become a big exporter of capital. In spite of certain rigidities of the model, such results are not far from the hard facts faced by many developing countries.

(9) The process generated by the I.B.R.D. is essentially a compound-interest process. It was shown that this would in fact be the case whenever a persistent attempt is made to maintain a given rate of growth from the outset. On the other hand a no-loan process could-at least in principle - lead to accelerated growth, with a rate eventually higher than any rate consistent with safe-borrowing.

(10) We had tried to reconcile this compound interest process arising from a series of loans with our single-loan approach. However, there is more to be said about the role of the m.p.s. We shall deal with this problem in part IV of this memo.

APPENDIX (A)

SOLUTIONS OF FIRST ORDER DIFFERENCE EQUATIONS

The general form of the first-order difference equation which is linear (i.e., of the first degree) with constant coefficients, takes the form

$$Y_t - a\,Y_{t-1} = F(t) \tag{A/1}$$

where F(t) is some function of t. The first step to solve this equation is to find some particular solution: $Y_t = \hat{Y}(t)$, which satisfies (A/1), in the sense that

$$\hat{Y}(t) - a\hat{Y}(t-1) = F(t) \tag{A/2}$$

If we subtract this equation from the former, and define

$$y_t = Y_t - \hat{Y}(t) \tag{A/3}$$

we obtain the corresponding homogeneous form:

$$y_t = a\,y_{t-1} \tag{A/4}$$

The solution of this latter equation is:

$$y_t = y_o\,a^t \tag{A/5}$$

Then the general discrete solution of (A/1) is:

$$Y_t = \hat{Y}(t) + y_o\,a^t \tag{A/6}$$

To obtain these solutions we introduce the lag operator L, and the difference operator Δ :

$$L = 1 - \Delta \qquad LY_t = Y_{t-1} \tag{A/7}$$

Suppose that we possess a function in L:

$$H(L) = H_1(L)/H_2(L) \tag{A/8}$$

where,

$$H_i(L) = \sum_{j=0}^{m_i} b_{ij}\,L^j \qquad (i=1,2) \tag{A/9}$$

We transform these functions into functions of Δ :

$$H_i(L) = \sum_{j=0}^{m_i} c_{ij} \Delta^j = h_i(\Delta) \qquad (i=1,2) \qquad (A/10)$$

Further, using a process of division we can write:

$$\frac{1}{h_2(\Delta)} = \sum_{k=0}^{\infty} d_{2k} \Delta^k \qquad (A/11)$$

Hence, we can rewrite (A/8) as:

$$h(\Delta) = h_1(\Delta)/h_2(\Delta) = (\sum_{j=0}^{m_1} c_{1j} \Delta^j)(\sum_{k=0}^{\infty} d_{2k} \Delta^k)$$
$$= \sum_{i=0}^{\infty} e_i \Delta^i \qquad (A/12)$$

To apply these rules to difference equations we express the left-hand side as a function in the lag operator :

$$D(L) Y_t = F(t) \qquad (A/13)$$

The particular solution will be

$$\hat{Y}(t) = \frac{1}{D(L)} \cdot F(t) \qquad (A/14)$$

or,

$$\hat{Y}(t) = h(\Delta). F(t) \qquad (A/15)$$

Then follows the general solution.

Returning back to the first-order linear equation (A/1), we can write:

$$D(L) = 1 - aL = 1 - a + a\Delta \qquad (A/16)$$

The reciprocal of this function can be evaluated by direct division to give:

$$h(\Delta) = \frac{1}{D(L)} = \frac{1}{1-a}\left[1 - (\frac{a}{1-a})\Delta + (\frac{a}{1-a})^2 \Delta^2 - \dots\dots\dots\right]$$

Hence, the particular solution of (A/1) is

$$\hat{Y}(t) = \frac{1}{1-a}\left[\sum_{i=0}^{\infty}(-1)^i (\frac{a}{1-a})^i \Delta^i\right] F(t) \qquad (A/17)$$

Its final shape will depend on the exact form of F(t). Let us consider some special cases:

(1) A quadratic:

Suppose that,

$$F(t) = b_o + b_1 t + b_2 t^2 \qquad (A/18)$$

We know that

$$\Delta t = 1, \qquad \Delta^i t = 0 \qquad (i \geqslant 2)$$

$$\Delta t^2 = 2t-1, \qquad \Delta^2 t^2 = 2, \qquad \Delta^i t^2 = 0 \qquad (i \geqslant 3)$$

Substituting in (A/17) we obtain:

$$\hat{Y}(t) = \frac{1}{1-a} \left[(b_o+b_1t+b_2t^2) - \left(\frac{a}{1-a}\right) \left\{ b_1+b_2(2t-1) \right\} + \left(\frac{a}{1-a}\right)^2 2b_2 \right]$$

This can be rewritten as:

$$\hat{Y}(t) = c_o + c_1 t + c_2 t^2 \qquad (A/19)$$

where,

$$\left.\begin{aligned} c_o &= \left(\frac{1}{1-a}\right) b_o - \left(\frac{1}{1-a}\right)^2 a\, b_1 + \left(\frac{1}{1-a}\right)^3 a\,(a+1)\, b_2 \\ c_1 &= \left(\frac{1}{1-a}\right) b_1 - \left(\frac{1}{1-a}\right)^2 2ab_2 \\ c_2 &= \left(\frac{1}{1-a}\right) b_2 \end{aligned}\right\} \qquad (A/20)$$

(2) A Linear Function:

$$F(t) = b_o + b_1 t \qquad (A/21)$$

This is a special case of the previous one, in which $b_2 = 0$. Hence we write:

$$\hat{Y}(t) = c_o + c_1 t \qquad (A/22)$$

where,

$$c_o = \left(\frac{1}{1-a}\right) b_o - \left(\frac{1}{1-a}\right)^2 a\, b_1; \qquad c_1 = \left(\frac{1}{1-a}\right) b_1 \qquad (A/23)$$

(3) A Constant:

$$F(t) = b_o \qquad (A/24)$$

This is still a more special case, whose solution is:

$$\hat{Y}(t) = c_o = \frac{b_o}{1-a} \qquad (A/25)$$

In all the above cases, we use (A/6) as a general solution, where

$$y_o = Y_o - \hat{Y}_o, \qquad \hat{Y}_o = c_o \qquad (A/26)$$

The value of c_o is calculated according to the given formulae.

(4) A Constant, with a = 1:

In this case it is ΔY rather than Y which is considered as a constant. The difference equation is:

$$Y_t = Y_{t-1} + b_o \qquad (A/27)$$

This indicates that the particular solution is linear in time. We assume that:

$$\hat{Y}(t) = Bt$$

Substituting we get

$$Bt = B(t-1) + b_o$$

or,

$$\hat{Y}(t) = b_o t \qquad (A/28)$$

This means that, $\hat{Y}_o = 0$, and (A/6) gives:

$$Y_t = Y_o + b_o t \qquad (A/29)$$

as the general solution.

(5) An Exponential:

Let

$$F(t) = m\, b^t \qquad (A/30)$$

Using (A/14), and evaluating the reciprocal of D(L) as:

$$H(L) = c_o + c_1L + c_2L^2 + \ldots\ldots$$

we can obtain the particular solution as follows:

$$H(L)\ m.b^t = m\ (c_o + c_1L + c_2L^2 + \ldots\ldots)b^t$$

$$= m\ (c_o\ b^t + c_1\ b^{t-1} + c_2\ b^{t-2} + \ldots\ldots)$$

$$= m\ b^t\ (c_o + c_1\ b^{-1} + c_2\ b^{-2} + \ldots\ldots\ldots)$$

This gives a new polynomical in $\frac{1}{b}$ in place of L Hence,

$$\hat{Y}(t) = m\ b^t\ H(\frac{1}{b}) \qquad (A/31)$$

In the special case, D(L) = 1 - aL, we have,

$$\hat{Y}(t) = m\ b^t \sum_{i=0}^{\infty} (\frac{a}{b})^i$$

or,

$$\hat{Y}(t) = \frac{m}{b-a}\ b^{t+1} \qquad (A/32)$$

This means that

$$\hat{Y}_o = \frac{mb}{b-a} \qquad (A/33)$$

The general solution will be,

$$Y_t = \frac{m}{b-a}\ b^{t+1} + (Y_o - \frac{mb}{b-a})\ a^t \qquad (A/34)$$

<u>Example (1)</u>

Suppose that $Y_o = m$

$$\therefore Y_t = \frac{m}{b-a}\ b^{t+1} + m\ (1 - \frac{b}{b-a})\ a^t$$

$$\therefore Y_t = \frac{m}{b-a}\ (b^{t+1} - a^{t+1}) \qquad (A/35)$$

(6) <u>A Special Case:</u>

Suppose that:

$$Y_t = a\ Y_{t-1} + m\ a^t \qquad (A/36)$$

The solutions given above will be meaningless, and we have to look for an alternative. Given Y_o we can easily find that

$$Y_1 = Y_o\, a + ma$$

$$Y_2 = Y_o\, a^2 + 2m\, a^2$$

and so on. Hence the general solution will be:

$$Y_t = Y_o\, a^t + m\, t\, a^t \qquad (A/37)$$

<u>Example (2):</u>

Suppose again, $Y_o = m$

$$\therefore\ Y_t = m\,(t+1)\,a^t \qquad (A/38)$$

<u>Example (3):</u>

To solve:

$$Y_t = a\,Y_{t-1} + m\,b^t - n\,c^t \qquad (A/39)$$

given $Y_o = m$, we apply (A/35):

$$Y_t = \frac{m}{b-a}\,(b^{t+1} - a^{t+1}) - \frac{n}{c-a}\,(c^{t+1} - a^{t+1}) \qquad (A/40)$$

If <u>b = a,</u> we have to use (A/38) as well:

$$Y_t = m\,(t+1)\,a^t - \frac{n}{c-1}\,(c^{t+1} - a^{t+1}) \qquad (A/41)$$

Similarly for the case <u>c=a</u>. If both b = c = a then we are back to example (2).

APPENDIX B.

THE I.B.R.D. NUMERICAL EXAMPLE

The following set of tables gives the main aggregates assuming the following values of the parameters (as indicated in section VII):

$s = 0.10$ $\sigma = 0.20$ $\gamma = 3.0$
$g = 0.05$, $r = 0.06$ $Y_o = 1000$

Given the rate g, the evolution of income Y can be estimated, as in the first column in table (B/1). Using the annual change in income, we calculate both savings and investment:

$$S_t = S_{t-1} + \sigma (Y_t - Y_{t-1})$$

$$I_t = \gamma (Y_{t+1} - Y_t)$$

These are given in Table (B/2). The second column of (B/1) gives consumption: $C = Y - S$; while the third gives the deficit $D = I - S$, which is negative whenever there is a surplus. Adding the consecutive values of D, we obtain the cumulative deficit, given in the last column of Table (B/1). On the other hand, the debt outstanding at the end of each year:

$$Q_t = (1 + r) Q_{t-1} + D_t$$

For this purpose we calculate rQ_{t-1} to show the indebtedness incurred as a result of interest charges.

In Table (B/2) we indicate two alternatives of the components of the foreign trade gap, which is equal to D also. In the first variant, exports are assumed to increase annually by 4% as suggested by Avramovic It follows that imports should be:

$$M_1 = X_1 + D$$

In the second variant we begin by calculating imports on the assumption the direct and indirect import components are 0.10 for consumption and 0.40 for investment. It follows that:

$$M_2 = 0.10\ C_t + 0.40\ I_t, \qquad X_2 = M_2 - D$$

we then compute the rate of growth of X_2, as indicated in the last column of Table (B/2).

To estimate the impact of foreing loans, we calculate income Y' assuming that D = 0 always. Savings are calculated as before; and investment is put equal to saving:

$$I'_t = S'_t = S'_{t-1} + \sigma (Y'_t - Y'_{t-1})$$

This can be used to calculate income:

$$Y'_t = Y'_{t-1} + \gamma I'_{t-1}$$

and consumption:

$$C'_t = Y'_t - S'_t$$

The comparison with the previous estimates is based on relating the excess of these latter to the present estimates as shown in the last 3 columns of Table (B/3).

Table (B/1) - Evolution of Income, Consumption, Deficit and Debt

t	Y	C	D	rQ_{-1}	Q	Cum. D.
0	1000.0	900.0	50.0	-	50.0	50.0
1	1050.0	940.0	47.5	3.0	100.5	97.5
2	1102.5	982.0	44.9	6.0	151.4	142.4
3	1157.6	1026.1	42.1	9.1	202.6	184.5
4	1215.5	1072.4	39.2	12.2	254.0	223.7
5	1276.3	1121.0	36.2	15.2	305.4	259.9
6	1340.1	1172.1	33.0	18.3	356.7	292.9
7	1407.1	1225.7	29.6	21.4	407.8	322.5
8	1477.5	1282.0	26.1	24.5	458.4	348.7
9	1551.3	1341.0	22.4	27.5	508.3	371.1
10	1628.9	1403.1	18.6	30.5	557.4	389.7
11	1710.3	1468.2	14.5	33.4	605.3	404.1
12	1795.9	1536.7	10.2	36.3	651.8	414.4
13	1885.6	1608.5	5.7	39.1	696.6	420.1
14	1979.9	1683.9	1.0	41.8	739.4	421.1
15	2078.9	1763.1	- 3.9	44.4	779.9	417.1
16	2182.9	1846.3	- 9.1	46.8	817.5	408.0
17	2292.0	1933.6	- 14.6	49.1	852.0	393.4
18	2406.6	2025.3	- 20.4	51.1	882.7	373.0
19	2527.0	2121.6	- 26.3	53.0	909.4	346.7
20	2653.3	2222.6	- 32.7	54.6	931.3	314.0
21	2786.0	2328.8	- 39.3	55.9	947.8	274.7
22	2925.3	2440.2	- 46.3	56.9	958.4	228.5
23	3071.5	2557.2	- 53.6	57.5	962.4	174.9
24	3225.1	2680.1	- 61.3	57.7	958.9	113.6
25	3386.4	2809.1	- 69.3	57.5	947.1	44.3
26	3555.7	2944.6	- 77.8	56.8	926.1	- 33.5
27	3733.5	3086.8	- 86.7	55.6	895.0	-120.1
28	3920.1	3236.1	- 96.0	53.7	852.7	-216.1
29	4116.1	3392.9	-105.8	51.2	798.1	-321.9
30	4321.9	3557.5	-116.1	47.9	729.9	-438.0
31	4538.0	3730.4	-126.9	43.8	646.7	-564.9
32	4764.9	3911.9	-138.2	38.8	547.3	-703.2
33	5003.2	4102.6	-150.2	32.8	430.0	-853.4
34	5253.3	4302.6	-162.7	25.8	293.1	-1016.0
35	5516.0	4512.8	-175.8	17.6	134.9	-1191.8
36	5791.8	4733.4	-189.6	8.1	- 46.6	-1381.4
37	6081.4	4965.1	-204.1	..	..	..
38	6385.5	..	..			
39	6704.7					
40	7040.0					

Table (B/2) - Components of The Two Gaps

	I	S	X_1	M_1	X_2	M_2	Growth rate, X_2
0	150.0	100.0	100.0	150.0	100.0	150.0	%
1	157.5	110.0	104.0	151.5	109.5	157.0	9.5
2	165.4	120.5	108.2	153.1	119.5	164.4	9.1
3	173.6	131.5	112.5	157.6	129.9	172.0	8.7
4	182.3	143.1	117.0	156.2	141.0	180.2	8.5
5	191.4	155.3	121.7	157.9	152.5	188.7	8.1
6	201.0	168.0	126.5	159.5	164.6	197.6	7.9
7	211.1	181.4	131.6	161.2	177.4	207.0	7.7
8	221.6	195.5	136.9	163.0	190.7	216.8	7.5
9	232.7	210.3	142.3	164.7	204.8	227.2	7.4
10	244.3	225.8	148.0	166.6	219.4	238.0	7.3
11	256.6	242.1	153.9	168.4	235.0	249.5	7.1
12	269.4	259.2	160.1	170.3	251.2	261.4	6.9
13	282.8	277.1	166.5	172.2	268.3	274.0	6.8
14	297.0	296.0	173.2	174.2	286.2	287.0	6.7
15	311.8	315.8	180.1	176.2	304.9	301.0	6.5
16	327.4	336.6	187.3	178.2	324.7	315.6	6.4
17	343.8	358.4	194.8	180.2	345.5	330.9	6.4
18	361.0	381.3	202.6	182.3	367.3	346.9	6.3
19	379.0	405.4	210.7	184.4	390.1	363.8	6.2
20	398.0	430.7	219.1	186.4	414.2	381.5	6.2
21	417.9	457.2	227.9	188.6	439.3	400.0	6.0
22	438.8	485.1	237.0	190.7	465.8	419.5	6.0
23	460.7	514.3	246.5	192.9	493.6	440.0	5.9
24	483.8	545.0	256.3	195.0	522.8	461.5	5.9
25	508.0	577.3	266.6	197.3	553.4	484.1	5.8
26	533.3	611.1	277.2	199.4	585.6	507.8	5.8
27	560.0	646.7	288.3	201.6	619.4	532.7	5.8
28	588.0	684.0	299.9	203.9	654.8	558.8	5.7
29	617.4	723.2	311.9	206.1	692.0	586.2	5.7
30	648.3	764.4	324.3	208.2	731.2	615.1	5.6
31	680.7	807.6	337.3	210.4	772.2	645.3	5.6
32	714.7	853.0	350.8	212.6	815.3	677.1	5.6
33	750.5	900.6	364.8	214.6	860.7	710.5	5.6
34	788.0	950.7	379.4	216.7	908.2	745.5	5.5
35	827.4	1003.2	394.6	218.8	958.1	782.3	5.4
36	868.8	1058.4	410.4	220.8	1010.5	820.9	5.4
37	912.2	1116.3	426.8	222.7	1065.5	861.4	5.4
38							
39							
40							

Table (B/3)- Comparison With the No-loan Case

t	Y'	Growth rate, Y' %	C'	S' = I'	(Y - Y')/Y' %	(C - C')/C' %	(I - I')/I' %
0	1000.0	-	900.0	100.0	-	-	50.0
1	1033.3	3.3	926.6	106.7	1.6	1.4	47.6
2	1068.9	3.4	955.1	113.8	3.1	2.8	45.3
3	1106.8	3.5	985.4	121.4	4.6	4.1	43.0
4	1147.3	3.6	1017.8	129.5	6.0	5.4	40.8
5	1190.4	3.8	1052.3	138.1	7.2	6.5	38.6
6	1236.4	3.9	1089.1	147.3	8.4	7.6	36.5
7	1285.5	4.0	1128.4	157.1	9.4	8.6	34.4
8	1337.9	4.1	1170.3	167.5	10.4	9.5	32.3
9	1393.8	4.2	1215.0	178.8	11.3	10.4	30.1
10	1453.4	4.3	1262.7	190.7	12.1	11.1	28.1
11	1516.9	4.4	1313.5	203.4	12.8	11.8	26.2
12	1584.7	4.5	1367.8	216.9	13.3	12.3	24.2
13	1657.0	4.6	1425.6	231.4	13.8	12.8	22.2
14	1734.2	4.7	1487.4	246.8	14.2	13.2	20.3
15	1816.4	4.7	1553.1	263.3	14.5	13.5	18.4
16	1904.2	4.8	1623.4	280.8	14.6	13.7	16.6
17	1997.8	5.0	1698.2	299.6	14.7	13.9	14.7
18	2097.7	5.0	1778.2	319.5	14.7	13.9	13.0
19	2204.2	5.1	1863.4	340.8	14.6	13.9	11.2
20	2317.8	5.2	1954.2	363.6	14.5	13.7	9.5
21	2439.0	5.2	2051.2	387.8	14.2	13.5	7.7
22	2568.2	5.3	2154.6	413.6	13.9	13.3	6.1
23	2706.1	5.4	2264.9	441.2	13.5	12.9	4.4
24	2853.2	5.4	2382.6	470.6	13.0	12.5	2.8
25	3010.1	5.5	2508.1	502.0	12.5	12.0	1.2
26	3177.4	5.5	2641.9	535.5	11.9	11.5	-0.4
27	3355.9	5.6	2783.7	571.2	11.2	10.9	-2.0
28	3546.3	5.7	2937.0	609.3	10.5	10.2	-3.5
29	3749.4	5.7	3099.5	649.9	9.8	9.5	-5.0
30	3966.0	5.8	3272.8	693.2	9.0	8.7	-6.5
31	4197.1	5.8	3457.7	739.4	8.1	7.9	-7.9
32	4443.6	5.9	3654.9	788.7	7.2	7.0	-9.4
33	4706.5	5.9	3865.2	841.3	6.3	6.1	-10.8
34	4986.9	6.0	4089.5	897.4	5.3	5.2	-12.2
35	5286.0	6.0	4328.8	957.2	4.3	4.3	-13.6
36	5605.1	6.0	4584.1	1021.0	3.3	3.3	-14.9
37	5945.4	6.0	4856.3	1089.1	2.3	2.2	-16.3
38	6308.4	6.0			1.2		
39	6695.6	6.1			0.1		
40	7108.6	6.2			-1.0		