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ON ECONOMIC BALANCES AND ALTERNATIVE METHODS OF VALUATION

by

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1- INTRODUCTION:

The elaboration of a national economic plan usually involves the construction of some or all of the following economic balances:

1.Commodity balances

2.Input-output tables (projected)

3.National budget.

It is evident that the values implied in one of these balances for the components of any of the others should be exactly those which figure out in the latter. If for example commodity balances are constructed in physical terms - as they in fact should - the values of the various resources and uses should express the exact values of the corresponding flows in physical terms. The values of those flows can be calculated in different ways; the specific way chosen in a certain type of balances depends on the type of analysis envisaged through their use. By the same token of consistency of all sets of estimates relating to a given period (whether realized or projected), care should be taken that the flows calculated in a certain way would remain unchanged if they are valued in terms of another set of prices, so that differences in the values of those flows reflect solely differences in prices.

Further, economic balances are not mere accounting frames which satisfy certain accounting or balance equations. They should basically ensure the fulfillment of more basic relationships implied by the structure of the given economy, a fact that has to be taken into account in the construction of the analytical models (mathematical or not) used in building up of the balances. Hence, the choice of a given valuation principle is to be made in the light of the requirements of the economic analysis envisaged.

In the construction of the national budget, attention is paid to the national aggregates, and emphasis is put on income or (product) rather than production. The subdivision of output into current vs. capital uses, as well as into domestic vs. foreign components, is based on the requirements of economic theory which provides an explanation of the factors determining each of these components. Therefore, differentiation between business, households, administration and the rest of the world is based on differences in rules governing economic behaviour. Since demand for any type of goods is determined according to the prices charged for it in the market, estimates at the so-called " market prices" are required. On the other hand, in the analysis of the factors affecting production, only payments paid to production factors in connection with

their contributions to output are relevant. Indirect taxes imposed on output are not determined within the framework of production processes, and it is necessary therefore to introduce another concept, viz., that of "factor costs". The well-known relationship between the two concepts is:

Gross domestic [1] product at factor costs. Gross domestic product at market prices - Indirect taxes of subsidies.

This latter concept is the one relevant to the measurement of income (after allowing for capital consumption), which is the decisive factor in determining demand relations.

Now, if we approach the problem from the viewpoint of production rather than income, (e.g., in commodity balances or input-output tables) the problem of valuation has to be treated again in connection with purposes of analysis. The question might be raised whether the two concepts discussed above are still relevant. Further, if the coverage of the system is incomplete (e.g., commodity balances which concentrates on a number of commodities, excluding others), are we free to device some concept without regard to its implications for the rest of the system ?

It is our belief that a discussion of the merits of any valuation principle, or the characteristics of a partial system of balances can be only systematically discussed within the framework of complete interflow-tables. Further the passage from one principle to the other can best be studied in terms of the relationships between different variants of those tables. It is true that interflow tables had been closely related to more refined mathematical techniques.[2] It is also a fact that many planners (as well as economists) prefer not to have much ado about mathematics, either because of natural dislike, or because of fear that the degree of reliability of available data does not warrant the use of those more refined techniques. This is not the place to indicate the dangers of the one view or the fallacies of the other. What we want to emphasise is that, given the amount of detailed information usually involved in the elaboration of a national plan, much of the labour and all of the errors occuring in the construction of the various types of balances which express that information can be avoided through the systematic use of interflow tables.

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1) Since we shall base most of our arguments on the conditions in a closed economy, we need not worry about differentiation between domestic and national concepts.

2) See for example, R.Frisch: "From national accounts to macro-economic decision models", pp. 1-26, in Income & Wealth, Series IV, Bowes & Bowes, London, 1955.

2. ALTERNATIVE CONCEPTS OF VALUATION:

As indicated before, two main concepts are involved in the valuation of national product in its entirty:

1. The factor cost concept, and

2. the market price concept.

This distinction- in spite of its worldwise acceptance- is somewhat misleading.

In order that the first concept as defined before should qualify for the measurement of costs in the proper economic sense we have to assume a special type of markets, and further to require the existence of a very special case of that market: We have to assume the case of equilibrium in a purely competitive market. The underlying notion is the elaboration of an indicator by which to measure the actual contribution of the various factors contributing to production, excluding any remunerations which are not warranted by production needs, but are due to "market" conditions. Ideally, the term should exclude all incomes accruing to monopolistic powers, as well as abnormal revenues obtained by some factors as a result of deviations from equilibrium conditions (e.g., quasi-rent). If all these imperfections in the market disappear, the only deviation occuring in the market due to extraneous factors would be that due to the interference of the State in the form of taxation. However, even if this is true, the exclusion of commodity taxes does not nullify their effects through the structural relationships in the economy, hence the need for the market value concept.3) Further, the exclusion of these taxes is not the only modification required, since this does not take care of the distortions in the market arising from the economic effects of taxes on certain types of income or of capital.

It is not our intention to dwell on this problematic question of national accounting. Our preoccupation here is with the definitions which would be considered as corresponding to those defined above, when the production of a given sector is under or commodity is under consideration. To simplify the argument, let us consider that a given sector specialises in the production of a given commodity or number of commodities, and that each commodity is exclusively produced by one sector.

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3) Probably this has been the reason for the fact that Eastern Countries usually pass from measurements in physical terms to values at market values directly. In those economies little can be said by way of justifying the separation of internal and external powers ruling the market.

The costs of production of the commodity include all payments to inputs, whether secondary (produced by some other sector) or primary (factors of production), including the producer's own services. The whole income of the producer is included even though he might be receiving abnormal profits.

Excluding storage at the source, all production is sold at the same period, either to other production sectors or to final demand sectors. At this point two things occur outside the circuit of production, i.e., within the boundaries of the market:

1. Commodity taxes have to be paid or subsidies taken. These might take various names: excise duties, sales taxes, etc.... But it is customary to charge them to the producer, who in turm charges them to the buyer. Thus, for all practical purposes, we can assume that the producer sells at a price which exceeds his costs by the amount of the tax (a subsidy being considered as a negative tax, hence subtracted)[4] Since this event takes place during marketing of the commodity rather than during its production, it can be considered that we are dealing with a market price; in fact the first of a series of market prices, noticing that a number of dealers exchange the commodity before it reaches its final destination, each adding a certain markup for his services, thus introducing another market price for the same commodity.

2. The second element is the fact that the passage of the commodity from the producer to its final user is accomplished through the media of those who help to transport it over space and time. In each step extra margins are added within the market, i.e., within the production circuits of sectors involved in the marketing processes. These comprise:

a) Rail, air, water, truck and pipeline transportation

b) Warehousing and storage

c) Retail and wholesale trade agents

d) Financial agents (banks, insurers, etc...).

The size of each of these items is determined by the relative position of the ultimate user with respect to the producer. Thus they form a part of the price paid by the purchaser, and their total when added to the price charged by the producer form the purchaser's price, which is the final step in market prices. It is clear that this price differs from one user to the other. When the producer is using his own production, also in the case of services, the commodity passes directly and the two market prices, selling and purchasing, are identical. The longer the distance between the producer and the user, the larger the difference between the two.

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4) Further complications are involved when various tax rates are charged against different users, or exemptions awarded to some. But we shall ignore this case for purposes of simplicity.

To summarize, the following items appear in the structure of the final price of the commodity:

i) value of secondary inputs, evaluated at the price which the producer pays for their purchase;

ii) payments by the producer to the services of the primary factors, including his own services;

iii) Indirect taxes (less subsidies) on the production of the commodity;

iv) Marketing margins on the commodity (trade, transport, and finance).

The three types of prices distinguished above can be defined as follows:

I) costs of production = (i) + (ii)
= value of inputs at purchaser's prices + factor payments

II) Producer's selling price (a market price) = (i)+(ii)+(iii)
= (I) + (iii)
= Costs of production + indirect taxes on production

III) Purchaser's price (another market price) = (i)+(ii)+(iii)+ (iv)
= (II) + (iii)
= Producer's selling price + trade margins.

Still other concepts can be devised:

IV) Purchaser's price net of taxes = (i)+(ii)+(iv) = (I) + (iv)
= costs of production + marketing margins

This concept ignores taxes on production, although it can be seen to contain taxes on inputs. If for purposes of consistency we exclude taxes on inputs, another producer's concept might be invented.

V) Partial costs of production = value of inputs net of their taxes (but including their own margins) + (ii), payments to primary factors.

3. THE PROBLEM:

The problem which we want to discuss in this paper is: What are the interrelations between valuation concepts of national product, and those of commodities ? Since in any case the national product concepts exclude the double counting due to secondary inputs, we have to subtract their values as estimated in the calculation of the value of the commodities. Thus, in concepts (I) - (IV), we have to subtract the purchasing value of inputs (defined according to concept III), while in (V) we have to subtract the value of inputs less their taxes (i.e., defined according to V). But this subtraction yields different results:

1. In the case of concepts (I) and (V) the remainder will be factor payments (iii) only, hence when adding up for all commodities and all sectors we obtain national product at factor cost.

2. When use is made of concept (IV), we also obtain national product at factor cost. However, when adding the products of a single sector, subtracting only inputs required for production, it will be clear that the value added by the sector will be containing trade margins which are produced by other sectors. This raises another type of double-counting, which can be treated in either of the following ways:

   a) To consider the margins as inputs in the specific producing sector. This ensures that the value added in that sector will be exactly attributed to it. Therefore, we have to estimate the value added in the marketing sectors in exactly the same manner.

   b) To include the margins as value added in the producing sectors. To eliminate double-counting, the value added in marketing sectors is composed of the difference between the value of all their services rendered directly to final users, and all inputs used in the production of these services and those relating to the marketing of the production of other sectors.

   It is clear that the second of these approaches, though yielding the same overall total, is not fair in its treatment of the various sectors, and the former approach is therefore preferable.

3. If either of concepts (II) and (III) is used, the aggregation yields total value added at market prices. However, whereas concept (II) yields estimates for various sectors which measure their actual contributions to the aggregate, concept (III) raises problems similar to those faced in using (IV), hence solved in the same manner.

The questions raised earlier can be now restated as follows:

- Can we consider the value of output at production costs (complete or partial) as being value at factor costs, because it leads to national product at factor cost ?
- Further, can we consider values at selling or purchasing prices as market values ? And, if so, which is the most relevant concept ?

4- CURRENT TERMINOLOGY:

As mentioned before, there is no much dispute about concepts relating to national product. But lesser agreement exists in dealing with sectoral productions. Probably the key element in the whole problem is the definition of trade or marketing margins. Viewed from the side of production sectors, all margins defined in Sec.(2) above, as well as indirect taxes are included in trade margins. This view obtains a wide acceptance, both in the Anglo-Saxon and the French schools. In principle two major concepts are identified:

1. Concept (I) defined above, which is called value at producer's price.
2. Concept (III), which is called value at purchaser's price.

In some cases a third concept is used, namely,

3. Concept (II), which is called value at market price.

Thus, a recent U.N. publication [5] stipulates that: " Net indirect taxes (i.e., taxes minus subsidies) form part of the margin between the producer's price and the purchaser's price. .... market prices .... are producer's prices plus net indirect taxes." Many other writers hold similar views. Thus Chenery [6] considers that "When output is valued at market prices, indirect taxes must also be treated as a primary input." Similarly J.Sandee [7] decomposes consumer's value into: producer's value, trade margins, sales taxes and transport costs. Moore [8] also defined margins so as to include excise taxes.

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5) U.N: Problems of Input-output Tables and Analysis, p.40 - Series F, No.14, 1966

6) H.B.Chenery & P.G.Clark : Interindustry Economics, p.17; John Wiley & Sons, 1959

7) J.Sandee: "Input-output accounts", Ch. 10 in, T.Barna (ed.): The Structural Interdependence of the Economy, p.226-John Wiley & Sons, Milano, 1955.

8) F.T.Moore: "A survey of current interindustry models", in N.B.E.R.: Input-Output Analysis; An Appraisal, p.239, Princeton University Press, 1955.

In spite of this overwhelming agreement, one can spot some differences in the practices adopted by certain writers. For example Richard Stone has recently 9) differentiated between flows at purchaser's values and at producer's values according to whether or not they include trade and transport services. In other words, both concepts include indirect taxes on production (as well as on inputs). To get rid of the cumulative effects of indirect taxes, he excludes all indirect taxes on production and on inputs required both directly and indirectly 10) to arrive at what he calls producer's value at factor cost. This latter concept defines the value if the system was free from any taxation whatsoever, a concept which is consistent with the factor cost principle used in connection with national product. It is clear that the concept is not directly observable; it is a purely analytical concept which has to be calculated after the compilation of the table at producer's value.

Now, if it is true that the latter concept is the one corresponding to the factor cost principle of national accounts, then what about concepts (IV) and (V) defined before? Further, can we therefore consider concepts (I) to (III) as variants of market prices? One might argue that concept (I), viz., the cost of production concept is difficult to consider as a basis for a market price valuation. We shall prove the reverse later (Section 9). But even if we leave this concept for the moment, we might question the advisability of defining one single price (the producer's selling price) as being "the" market price, hence implying that the others are not market prices. It is true that the purchaser's price need not be "a" market price, in the sense that it might include some extra costs (e.g., freight and insurance) over and above what is quoted for the commodity in the last stage of the market. But then, the term "price" itself is misleading and it is preferable to call it "value" as Stone did, or "cost". As it will be shown later, all three variants - when considered within the framework of a complete interflow table - lead to the calculation of national product at market prices.

It seems however, that the current terminology does not convey the underlying notions of the various concepts. To avoid any misunderstanding we propose the following terminology:

1) Concept (I) will be taken to stand for "producer's costs of production." They represent the price which if charged will cover all expenses for inputs employed by the factory, firm, establishment, etc..

2) Concept (II) will be called "producer's selling price". The word selling is added in order to differentiate this concept from (I) currently called producer's price. This concept is meant to include all indirect taxes whether paid at the point of production or directly by the consumer (e.g. purchase taxes).

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9) R.Stone: Input-output and National Accounts -Ch.III, O.E.E.C.,1961
10) See section 11 below

3) Concept (III) will be called "user's price". The term user replaces the current term "purchaser", simply because we want to distinguish those purchasers who are buying the commodity with the intention of using them either for intermediate or final uses. In fact the term price does not in this connection necessarily stand for a " price quotation" in the market. A better terminology would have been "value" or "cost" as suggested before. However, the adoption of the term user is considered to remove any ambiguity.

Now, if we start by the second concept, it will be seen that it expresses, in most cases, an actual situation in the market. In this version, taxes on commodities are allocated to their producers, while margins allocated to users. Concept (I) is a hypothetical one, in the sense that it means a reallocation of taxes so as to be debited directly to users. On the other hand, concept (III) is also hypothetical since it implies reallocation of margins to producers. The main difference between these two latter concepts is that in the former the item which is reallocated belongs to the treasury, which lies among the primary factors, while in the latter the items reallocated belong to sectors normally included in business. This might involve different treatments as will be shown in section 7-10 below. But the important thing is that both of them introduce alternative treatments of items entering into the set of market prices involved.

5- THE PLANNING COMMITTEE CONCEPTS:

Let us consider first the practices developed by the U.A.R. Planning Committee. [11] Referring to the first five-year plan's document,[12] we can distinguish a number of concepts which are summarized in the table below. The English version of the document has been compared with the original Arabic to make sure that no additional errors were introduced through translation.

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11) The problem has been recently considered by Mr. F.Afia in Memo.698, Ministry of National Planning (in Arabic). The author is also indebted tor Mr. Afia for valuable comments made on the present paper.

12) U.A.R.National Planning Committee: General Frame of the 5-Year Plan for Economic and Social Development July 1960 - June 1965; Cairo, 1960, Government Printing Offices.

Concept	The N.P.C. corresponding concepts	References to them in General Frame
I	1- " Selling prices of the factory, the farm or the establishment, thus excluding indirect taxes and including government subsidies"	p. 54
	2- " Value at factor cost, which is market prices excluding taxes and including government subsidies"	Tables 29, 30 & 37
II	3- " Producer's price, which is in a sense ... 4- ... market price at some stage or another" (see 2 above)	p. 39
III	5- " Users' costs,i.e., market prices inclusive of commodity taxes"	p. 55
	6- " Users' price"	Table 37
	7- " Value at market price"	p. 54 and Table 28
IV	8- " Value of resources to their users at factor cost excluding commodity taxes" or, simply,	p.54
	9- " Value at factor cost"	Table 28
V	10- "Value at factor cost" [13]	

The above table exhibits many undesirable features:

1. The lack of a strict terminology that is used throughout the same book.
2. The use of the same term to signify different meanings, even in the same page. Section Three of the document, especially pp. 54 and 55,and table 28 and the following tables are the most striking examples in this connection.

13) This concept seems to have entered into circulation at a later stage; see Memo.698, quoted in footnote 11 before.

3. A more serious problem is that the values standing for the same concept, and the values of the different concepts when compared together, do not fit together. In some cases, footnotes would indicate the source of discrepancy, thus reflecting the existence of more than one conceptual framework.[14] But in others no logical explanation is given, and there seems to exist certain errors which have been ambiguously treated by means of certain balancing items. A striking example of this latter error relates to the definition of "commodity taxes" in connection with commodity balances. Since the treatment affects the basic conceptual framework of these balances, we shall consider it in the following section.

6- CONTENTS OF COMMODITY TAXES:

In connection with the commodity balance between resources and uses, Section three of the General Frame gives in Tables 29 and 30 details of balances for commodities and commodity groups classified according to sectors, distinguishing agriculture as one sector, and 23 industrial sectors (including electricity). Table (28) summarises the base year estimates, while (28a) summarises the fifth year planned figures. Let us consider the latter of these tables to see what type of balance was envisaged for the plan targets. 15) Ignoring the first 24 rows for the moment (giving the totals by sector) the remainder of the table runs as follows:

1. Sub-total. This is the total of resources and uses of all products of agriculture and industry according to concept (I), i.e.,before inclusion of indirect taxes.
2. Studies, plans and management of investment.
3. Construction and machine installation.
4. Trade margins.
5. Total at factor cost = sum of previous four items. This is in fact concept (IV)

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14) See for example, footnote 1 to Table 74, pp. 213 -219 of the General Frame.

15) Table (28) which relates to the base year includes other sources of errors, due to approximations. This can be seen when comparing its estimates with other estimates in the same document in other sections. The whole problem had been discussed by the present author in his: "Planning for Economic & Social Development",(in Arabic),Cairo 1963, esp. pp. 155-156

6. Net commodity taxes

7. Total at market price = Total at factor cost + net commodity taxes.

The columns of the table run as follows:

A. Local production (at production costs)

B. Imports CIF

C. Total. This gives the total of "Value of available commodity resources"

The remaining columns relate to "Uses", and they are:

D. Exports

E. Final consumption

F. Commodity production requirements

G. Investments.

H. Increase in stocks.

The data for 1964/65 are:

Item	Commodity Resources			Uses				
	Local Prod.	Imports C.I.F.	Total	Exports	Final Cons.	Com. Prod. Requirs.	Investment	Stocks incr.
Sub-Total	2384.4	214.9	2599.3	214.7	824.6	1310.3	137.3	112.4
Invt.Studies	1.5	-	1.5	-	-	-	1.5	-
Machine Inst.	49.5	-	49.5	-	-	-	49.5	-
Trade Margin	220.9	24.0	244.9	14.6	134.4	62.9	30.0	3.0
Total at F.C.	2656.3	238.9	2895.2	229.3	959.0	1373.2	218.3	115.4
Net Comm.Taxes	236.6	107.5*		3.0	127.2	124.9	16.1	4.6
Total at M.P.	2892.9	346.4	-	232.3	1086.2	1498.1	234.4	120.0

*) Less 8,000,000 pounds value of taxes on exported cigarettes.

§) Since commodity taxes on exports appear in this table, the total balance cannot be shown. They are imposed on production and imports in the columns showing commodity resources and on exports shown in the corresponding columns for uses."

First of all we notice that what is meant by "commodity sectors" are the sectors: agriculture, industry and electricity, and construction, which has been shown in section two of the G.F. (footnote to table 18) to include "costs of machine installation and fixed equipment". In section two, the total production of these sectors at the so-called "producer's price" (i.e., concept II), was estimated as follows:

Agriculture	736
Industry (& electricity)	1814
Construction	122
Total Commodity Sectors	2672
If to this we add, Trade margins on them	220.9 (given before)
then, the total at user's price is	2892.9

exactly as given in the above table. If this is so, the table should have included somewhere in its details, the value of production in the three sectors. Since the first row in the above table gives the "sub-total" as being the total of the two sectors agriculture and industry, this means that the total value of production in the sector "construction" is included somewhere between that row and the last row in the table. Now, we have already checked the "trade margins" item. On the other hand the second and third rows include only a part of that production, viz., "the value added in construction", which is found in Section Two of the G.F. to be 51.0 = 1.5 + 49.5. Hence there is no alternative but that the "balance item net commodity taxes" includes the remainder,71, which is the value of the so-called "value of production requirements" i.e., secondary inputs of the construction sector.

Failing to account for this fact, the constructors of the table have added the second footnote to the table. Now, if it is true that the reason for the two sides not balancing is that the commodity balances on exports are included in the right-hand side of the table,but not on the left-hand side, then we should have expected that value at users' prices (or what has been called market prices) in the right-hand side should exceed that of the left-hand side by the amount of duties on exports. However, comparing the two totals we find that resources exceed uses by 71.3, if we allow for export duties. Taking into consideration that some of the taxes appearing in the uses side are not included in the resources side (e.g.,1.3 paid by consumers on final consumption of electricity and included in the total of 127.2 as shown in Tab.37 of Section Five), it can be seen that the discrepancy between the two sides even exceeds the amount of inputs to construction. On the other

hand the total 120.0 millions [16] of increase in stocks appearing in Table (28a) quoted above, differs from 117.6 estimated for the same item in Section Ten on the National Budget (p.204). In other words, the balancing item net commodity taxes conceals a number of errors in the estimates.

It is clear also that the imposition of certain duties on exports in the uses side without a counterpart in the production column does not justify the suggested remark in the tables. Two ways are open:

1. To drop export duties completely from both sides, and ensure their equality, adding a footnote to indicate their magnitude; or
2. To add their value to taxes on production, and mentioning their magnitude in a footnote, exactly as has been done with respect to taxes on exported cigarettes (though the latter was originally imposed on imports and redeemed on exportation)

In either case the basic assumption is that the two sides of the balances should be equal when estimated at user's price. The equation might be raised as to whether this is a realistic assumption to be made.

To answer this question we might refer to other studies on this problem. The general practice is to estimate all uses at user's prices. In such a case all available statistical information on expenditures is obtained and the estimates directly made. On other hand, since information on production at production costs and imports CIF is readily available, these values are presented on the resources side. To balance the two sides, a separate item (for each commodity or group of commodities) indicates indirect taxes on each, while another indicates trade margins. The equality of the two sides is ensured by estimating exports FOB.[17] It is clear that this treatment enables an analysis of the various types of demand, since they are represented at the prices determining their demand in the market. On the other hand the same would apply to the analysis of production and imports at the indicated prices.

16) It is difficult to reconcile this estimate for the fifth year with that for the five-years period given for changes in stocks in Table (1) p.24, to the same amount. It implies that no changes in stocks occur during the first four years, an unrealistic assumption to be made. In Section Ten,p.205 it is indicated that of this 30 millions are increases of stocks of producers' goods, and the rest of consumers' goods, and increased exports were suggested as a solution. Such assumptions are difficult to accept at such a stage of industrial development,something that turned to be proved by hard experience.

17) See for example, Ph.Berthet: Proposals for an Intermediate System of National Accounts for use in African Countries - E/CN.14/NAC/7, an ECA document, 1962 Table 2, App.1
See also, M.Courcier: Manuel de Comptabilité Nationale,Ministère de Coopération,Paris, 1962, pp. 145 and 155.

Going back to the inconsistencies observed in Tables (28), the reason for them can be explained as follows. The total of intermediate consumption at user's cost for all commodity sectors in the fifth year is found from table (32), p.78, to be 1569.1. Out of this 71.0 is taken by the construction sector. Hence the remaining sectors (agriculture and industry) take the rest, 1498.1, which is exactly the total of intermediate demand as given in Table (28a) quoted above. If they are excluded from intermediate demand, the inputs of the construction sector should be included in the final demand "investment".18) But this ensures the balance of the two sectors, agriculture and industry, as estimated by the "sub-total" appearing in Table (28a). Now to complete the column of investment, we have to add only the value added in construction which is given in the next part of the table. But if we add this part only in local production in the side of resources, the requirements of production of that sector have still to be accounted for. If they are included in the row for value at "user's factor cost", they have to be also included in the uses which would be difficult to ascribe to either investment, since they have beem already included, or intermediate demand, since they are not the production of the sector construction itself. The solution of that difficulty was through the inclusion of that part in indirect taxes, but then it stops to deserve its meaning, and the footnote given becomes misleading. If we insist to consider the construction sector as a commodity producing sector, we should identify its products, represent the whole of its production as resources, consider its inputs as intermediate consumption (as has been done in Table 32) and introduce in the investment column its products. But this would raise some difficulties in decomposing investment into production costs, trade margins and taxes. But if we really want to consider investment activity as a sector receiving inputs from production sectors directly and constructing all its components, then we have to do without construction as a commodity sector falling within the current production sphere, and replace it by a services sector, viz. that of contractors which offers its services to those who are providing it with materials,i.e., investors; exactly as we treat services of tailors with respect to consumption of textile materials. Whichever approach is chosen, the whole treatment should be consistent. Once more the treatment within the framework of an interflow table is found more convenient since it ensures the consistency of the whole set of estimates. The following sections will show what we mean by that.

7- THE INTERFLOW TABLE AT USER'S VALUES:

Let us consider a closed economy of three production sectors and one single final demand sector. The third production sector is

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18) A part of the output of construction usually goes as intermediate consumption of other sectors. However the row for that sector in Table (32) is empty, which means that the total output went directly to investment.

that of trade, finance and transport services, i.e., the recepients of trade margins. Suppose that for a total production whose production costs are 100, an ad valorem tax of 20 is imposed on all products of sector (a), and of 25 on sector (b), while nothing is imposed on (c), being a services sector, normally not liable to indirect taxes. In other words the producer's selling prices (for the 100) are 120, 125 and 100 respectively. These latter prices are the same for all types of users without any differentiation as regards taxation which is merely a simplifying assumption, not affecting the main line of argument.

Further, suppose that the trade margins are also similar for all types of users and for all products of a given sector. Let these margins be 15%, 16% and 0 on the selling price of the sectors respectively. Here again sector (c) being a services sector, delivers its products to their users without any intermediary. This means that for the product costing 100, the margins are : $\frac{15}{100}$ x 120 = 18, for (a), and $\frac{16}{100}$ x 125 = 20 for (b). These assumptions can be summarised as follows:

Sector	Costs of Production	Indirect Taxes	Producer's Selling Price	Trade Margins	User's Price
a	100	20	120	18	138
b	100	25	125	20	145
c	100	-	100	-	100

The set of simplifying assumptions are:

1. The economy is closed, which helps us to avoid problems due to foreign trade.
2. Final demand is grouped in one sector, including both current expenditures of administration and households, as well as investment.
3. Three production sectors only are distinguished. The extension to a larger number is simple.
4. No separate treatment is given to saving; but this does not affect inter-sectoral commodity flows.
5. Similarly no direct taxes are involved
6. Indirect taxes relate to all products of a given sector without differentiation among the various users. Only ad valorem taxes are considered at this stage.
7. Similarly trade margins are the same for all types of users.

It can be shown that the relaxation of any of these simplifying assumptions will not affect the basic arguments involved.

Taking that into consideration, we can summarize the activities of the whole economy in a simple interflow table. Table (1) records all transactions at purchaser's or user's values:

Table (1) - Interflow Table at User's Price

Deliveries of	Purchases of					Total Deliveries
	Receiving Sectors			Total Inter-mediate	Final Demand	
	a	b	c			
Delivering Sectors:						
a	27.6	138.0	13.8	179.4	96.6	276.0
b	145.0	87.0	43.5	275.5	159.5	435.0
c	36.0	60.0	-	96.0	40.0	136.0
Total Inputs	208.6	285.0	57.3	550.9	296.1	847.0
Factor Services	27.4	75.0	78.7	181.1	68.9	250.0
Indirect Taxes	40.0	75.0	-	115.0	-	115.0
Total Purchases (Output)	276.0	435.0	136.0	847.0	365.0	1212.0

The main characteristics of the economic structure represented by the above table are:

1. The first rows represent the flows of goods and services at user's value purchased (and simultaneously used) by each of the production sectors and by the final demand sectors. In other words, the first two rows show the values of intermediate and final demand on the products of sectors (a) and (b) at the values which they cost their purchasers. Thus the inputs in the first three columns represent the total cost of materials to the producers in these sectors. Further, the flows into final demand are evaluated at the prices which virtually determine the volume of demand.

2. Coming to the third row, that of sector (c) which is responsible for the marketing services, we notice that the services they render with respect to the flows in the first two rows are already incorporated in their values. Hence all flows appearing in this row are independent of the first two. For final demand, they show

those services which are rendered directly to final users; e.g., passengers' transport, financial services, etc... (the 40 in the table). For intermediate demand these flows represent in fact the margins paid by all users in order to obtain the products of the other two sectors. In other words they actually include those margins which were already incorporated in the first two rows (as well as margins on final demand). This means that this amount is in fact counted twice, exactly as in the inputs which are (physically) included in total output.

3. The fourth row represents the total of inputs in the same way we define the value of production. In fact this row includes besides actual inputs to the production sectors (those in a and b), the values of the margins which are not due to the activities of the given sectors, and hence have to be subtracted from the total value of output in order to arrive at the actual value added by the sectors.

4. Hence the fifth and sixth rows give the values of the value added due to the activities of the various sectors, subdivided into two parts: payments made to the primary factors in connection with their services, and indirect taxes paid by each sector according to the size of its output. It can be seen that final demand sectors generate some value added directly, in the shape of e.g. wages and salaries to domestic services, and government employees.

5. The total of the fifth row determines the total gross value added at factor cost. Out of this 181.1 represents that part of it which is due to the activity of the business sector, while 68.9 is due to the activities of the final demand sectors.

6. The total of the sixth row gives the (net) indirect taxes generated in the economy; When added to the previous row, we obtain a total of 250 + 115 = 365 which is equal to the total of the final demand column, both equal to national product at <u>market prices</u>.

7. The table satisfies the familiar identities, viz., the equality of the total of a given row and that of the corresponding column for the same sector. It is due to this that the total of final demand is equal to the sum of totals in the fifth and sixth rows. This equality for the production sectors justifies the treatment of all flows at user's prices: to each output we add all trade margins and all indirect taxes relating to it.

8. The gross value added in the business sector (at market prices) is 181.1 + 115.0 = 296.1, which is the amount shown in the fourth row of the final demand column. This implies that this value added is just equal to the amount purchased by final demand from the business sector. Similarly the fifth row item in that column

shows that the value added in the final demand sectors is just equal to the final demand on that sector, whose members are primary factors. The latter of these equalities is obvious. But the former one is due to the absence of an explicit treatment of savings coupled with differentiation between consumption and investment in the consolidated final demand column, as well as for the absence of foreign trade. However, this does not affect the validity of the remaining characteristics, and it can be taken into account within the framework of a more elaborate interflow table.

Thus it can be seen that the above table preserves the basic identity:

GNP at market prices = GNP at factor cost + indirect taxes (net)

Further, in spite of the fact that total production is evaluated at user's prices, the total of 847 shown in the table (for the business sector) includes a certain amount of duplication, due to the fact that trade margins are counted twice, once in the first two rows, and another time in the third row. This duplication is removed by counting this amount once more in inputs (96) so that when deducting the inputs thus valued (550.9), we obtain the value added (in the business sector) exactly.

It is clear that the input-output matrix represented by the above table is also consistent. However, in practice, row (c) introduces certain disparities when the rates of trade margins differ as between various types of users. The entries in (c) are not necessarily related to the other entries in the same column, a phenomenon which is inconvenient for input-output analysis. But what are the implications with respect to commodity balances ?

Suppose that each sector produces a unique group of commodities. We can assume that a more detailed information on commodity flows simply means that we differentiate more sectors; but the underlying principles remain the same. The summary balances can be represented as follows:

Table (1-a) Commodity and services balances at user's prices

Sectors (Commodity groups)	Resources (local production only)				Uses	
	Produc.at Prod.costs	Indirect Taxes	Trade Margins	Total Prod at User Pr.	Interme-diate demand	Final demand
Com.Sectors						
a	200.0	40.0	36.0	276.0	179.4	96.6
b	300.0	75.0	60.0	435.0	275.5	159.5
a + b	500.0	115.0	96.0	711.0	454.9	256.1
c	136.0	-	-	136.0	96.0	40.0
Grand Total	636.0	115.0	96.0	847.0	550.9	296.1

The first two rows summarise the balances for commodity sectors, while the services sector (c) is given in the second part. Total resources are represented by local production only since no foreign trade is considered. The uses are given at user's prices. The above table is similar to the one given in the General Frame, apart from the fact that for each commodity group we now have both taxes and margins included instead of representing these items as totals for all sectors. On the other hand, all uses are directly given in user's prices rather than production costs. The equality of both commodity group details and the grand totals are ensured as a consequence of the balance equations satisfied by the interflow matrix.

8- VALUATION AT PRODUCER'S SELLING PRICE:

Let us now allocate the margins to the receiving sectors (as they should) rather than to delivering sectors. Since indirect taxes remain included in the items of the delivering columns, we have to evaluate all flows at producer's selling prices, as in Table (2):

Table (2)- Interflow Table at producer's selling prices

Deliveries of	Purchases of Reciving Sectors			Total Intermediate	Final Demand	Total Deliveries
	a	b	c			
Delivering Sectors:						
a	24.0	120.0	12.0	156.0	84.0	240.0
b	125.0	75.0	37.5	237.5	137.5	375.0
c	23.6	30.0	7.8	61.4	74.6	136.0
Total Inputs	172.6	225.0	57.3	454.9	296.1	751.0
Factor Services	27.4	75.0	78.7	181.1	68.9	250.0
Indirect Taxes	40.0	75.0	-	115.0	-	115.0
Total Purchases (Output)	240.0	375.0	136.0	751.0	365.0	1116.0

Considering the first two rows, we find that they represent inputs to the business sectors at selling rather than purchase prices. But this does not cover the total amount which they did cost to the using sectors; and it is therefore necessary to add in the third row trade margins on those inputs. This ensures that total inputs represented in the fourth row virtually estimate intermediate consumption whose total is 454.9 as given in Table (1-a). The same applies to the elements of

final demand, whose total is still 296.1 as before. Each of the first two items is netted from its margins (12.6 and 22.0 respectively), hence the item in (c)adds up to 74.6, which is composed of the 40.0 delivered directly to final demand plus the total margins, 34.6, on the former two. Once more the total of final demand is the national product at market prices. This is due to the fact that we still incorporate in the values of outputs indirect taxes imposed on them, while the margins are taken care of in (c).

The total of production is now found to be 751.0, which falls below the corresponding estimates in Table (1) by 96.0, the total of margins on intermediate consumption which was duplicated there. Since total intermediate consumption was estimated at 454.9 without duplication, the gross value added in the business sector remains 296.1 as before. On the other hand, knowing that total indirect taxes is 115.0, we can calculate value added at factor cost which is 181.1 plus 68.9 = 250.0 as before. In other words, exactly as in Table (1), the information in Table (1) gives estimates at market prices for the value added in the system.

But what are the implications for input-output tables and commodity balances ? It is clear that in the input-output matrix of the table all inputs hold fixed relations to the output without being affected by the type of use. The only item which might vary independently of the volume of production is indirect taxes. But if we assume that selling prices would remain unaffect in spite of a change in tax rates (an assumption which is not quite realistic) then we are implying that the share of factor services will be affected in a compensatory way. This would happen only if entrepreneurs react to taxes passively, so that taxes have to be deducted from what they consider their gross income. But, in any case the establishment of a new equilibrium would go even beyond this simple reaction, especially if substitution is allowed. It can be assumed that, given a specific tax structure, and specific rates of remuneration for primary factors, the columns in Table (2) can faithfully display the behaviour of production sectors without any distortions from the user's side.

As to balances, it can be seen that for each commodity group we can only build up the balances in producer's selling prices. To obtain the "totals" for the commodity sectors or the economy at user's prices, we have to add another row for trade margins although they are included in the balance for sector (c). However this implies that for the latter what should be recorded in final uses are only those services which have been sold directly to the consumers, treating all margins as intermediate demand. This can be seen from the following table:

Table (2-a) Commodity and Services balances at producer's selling prices.

Sectors (Commodity Groups)	Resources (local production only)				Uses	
	Produc.at Prod.Costs	Indirect Taxes	Trade Margins	Total Prod. at Sell.Pr.	Inter-mediate Demand	Final Demand
Comm.Sectors						
a	200.0	40.0	-	240.0	156.0	84.0
b	300.0	75.0	-	375.0	237.5	137.5
a + b	500.0	115.0	-	615.0	393.5	221.5
c	136.0	-	-	136.0	96.0	40.0
Trade Margins (a+b) at user's price	-	-	96.0	96.0	61.4	34.6
	500.0	115.0	96.0	711.0	454.9	256.1
Grand Total at						
-Pr.Sell.Pr.	636.0	115.0	-	751.0	489.5	261.5
-User's Price	636.0	115.0	96.0	847.0	550.9	296.1

It has to be remembered that the value of the production of the "services" sector (c) is the same for all three concepts. Hence we can treat its balance in Table (2-a) as in (I-a), which means that we do not represent the total of 74.6 in final demand (as we did in Table 2 to ensure its balance), but only its direct services 40 to final demand. Consequently we have to register in intermediate demand all trade margins 96 and not only 61.4 relating to trade margins on intermediate goods. A check is provided in the fact that when we account for trade margins in the table and treat, at the same time, sector (c) in the above manner, the totals at user's prices, both for commodity sectors and for the whole economy, will be the same as those given in Table (1-a).

But it can be seen that this consistency at user's prices hits the aggregates at producer's selling price. The last row but one in Table (2-a) gives the value of output as 751 which is the same as in Table (2). But it reduces final demand by its trade margins, 34.6, to 261.5 instead of 296.1; and it raises intermediate demand by the same amount, as compared with the fourth row of Table (2). The reason for all this is that trade margins are

virtually the production of one of the sectors included in the balances. Table (2) has treated this problem in one way. Another way of reconstructing the latter Table and at the same time satisfying the provisions of Table (2-a) with respect to the final demand of (c), is to break (c) into two mutually exclusive parts: trade margins on commodity sectors, and direct services. This helps to remove the duplication due to the repetition of margins in order to arrive at totals at user's prices shown in the last row of Table (2-a):

Table (2-b) Another version of the balances at producer's selling prices.

Sectors (Commodity Groups)	Resources (local production only)				Uses	
	Prod.at Prod.Costs	Indirect Taxes	Trade Margins	Total Prod. at Sell.Pr.	Inter-mediate Demand	Final Demand
Comm.Sectors						
a	200.0	40.0	-	240.0	156.0	84.0
b	300.0	75.0	-	375.0	237.5	137.5
Sector (c):						
-Trade Margs.	-	-	96.0	96.0	61.4	34.6
-Direct Serv.	40.0	-	-	40.0	-	40.0
Grand Total	540.0	115.0	96.0	751.0	454.9	296.1

In this table we can also calculate the total for commodity sectors at user's prices.

9- VALUATION AT PRODUCTION COSTS:

A further step can be taken in order to net the value of the flows from additions due to marketing rather than production activities. This is done by subtracting all indirect taxes as well as margins, both from inputs and outputs. But since the taxes and margins on inputs are included in the costs of production, they should be accounted for in some other place in the column of inputs. Trade margins on inputs are due to the activities of (c) and they should therefore be recorded as inputs from that sector together with any direct services from it (exactly as in table 2).

Also their indirect taxes have to be recorded in the taxes row, i.e., together with primary factors. A similar treatment is made for final demand. Final users appear therefore to pay their own trade margins as well as some indirect taxes as shown in the following table:

Table (3) - Interflow Table at Production Costs

Deliveries of	Purchases of Receiving Sectors a	b	c	Total Intermediate	Final Demand	Total Deliveries
Delivering Sectors						
a	20.0	100.0	10.0	130.0	70.0	200.0
b	100.0	60.0	30.0	190.0	110.0	300.0
c	23.6	30.0	7.8	61.4	74.6	136.0
Total Inputs	143.6	190.0	47.8	381.4	254.6	636.0
Factor Services	27.4	75.0	78.7	181.1	68.9	250.0
Indirect Taxes	29.0	35.0	9.5	73.5	41.5	115.0
Total Purchases (Output)	200.0	300.0	136.0	636.0	365.0	1001.0

In other words those margins or indirect taxes in the system which are not included in the estimates of output (totalling 636) in relation to inputs, are included in the final demand column, so that its total is as before national product at market prices. Since the total of indirect taxes remains as before, 115, the value at factor cost is also preserved. However, subtracting total inputs (381.4) from total production (636.0), we find that the value added in the business sector, is 254.6 which exceeds value added at factor cost (181.1) by the part of indirect taxes on intermediate goods (73.5) and not by the whole amount of taxes. To arrive at value added at market prices we have also to add 41.5, the value of taxes on goods consumed finally. Thus in spite of the fact that we estimated outputs free from taxes, we had to account for those taxes by some other means: for users rather than producers, which is a mere reallocation of the items of the values transacted in the market. In other words, we are still within the limits of the market prices principle, rather than the factor cost principle.

The present principle of valuation (usually referred to as producer's prices) is generally considered more suitable for construction of input-

output relations. Both secondary inputs and outputs are evaluated at prices which are not affected by the distribution of final output among members of final demand, something not related to conditions of production. But it has to be remembered that the value added calculated as the difference between outputs and inputs in Table (3) includes taxes on inputs. If a change is expected in taxation, then we have to take care of those effects on the value added. On the other hand, the values of uses in that table are not directly relevant to the determination of their market demand, a fact which has to be taken into consideration in open models.

To construct balances, we have to treat (c) as in Table (2-a) for the same reasons given in the previous section. It has to be remembered also that this sector does not pay taxes. To restore the picture at user's prices, we include in the following table both trade margins and indirect taxes on commodity sectors:

Table (3-a) Commodity and Services balances at costs of production

Sectors (Commodity Groups)	Resources (local production only)				Uses	
	Produc.at Prod.Costs	Indirect Taxes	Trade Margins	Total Prod. at Prod. Cost	Inter-mediate Demand	Final Demand
Comm.Sectors						
a	200.0	-	-	200.0	130.0	70.0
b	300.0	-	-	300.0	190.0	110.0
a + b	500.0	-	-	500.0	320.0	180.0
c	136.0	-	-	136.0	96.0	40.0
Indirect Taxes	-	115.0	-	115.0	73.5	41.5
Trade Margins	-	-	96.0	96.0	61.4	34.6
Grand Total at						
-Prouc.Costs	636.0	-	-	636.0	416.0	220.0
-user's prices	636.0	115.0	96.0	847.0	550.9	296.1

As in Table (2-a), total final demand is accounted for at user's prices. But to obtain that we have to treat (c) in the same way. The total at production costs, 220, is then found to be smaller than that value by 76.1 which is the total of trade margins and indirect taxes incorporated. A table similar to (2-b) can be constructed, but it will not

involve any further complication since the extra item, which is indirect taxes, does not belong to any production sector.

To conclude, we can say that whichever of the three concepts (I)-(III) we use to evaluate production, we are liable to obtain final demand at user's prices. The question might be raised as to whether this is a normal conclusion for concept (I) in particular. To put it in another way, what could have happened if we dropped taxes altogether from our consideration ? But this leads in fact to concept (V) to which we now turn.

10- IMPLICATIONS OF PARTIAL PRODUCTION COSTS CONCEPT:

Consider once more Table (3), and let us assume that we omit only the row for indirect taxes, thus seemingly freeing the system from "all" indirect taxes. It can be seen that the table will become imbalanced, since the column totals will become different from the row totals (which remain the same as the corresponding colum and row totals in 3). The reason can be easily seen to be that whereas we have estimated production net of its own taxes and those of its inputs, the inputs themselves have not been treated in the same way. Together with final demand components they are uses of a production evaluated at full production cost. To ensure consistency in the table, we have to evaluate all commodity flows, whether outputs, inputs or final demand, at the same price, viz., partial production costs. To obtain this latter we divide the values of productions at partial costs (200 - 29 = 171; 300 - 35 = 265; 136 - 9.5 = 126.5) by the corresponding values at full costs, and multiply each row for a sector by the corresponding ratio. But when adding up the columns they will be less than the corresponding rows as follows:

Table (4) Interflow Table at Partial Costs of Production

Deliveries of	Purchases of					Total Deliveries
	Receiving Sectors			Total Intermediate	Final Demand	
	a	b	c			
Delivering Sectors						
a	17.1	85.5	8.6	111.2	59.8	171.0
b	88.3	53.0	26.5	167.8	97.2	265.0
c.	22.0	27.9	7.2	57.1	69.4	126.5
Total Inputs	127.4	166.4	42.3	336.1	226.4	562.5
Factor Services	27.4	75.0	78.7	181.1	68.9	250.0
Residual (Taxes)	16.2	23.6	5.5	45.3	28.2	73.5
Total Purchases (Output)	171.0	265.0	126.5	562.5	323.5	886.0

The difference between row and column totals is indicated in the last row but one, under the name "residual (taxes)". It is true that we have omitted now all the 115 indirect taxes shown in Table (3), hence the grand total of the table is smaller by that amount (1001-886). In spite of that we have a new set of taxes whose total is 73.5, which is in fact the taxes on intermediate consumption as can be seen from Table (3). In other words, the concept of partial costs succeeded only in reallocating taxes on direct inputs between production sectors (45.3) and final demand (28.2). For final demand this means that besides the original 41.5 taxes on goods evaluated at full production cost, there is an extra amount of taxes on the inputs of those goods. Similarly, it can be concluded that, besides the 73.5 taxes paid directly on the inputs of production, there is in fact another 45.3 which is taxes on the inputs of those inputs. We are exactly in the same situation which arises when we want to calculate all direct and indirect requirements of production.

Hence, we conclude that the partial production concept goes only a part of the way towards netting the whole system from indirect taxes. Final demand is virtually less than the value at market prices, but it is still larger than the value at factor costs. Further steps have to be taken in order to exclude all indirect taxes involved. The reader can repeat the same process once or twice, by treating Table (4) as if it were at full costs, and other tables in partial costs. It can be seen that in the first of these steps, the residual taxes will be 45.3, although we would have already omitted 73.5. The process goes on indefinitely but it can be seen to be convergent, in the sense that every time we have to reduce the values by an amount smaller than in the round before, owing to the "leakage" due to final demand. But is it not possible to use the technique familiar in estimating direct and indirect requirements? This is in fact the case as can be seen from the following section.

11- THE FACTOR COST PRINCIPLE:

Let us divide the elements of the first three rows in Table (2) by the values of the corresponding outputs at producer's selling price, to obtain the following matrix of technical coefficients:

$$\underline{\underline{A}} = \begin{bmatrix} 0.1000 & 0.3200 & 0.0882 \\ 0.5208 & 0.2000 & 0.2757 \\ 0.0983 & 0.0800 & 0.0574 \end{bmatrix}$$

Subtracting this matrix from the unit matrix $\underline{\underline{I}}$, and calculating the inverse we obtain

$$(\underline{\underline{I}}-\underline{\underline{A}})^{-1} = \begin{bmatrix} 1.5172 & 0.6398 & 0.3291 \\ 1.0737 & 1.7405 & 0.6095 \\ 0.2493 & 0.2145 & 1.1469 \end{bmatrix}$$

As is well known each of the three columns gives the total direct

and indirect requirements of production from the production sectors to obtain one unit of the output of the respective sector which is not required any more as inputs, but is free for any type of use outside the production system (i.e., final demand).[19]

Now if we add up the columns of A, we obtain the total inputs required directly for the production of one unit of output of the corresponding sector. Dividing the remaining rows of Table (2) by the column totals, we obtain similar coefficients for value added, which is the remainder from unity (0.2809, 0.4000, 0.5787), divided into factor payments and indirect taxes as shown in the following table :

Decomposition of one unit of output

Components	Sector		
	a	b	c
Secondary Inputs	0.7191	0.6000	0.4213
Primary Inputs	0.1142	0.2000	0.5787
Indirect Taxes	0.1667	0.2000	0
Output	1.0000	1.0000	1.0000

If we multiply the value added component of each sector (sum of second and third rows) into the total requirements (direct and indirect) calculated in the inverse matrix of that sector from a certain delivering sector, we obtain the total value added generated in the process of producing all those requirements. But adding up for all direct and indirect inputs required for one unit of output, we should obtain a total value added just sufficient to meet final demand on that unit, i.e., unity. For example, we have for sector (a):

Total direct and indirect valued added = 0.2809 x 1.5172 + 0.4000 x 1.0737 + 0.5787 x 0.2493 = 1.0000

Similarly for the other two sectors.

19) The diagonal elements include the unit output, hence they are normally greater than one. Other elements exceed one in our inverse owing to the high interdependence assumed in the original matrix.

It is clear that if we subdivide the value added coefficients into their two components: factor payments and indirect taxes, and repeat the same process, we would be in fact subdividing the unit value added obtained directly and indirectly into these two components. Thus using the row of factor payments in the last table we find that for sector (a):

Total direct and indirect factor payments =

$$0.1142 \times 1.5172 + 0.2000 \times 1.0737 + 0.5787 \times 0.2493 = 0.5324$$

Also, when using the row of indirect taxes, we obtain for the same sector :

Total direct and indirect contents of indirect taxes =

$$0.1667 \times 1.5172 + 0.2000 \times 1.0737 + 0 \times 0.2493 = 0.4676$$

which is obviously equal to 1.0000 - 0.5324. We can summarize these results as follows:

Direct and Indirect Contents of Output

Components	Sector		
	a	b	c
Factor payments	0.5324	0.5453	0.8232
Indirect Taxes	0.4676	0.4547	0.1768
Total	1.0000	1.0000	1.0000

In other words each unit of output at producer's selling price is "eventually" composed of these factors. Similar calculations can be made when we have more detailed information about the breakdown of primary inputs. For example if we differentiate labour, we can calculate the wage component of the value of output of each sector.

Since the output costing one unit in the market contains the ratio given in the last table as factor payments, it is clear that for any value of that output, its virtual contents of factor payments can be obtained by multiplying it by the relevant ratio. Therefore, if we multiply all the elements of the first row of Table (2) by 0.5324, we obtain the factor payments contents of those elements, i.e., their values at factor cost. Multiplying the second row by 0.5453, and the third by 0.8232, we can transform all inputs into their factor cost equivalent. Adding the row of factor payments as it were, we can check that the total of each column is exactly the same as that obtained for the corresponding row, which means that we have arrived at a representation of all commodity flows: inputs and outputs at factor costs, as shown in Table (5):

Table (5) Interflow Table at Factor Costs

Deliveries of	Purchases of Receiving Sectors a	b	c	Total Intermediate	Final Demand	Total Deliveries
Delivering Sectors						
a	12.8	63.9	6.4	83.1	44.7	127.8
b	68.2	40.9	20.4	129.5	75.0	204.5
c	19.4	24.7	6.4	50.5	61.4	111.9
Total Inputs	100.4	129.5	33.2	263.1	181.1	444.2
Factor Services	27.4	75.0	78.7	181.1	68.9	250.0
Total Purchases (Output)	127.8	204.5	111.9	444.2	250.0	694.2

Total output at factor costs is found to be 444.2, which is much less than the value at production costs. Total intermediate consumption at factor costs also is 263.1 which, when subtracted from output, gives the value added at factor cost, 181,1. The table represents therefore the situation if the economy would produce the same amounts of "real" output, at a new set of prices completely free from all indirect taxes.

Comparing output values in Tables (2)-(5) we can estimate the various tax components in producer's selling values. The results can be summarized as follows:

	Outputs a	b	c	Total	Total Inputs	Value Added
Value of Output at:						
a) Producer's Selling Price	240.0	375.0	136.0	751.0	454.9	296.1
b) Production Costs	200.0	300.0	136.0	636.0	381.4	254.6
c) Partial Prod.Costs	171.0	265.0	126.5	562.5	336.1	226.4
d)Factor Costs	127.8	204.5	111.9	444.2	263.1	181.1
Indirect Taxes on:						
e) Direct Inputs (b-c)	29.0	35.0	9.5	73.5		
f) Indirect Inputs (c-d)	43.2	60.5	14.6	118.3		
g) Total Inputs (b-d)	72.2	95.5	24.1	191.8		
h) Output (a-b)	40.0	75.0	-	115.0		
Total indirect taxes (a-d)	112.2	170.5	24.1	306.8		

The total amount of indirect taxation which is embodied in the system, even after deducting indirect taxes on output, is quite large. Taxes on direct inputs are 73.5, on the inputs of those inputs 45,3, and on all remaining chain of indirect inputs 73.0. Value added at factor cost is directly obtained only when the factor cost principle is used.

Using Table (5) we can calculate the matrix of technical coefficients which would be now expressed in terms of the new prices. It is evident that the diagonal elements will remain the same as before (apart from rounding-off errors) since they are changed in the same proportion as the corresponding output. They are found to be

$$\underline{\underline{A}} = \begin{bmatrix} 0.1002 & 0.3125 & 0.0572 \\ 0.5336 & 0.2000 & 0.1823 \\ 0.1518 & 0.1208 & 0.0572 \end{bmatrix}$$

The diagonal elements of the inverse should be also the same as before, as can be seen from :

$$(\underline{\underline{I}} - \underline{\underline{A}})^{-1} = \begin{bmatrix} 1.5179 & 0.6251 & 0.2129 \\ 1.1002 & 1.7407 & 0.4034 \\ 0.3854 & 0.3237 & 1.1467 \end{bmatrix}$$

This matrix can be used for the familiar types of problems, once we obtain values in factor costs. We shall have the occasion to use it in what follows.

12- EFFECTS OF CHANGES INDIRECT TAXES:

It has been always alledged that the reasons for using such concepts as production costs or partial production costs is to avoid complications arising as a result of taxes. But we have shown that none of those concepts is actually free from tax influences. Removal of taxes was found to be equivalent to the adoption of a new set of output prices. So let us study the problem in that spirit, i.e., as one of changes in prices.

Suppose we start with a base period in which prices are all equal to 1.00. Hence we consider that the coefficients obtained by dividing the elements in a table at producer's selling prices of the base period are in fact proportional to the physical flows (at base year prices). Suppose now that taxes are changed, and consequently prices were changed (in terms of base year prices) to p_i (i denoting the sector). The problem is then to calculate the p's which will take place if the only change in the economy

was that of tax rates.

The base period coefficients can be denoted by a_{ij} for secondary inputs, v_j for factor payments, and t_j for tax rates, and the sum of these coefficients is obviously the value of the unit output, i.e., unity. It is immaterial here whether the tax rate is related to the value or the volume of the commodity, since the two flows are proportionate at the given prices. The table of coefficients in the base period can be written as follows:

Base Period Coefficients

Deliveries From	Purchases of Sector:		
j =	1	2	3
Sector i =			
1	a_{11}	a_{12}	a_{13}
2	a_{21}	a_{22}	a_{23}
3	a_{31}	a_{32}	a_{33}
Factor Services	v_1	v_2	v_3
Indirect Taxes	t_1	t_2	t_3
Value of Output	1.0	1.0	1.0

In particular we can consider the base period as one with no indirect taxes (i.e., all flows evaluated at factor costs), with all $t_j = 0$. If taxes are changed from t_j to s_j, with at least one of the s's different from the original t, then it becomes essential to differentiate between specific and ad valorem taxes:

i) Specific Taxes:

It is clear that for one unit of base period value the new value will be p_j, and the tax paid will be s_j. We assume that the value added will remain unaffected, v_j. But the flows of secondary inputs have to be valued at the new prices. The situation can be summarized as follows:

Coefficients in New Values After Change of Taxes

Deliveries from	Purchases of Sector :		
	j = 1	2	3
Sector i =			
1	$p_1 a_{11}$	$p_1 a_{12}$	$p_1 a_{13}$
2	$p_2 a_{21}$	$p_2 a_{22}$	$p_2 a_{23}$
3	$p_3 a_{31}$	$p_3 a_{32}$	$p_3 a_{33}$
Factor Services	v_1	v_2	v_3
Indirect Taxes	s_1	s_2	s_3
Value of Output	p_1	p_2	p_3

It is evident that the new (relative) prices should ensure the equality of each column total to the corresponding price. For any column j, we should have :

$$p_j = p_1 a_{1j} + a_2 p_{2j} + a_3 p_{3j} + v_j + s_j \qquad (j = 1,2,3)$$

or, gathering all terms in the unknown p's in the left-hand side, and leaving the knowns in the right-hand side, the system of three equations can be written in matrix form as follows:

$$(p_1 \quad p_2 \quad p_3) \cdot \begin{bmatrix} (1-a_{11}) & -a_{12} & -a_{13} \\ -a_{21} & (1-a_{22}) & -a_{23} \\ -a_{31} & -a_{32} & (1-a_{33}) \end{bmatrix}$$

$$= (v_1+s_1 \qquad v_2+s_2 \qquad v_3+s_3)$$

Or, denoting the vectors involved by:

$$\underline{p} = (p_1 \quad p_2 \quad p_3)$$

& $\underline{s} + \underline{v} = (s_1+v_1 \qquad s_2+v_2 \qquad s_3+v_3)$

the system can be concisely expressed as:

$$\underline{p} \cdot (\underline{\underline{I}} - \underline{\underline{A}}) = (\underline{s} + \underline{v})$$

Its solution can be easily seen to be:

$$\underline{p} = (s + v) \cdot (\underline{\underline{I}} - \underline{\underline{A}})^{-1}$$

premultiply
In other words, we have to/multiply the inverse matrix by a row vector composed of the new value added coefficients (factors plus new taxes).

Example: Let us assume that the base period was tax free, hence the relevant matrix $\underline{\underline{A}}$ is the one derived from estimates at factor costs, namely the one given towards the end of the last section. Hence the vector $\underline{v}$ is the difference between the totals of the columns of that matrix and unity:

$$\underline{v} = (0.2144 \quad 0.3667 \quad 0.7033)$$

Further, since the tax is affected by the volume rather than the value, we calculate the tax rates by relating the total taxes (40; 75; 0) to the base period values (i.e., values at factor cost):

$$s_1 = \frac{40}{127.8} = 0.3130; \qquad s_2 = \frac{75}{204.5} = 0.3667; \qquad s_3 = 0$$

Hence,

$$\underline{s} + \underline{v} = (0.5274 \quad 0.7334 \quad 0.7033)$$

Multiplying this into the inverse $(\underline{\underline{I}} - \underline{\underline{A}})^{-1}$ given towards the end of last section:

$$\therefore \quad \underline{p} = (0.5274 \quad 0.7334 \quad 0.7033) \begin{bmatrix} 1.5179 & 0.6251 & 0.2129 \\ 1.1002 & 1.7407 & 0.4034 \\ 0.3854 & 0.3237 & 1.1467 \end{bmatrix}$$

$$= (1.8785 \quad 1.8345 \quad 1.2146)$$

These results can be checked, either by multiplying each commodity flow in Table (5) by the corresponding price, and comparing the results with Table (3); or by calculating the prices from these two tables directly:

$$p_1 = \frac{240.0}{127.8} = 1.8779; \qquad p_2 = \frac{375.0}{204.5} = 1.8337; \qquad p_3 = \frac{136.0}{111.9} = 1.2153$$

Apart from rounding-off errors, the estimates are the same up to the last decimal place but one.

ii) Ad Valorem Taxes:

The treatment of this type of taxes differs since for the same quantity of output (same value at base period prices), the tax will change proportionately with the value of that output, i.e., with its price. The coefficients will remain the same as before, except for taxes which become now: $p_j s_j$, rather than s_j. Thus the balancing condition for the column

total becomes:

$$p_j = p_1 a_{1j} + p_2 a_{2j} + p_3 a_{3j} + p_j s_j + v_j \qquad (j = 1, 2, 3)$$

The complete system runs as follows:

$$(p_1 \quad p_2 \quad p_3) \cdot \begin{bmatrix} (1-s_1-a_{11}) & -a_{12} & -a_{13} \\ -a_{21} & (1-s_2-a_{22}) & -a_{23} \\ -a_{31} & -a_{32} & (1-s_3-a_{33}) \end{bmatrix} = (v_1 \quad v_2 \quad v_3)$$

Thus, if we define a matrix $\underline{\underline{S}}$ whose diagonal elements are the tax rates sj, and the non diagonal elements zero, then:

$$\underline{p} = (\underline{\underline{I}} - \underline{\underline{S}} - \underline{\underline{A}}) = \underline{v} \qquad \& \qquad \underline{p} = \underline{v} \cdot (\underline{\underline{I}} - \underline{\underline{S}} - \underline{\underline{A}})^{-1}$$

We have to calculate the new matrix and the corresponding inverse, since we no longer have the familiar inverse.

Example: Suppose the tax rates were as before (relative to selling prices):

$$s_1 = \frac{40}{240} = 0.1667; \qquad s_2 = \frac{75}{375} = 0.2000; \qquad s_3 = 0$$

Using the matrix $\underline{\underline{A}}$ at factor cost, we can compute :

$$(\underline{\underline{I}} - \underline{\underline{S}} - \underline{\underline{A}}) = \begin{bmatrix} 0.7331 & -0.3125 & -0.0572 \\ -0.5336 & 0.6000 & -0.1823 \\ -0.1518 & -0.1208 & 0.9428 \end{bmatrix}$$

The inverse is then premultiplied by $\underline{v}$ only:

$$\underline{p} = (0.2144 \quad 0.3667 \quad 0.7033) \cdot \begin{bmatrix} 2.4294 & 1.3475 & 0.4078 \\ 2.3719 & 3.0499 & 0.7335 \\ 0.6950 & 0.6077 & 1.2203 \end{bmatrix}$$

$$= (1.8794 \quad 1.8347 \quad 1.2146)$$

Again the results check with the prices calculated directly.

The reader can assume different tax rates, calculate the inverse and construct the corresponding interflow table using (5) as a basis and checking the column totals after calculating the taxes according to the rates assumed.[20] The multiplier effects of the ad valorem basis can be immediately felt, if they are imposed on intermediate as well as final goods. The implications for the balances are obvious.

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20) The matrix calculated on the basis of selling prices can also be used in the same manner. In particular putting all $s_j = 0$ we find that the two types of solution coincide, and both yield the estimates of factor cost prices obtained before (relative to the given selling prices).