UNITED ARAB REPUBLIC

THE INSTITUTE OF NATIONAL PLANNING

التخطيط القومي

Memo. No. 584

Economic Models :
Analytical Models and the
decision-making problem

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12 June. 1965

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I - INTRODUCTION:

For a long time economists have been very keen to draw a distinct line between what should be considered as falling within the domain of economic science and what should not. It had been always emphatically stated that economics is concerned with the analysis of "what is" and not "what ought to be". This means that starting from a group of assumptions, an economic scientist can legitimately analyze their implications, and draw the conclusions that would follow logically from their realization. In this sense, economic science had its great value in problems of historical analysis theoretical discussion of potential cases, and, hence, in dealing with problems of forcasting and projection.

Then; where does the problem of economic policy lie? Strangely enough, it was still within the powers of economists to suggest economic policies, and to criticize them, although this right was denied to the science itself. In fact, it was considered as an "art", and hence by definition excluded from the science as such. At best it might appear as chapters under the general heading "applied economics". But how could economists go around the dilema?

The steps usually adopted can be tentatively formalized as follows:

1. The use of economic analysis in detecting a certain problem and determining its causes, (analysis).
2. Pursuing this analysis to indicate the direction in which the automatic forces of the given economy would work (forcasting).
3. Revealing the "undesirable" aspects of these tendencies: This is not derived from economic theory directly, but it follows from the economic implications of those tendencies as judged by an implicit (or explicit) set of "preferences" or "objectives". These preferences and objectives reflect the desires of the policy maker. (statement of preferences).

4. Suggesting the use of certain "policy-instruments" to realize these objectives. This is largely left to the intuition of the economist, and his experience, thus displaying the artistic colour of the approach. (*Selection of instruments*).

5. To "prove" the advisability of the suggested policy by introducing the postulated changes in the selected instruments, into the original model, and showing that *"if"* the suggested changes were introduced, *then* the logical outcomes would be in conformity with the stated preferences (*projection*).

In all such discussions, economists were very keen to stand away from any discussion or advocation of preferences. If the consumer is maximizing his satisfaction, that is his problem. The economist's role is to find out the implications of such an attitude as regards purely economic variables. If the economy as a whole advocates full-employment as an objective, then let us accept that as a point of departure. (As a matter of fact, one could very well question whether it should be really the aim of everybody around stould earn his living the hard way!!) Thus it has been agreed among economists that preferences should not be discussed. They belong to other fields of social behaviour, and hence any attempt at more than their description is non-economic. Consequently any advocation for their alteration, even when based on certain economic arguments, is not itself the concern of economics.

With this - drastic, and sometimes damaging - simplification, the role of economics in dealing with practical problems of policy-making becomes more or less determined. Suppose, for example, that a deflationary process is on its way. The first step is to find out the relevant theoretical model to determine which type of deflation is faced, and what are its actual causes. The second step is to warn policymakers as to the damaging consequences of the persistence of this tendency (e.g., effects on employment, foreign balance, national income, prices, relative incomes, government revenues, etc..). Suppose now that the policy-takers are keen not to let unemployment spread, nor to let the foreign balance fall beyond a certain limit, etc. Further, suppose that they are willing to accept measures related to monetary policy, and that they would accept fiscal measures that would not effect tax rates, or lead to the indulgance in types of expenditure which are competitive with the activities of the private sector. Given these preferences, the economist can refer to economic theory and suggest, e.g., a decrease in the rate of interest, an increase in government expenditure on armament, etc. To prove his point, he would indicate through the use of economic theory, the effects of these measures on the variables reflecting the preferences: national income, the level of employment, and so on.

If this approach is to be practically applied to concrete cases, theoretical discussions have to be coupled with econometric investigation. Values of the parameters have to be properly estimated, and concrete estimates have to be obtained for the suggested policy instruments. The results of these policy measures can be also appraised through further econometric investigations.

Normally these policy recommendations assume a given institutional structure, although they might eventually lead to some fundamental changes in these structures. But the important fact is that, even with a given institutional structure, the solution of a given problem is far from being unique. One can easily accept Lawrence Klein's argument that: "The usual experience in the field of economic policy is that there are about as many types of advice as there are advisors (sometimes even more!)"(1) This is a state of affairs which could not go on indefinitaly especially as the role of public economic policy has gained an increasing weight under

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(1) L. Klein: "The use of econometric models as a guide to economic policy" - Econometrica, 1947, P. 111

various economic systems, however liberal they pretend to be.

The need for an alternative and more realistic approach has become quite evident. To well-trained econometricians, the main source of the difficulty lies in an improper reading of available information; and that was the gest of Klein's treatment after the above-mentioned statement. The way out is clear: To try to find out the best way of extracting the maximum - correct - implications of that information. This means that:

1. A well-formulated theory has to be built and tested through econometric investigation.

2. Collection of data should be given enough care, and should be in conformity with the theoretical set-up as far as possible.

3. Proper methods of econometric analysis should be investigated, and research-workers should avoid copying highly refined statistical methods which proved useful in other fields of statistical inference, but which do not cope fully with conditions under which economic relationships act.

Being an econometrician myself, I strongly recommend this approach, but ... it is necessary but not sufficient. Two qualifications should be mentioned:

1. No amount, however large, of econometric analysis, can prove the correctness of an economic theory. Hence much gain can be obtained from paying more effort to that problem. Specifically, an attempt towards a more rigorous treatment of problems of economic policy within the domain of economic analysis, is quite essential.

2. More attention should be paid to the problem of elaborating preference functions. Economists should not passively yield to "politicians", "social reformers", etc., all the way. They should enable them to bring the composition and the parameters of their preferences to perfection. This is

not a call for the domination of economists; it is rather a call for the domination of scientific economic thinking. One should not forget that economics is a science of "society", and not simply of a single individual. This becomes all the more important when somebody assumes for himself the power (by delegation or otherwise) to "think" for others. If policy recommendations should satisfy preferences, every thing else should follow suit. It is this every thing which has to be brought to focus, and to be considered when preferences are elaborated for purposes of policy-making.

In this paper we shall deal with one point relating to the first problems We shall try to prove in a more rigorous way that the so-called "policy-models" are in fact one branch of economic theory itself, and hence belong to a certain category of analytical models. This is done through a consideration of this latter type of models, and showing that analytical models concerned with the study of the economic behaviour of individual economic units, are in essence policy-models. Once this fact is established, a more rigorous reconsideration of economic policy models would become possible.

II - TWO MAIN TYPES OF ANALYTICAL MODELS:

In previous occasions (1) I have treated analytical models as one single group, in contrast with policy models. Contents of such models were discussed with potential econometric investigation in the background. I would like to distinguish here two main types of analytical models, with the purpose of bringing the policy-making, or rather the decision-making problem into focus.(2)

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(1) M.M.El-Imam: Economic models: Definitions and classification - Memo. 96, INPC.

(2) For a similar - though with a different purpose - subdivision of approaches, see: P. Samuelson: Foundations of economic analysis; P. 258.

These two tapes can be labelled:

(1) Models of action; and, (2) Models of interaction.

In fact these two categories correspond to models concerned with the determination of the behaviour of various economic individuals, and models meant to offer a description of market situations arising from the interction of those behvaiours.

Thus, to the first group belong models of, e.g., consumer behaviour and producer behaviour, whether in the short run or in the long run; static or dynamic. To the second belong models of the markets of a single commodity or of several commodities, and aggregative models of the whole economy. The most familiar models of action might be called - to use Samuelson's terminology - models of "maximizing behaviour". If it is accepted that such models outline a line of action to be adopted by certain individual, given certain conditions and assumptions, then the analogy with the situation of general policy-making bodies becomes evident. The fact that the behaviour of such bodies could be safely assumed as independently determined and based on criteria which are exogenous to most analytical models, was responsible for the blurring of the treatment of policy models for a long time.

With this distinction in mind, let us consider the steps followed in the construction and use of analytical models. These steps can be briefly stated as follows:

1. Definition of the problem.
2. Listing of the variables considered as relevant to the analysis. This stage reflects the theoretical ability of the model-builder and largely determines the possibilities of his success.
3. The selection of those variables which should be explained by the model, viz. the endogenous variables; the remaining variables are considered as exogenous.
4. Statement of assumptions; this is required to define a reasonable degree of abstractness, while still preserving a sufficient amount of operational meaningfulness of the model, to make it worth while of any study at all.

5. Formulation of theorems which offer a solution of the problem. These together with the previous assumptions, complete what might be considered as the structural or basic form of the model.

6. Derivation of theoretical implication of the model, through various types of "solutions" of the model.

III - THE CONSTRUCTION OF INTERACTION MODELS:

Let us consider the above steps with regard to the construction of what we termed "interaction" models. The first step depends on the specific problem under study. Examples are: The study of the market of a given commodity; the determination of the level of activity; the derivation of the rate of growth of national income, etc ...

The completion of the list of variables is a process which is gradually accomplished according to the development of the elaboration of the problem; this step is usually taken together with steps 3 and 4. Thus we start with the statement of the variables immediately related to the problem. In attempting to explain them, other variables are introduced. We go on adding variables and relationships until the number of these latter is sufficient to explain all endogenous variables. This process can be systematically treated by certain tabular or graphical aids, as will be shown in another place.

The introduction of assumptions determines to a large extent the fate of the model. This step would cover decisions with regards to many points:

1. The institutional structure within which the problem is studied;
2. The level of aggregation;
3. The type of relationships to be introduced; in a sense the statement that a given variable x_i is exogenous (hence not needing a separate relationship) can be considered as one of the assmpitons.

4. The properties of the parameters to be included;

5. Hence, the shapes of the various functional relationships.

6. Any boundary or side condition to be satisfied by the solution of the model.

In formulating the set of equations to be included in the model, one can distinguish two main categories:

A) Equations that should hold by definition, or identities; there are 3 types of such equations:

1. Definitional equations, which define a new variable in terms of other variables.

2. Balance equations; which hold true as a result of the belance which should (a postriori) hold between the receipts and the payments of a given sector.

3. Equilibrium conditions, which belong in fact to the set of assumptions (thus failing to hold if the assumptions are incorrect). Examples are equality between supply and demand, or between desired and realised investments.

B) Equations which are based on theoretical assumptions, taking the shape of stochastic equations in econometric investigations. Here again 3 other types can be distinguished according to the source of assumptions made:

1. Institutional equations, describing certain relations originating from traditions and institutional structures.

2. Technical equations describing relations arising from the technical conditions of production.

3. Behaviouristic equations derived largely from assumptions about psychological and sociological factors affecting the behaviour of economic individuals. They reflect, in fact, a way of describing the manner in which decisions are taken.

Although the model can be considered as complete if the number of equations is sufficient to explain endogenous variables, we can in some cases introduce another set of equations which express the pattern of change of exogenous variables. For example, exports or government expenditure might be expressed as a given function of some other exogenous variable, such as world income, or time. One important type of such equations is demographic equations expressing the growth and structure of population or the supply of labour. Such equations would be virtually lying outside the economic model under study, and they can be considered as belonging to another model, economic or not.

IV - LINEAR ECONOMETRIC INTERACTION MODELS:

In spite of the fact that the assumption of linearity limits the range of analysis, it is frequently maintained, especially for purposes of econometric investigation. We shall give here a brief description of the various forms of interaction models, under the assumption of linearity. The main ideas still hold for non-linear cases.

Let us assume that our model contains H variables, $\underline{x}$, of which J are endogenous. Hence, the model should contain J linearly independent relationships explaining the current values of the J endogenous variables, i.e., the J jointly dependent variables $\underline{y}$. Some of these endogenous variables appear with certain lags, and hence should be regarded as being predetermined in the temporal sense, and we denote them by $\underline{z}^{*}$, being D in number. Together with the G exogenous variables, $\underline{z}^{**}$, they form the set of K predetermined variables, $\underline{z}$. If we denote the longest lag appearing for an endogenous variable by m, then $D \leq J.m$; it might be equal to 0, if no such lags appear. Our system of notation can be summarised as follows:

	Jointly Dependent Variables	Predetermined Variables			All variables
		Lagged Join. Dep. Vars.	Exogenous Vars.	Total Pred Vars	
Symbol of Variable	y	z^{*}	z^{**}	z	x
Coefficients	b	c^{*}	c^{**}	c	a
Number of Variables	J	D	G	K	H

For a single point t of observations(T in number), we have $x = \underline{x}(t)$, and similarly for its components. Dropping t, we can express the model as follows:(1)

$$\underline{\underline{A}} \cdot \underline{x}' = \underline{u}' \qquad (1)$$

where $\underline{u}$ is a vector of random disturbances, included for econometric purposes to account for deviations from reality. Under ideal conditions, such disturbances do not appear in the first group of equations. Now:

$$\underline{\underline{A}} = \left[\underline{\underline{B}} \quad \underline{\underline{C}}\right], \qquad \underline{x} = (\underline{y} \quad \underline{z})$$

Hence, the model can be rewritten as:

$$\underline{\underline{B}} \cdot \underline{y}' + \underline{\underline{C}} \cdot \underline{z}' = \underline{u}' \qquad (2)$$

Both (1) and (2) are alternative expressions of the <u>structural form</u> of the model. By virtue of the linear indpendence condition, the square (J. J) matrix $\underline{\underline{B}}$ can be assumed to be non-singular.

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(1) For typing purposes, we denote a matrix by the sign (=) underneath, and a vector by (-) . The transpose is denoted by ('); with the sign denoting rows for vectors, the transpose being columns.

To draw meaningful conclusions as regards the interaction of the various types of behaviour included in this form, we have to obtain a solution of the model. We have to derive a new form of the model, in which each jointly dependent variable is expressed as an explicit function of the magnitudes assumed to be given for the predetermined variables. This leads to three types of derived forms as will be shown in the next section.

V - THREE TYPES OF DERIVED FORMS:

A) THE REDUCED-FORM:

Given that $\underline{\underline{B}}$ is non-singular, we can solve (2) to obtain the reduced form:

$$\underline{Y}' = \underline{\underline{F}} \cdot \underline{z}' + \underline{V}' \qquad (3)$$

where,

$$\underline{\underline{F}} = - \underline{\underline{B}}^{-1} \cdot \underline{\underline{C}} \qquad \underline{V}' = \underline{\underline{B}}^{-1} \cdot \underline{u}' \qquad (4)$$

This means that each jointly-dependent variable is expressed as a function of all predetermined variables only(1). This form has some advantages over the structural form;

1. First, it enables us to estimate the effect of every predtermined variable on each jointly dependent variable. In keynesian terminology, it gives a set of multiplier effects, equal to $-\underline{\underline{B}}^{-1} \cdot \underline{\underline{C}}$. To use the input-output jargon, they represent the totality of direct and indirect effects of the predetermined variables. It helps us to deal with problems of comparative static analysis.

2. Second, it is more convenient for statistical purposes. The fact that the R.H.S. in (3) includes only predetermined variables, makes it possible to apply (in the absence of observation errors) classical methods of estimation, such as the method of least-squares.

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(1) If a constant term is not zero, we can consider one of the exogenous variables be the variable having the value 1 for all points of observation.

3. Given a set of values of predetermined variables, it gives a direct estimate of the values of all jointly dependent variables. This makes it a powerful tool in dealing with problems of forcasting and projection, but the forcasts would be only valid under the assumption of unchanged structure.

This last observation indicates an important disadvantage common to all derived forms: namely, that a change in any part of the structural form would be generally diffused into all parts of the derived system. This means that the knoweldge of $\underline{\underline{F}}$ does not prowide a complete substitute for the knoledge of $\underline{\underline{A}}$.

B) THE SEPARATED FORM:

If the system is dynamic in the sense that $\underline{z}^*$ is not empty, the question arises: what would be the time path of each endogenous variable, that would correspond to a certain initial disturbance, i.e., a once-and-for-all change in the values of exogenous variables?

Suppose that the system achieved a certain equilibrium, such that the consecutive values of all variables remain the same. Suppose now that some (or all) exogenous variables took a set of now values. Owing to the lagged effects on the jointly dependent variables, the new equilibrium levels would be only achieved (if ever) after a number of periods. This means that the initial values after the change would be out of equilibrium. Knowing a sufficient number of such values, we should be able to deduce the time path followed by each endogenous variable. To do this we adopt the following procedure.

Let the longest lag of any endogenous variable be m. The terms containing all current and lagged values of a given endogenous variable in the i-th equation are:

$$b_{ir}y_{rt} + c_{ir_1}\, y_{r,t-1} + c_{ir_2}y_{r,t-2} + \dots + c_{ir_m}y_{r,t-m}$$

with some or all of the coefficients possibly equal to zero. Introducing the "lag operator L", such that

$$x_{t-1} = L.x_t \qquad\qquad x_{t-s} = L^s.x_t \tag{5}$$

and putting:
$$b_{ir} = c_{ir_o}$$

we can rewrite the above expression in the terms of y_r as follow

$$\sum_{s=o}^{m} c_{ir_s}\, y_{r,\,t-s} = \sum_{s=o}^{m} c_{ir_s}\, L^s.y_{rt} = b^{*}_{ir} y_{rt} \tag{6}$$

say, where:

$$b^{*}_{ir} = \sum_{s=o}^{m} c_{ir_s}\, L^s \tag{7}$$

thus a polynomial in L of degree $\leq m$. The system can therefore be rewritten as followes:

$$\underline{\underline{B}}^{*}\underline{y}' + \underline{\underline{C}}^{**}\,\underline{z}^{**'} = \underline{u}' \tag{8}$$

Suppose that the exogenous variables attain a fixed level $\hat{\underline{z}}^{**}$. Then the actual levels of $\underline{y}$ would satisfy:

$$\underline{\underline{B}}^{*}.\,\underline{y}' + \underline{\underline{C}}^{**}\hat{\underline{z}}^{**'} = \underline{u}' \tag{9}$$

If B^{*} is the determinant of $\underline{\underline{B}}^{*}$, then it will be a high-order polynomial in L

$$B^{*} = \sum_{j=o}^{n} b_j.L_j = b_o\left(1 + \sum_{j=1}^{n} \frac{b_j}{b_o} L^j\right) = b_o\left(1 - \sum_{j=1}^{n} d_j.L^j\right) \tag{10}$$

Using the adjoint $\underline{\underline{D}}^{*}$ of $\underline{\underline{B}}^{*}$, whose elements are also polynomials in L, then:

$$(\underline{\underline{D}}^{*}.\underline{\underline{B}}^{*} = .B^{*}.\underline{\underline{I}} = b_o\left(1 - \sum_{j=1}^{n} d_j.L^j\right).\underline{\underline{I}}$$

Premultiplying (9) by $\underline{\underline{D}}^{*}$, dividing by b_o and re-arranging the terms, we obtain:

$$\underline{y}' = \left(\sum_{j=1}^{n} d_j L^j\right).\,\underline{y}' - \frac{1}{b_o}.\,\underline{\underline{D}}^{*}.\,\underline{\underline{C}}^{**}\hat{\underline{z}}^{**} + \underline{w}' \tag{11}$$

where,

$$\left(\sum_{j=1}^{n} d_j L^j\right) \text{ is a scalar,} \qquad \underline{w}' = \frac{1}{b_o} . D^{*} . \underline{u}'$$

This means that each jointly dependent variable is expressed as function of its own lagged values and of the exogenous variables. (incidentally, if $\underline{u}'$ is serially independent, $\underline{w}'$ will not be so).

Under the assumption that $\underline{z}$ is fixed over the period of study then $L^s \hat{Z}^{**}_{jt} = \hat{Z}^{**}_{jt}$ and we can therefore put L in $\underline{\underline{D}}^{*}$ equal to unity and denote the result by $\underline{\underline{D}}^{*}_{(1)}$. It is clear that it will be the adjugate of the matrix $\underline{\underline{B}}^{*}_{(1)}$ defined in the same manner.

Hence,

$$-\frac{1}{b_o} . \underline{\underline{D}}^{*}_{(1)} = -\left(1 - \sum_{j=1}^{n} d_j\right) . \underline{\underline{B}}^{*-1}_{(1)}$$

Thus we can rewrite (11) as follows:

$$\underline{y}' = \left(\sum_{j=1}^{n} d_j L^j\right) . \underline{y}' - \left(1 - \sum_{j=1}^{n} d_j\right) . \underline{\underline{B}}^{*-1}_{(1)} . \underline{\underline{C}}^{**} \hat{\underline{Z}}^{**\prime} + \underline{w}' \qquad (12)$$

This is a set of J equations in which each endogenous variable is separated from all other endogenous variables, and its current value (the jointly-dependent variable) is a function of its own lagged values, and of the exogenous variables, hence the name: separated form. If we know n initial values of y_{rt} we can substitute them, as well as the values of $\hat{z}$ to obtain the value of y_{rt} in the period (n+1) , hence all subsequent values.

C) THE RESOLVED FORM:

The fact that the separated form does not enable us to calculate the value of any variable y_r at a given point of time, t, unless we know all its n preceding values, makes it inconvenient for a general study of the time path of y_r in a general manner. Therefore we have to obtain a general solution of (12).

Now, if the variables $\underline{y}$ should ever attain an equilibrium set of values $\hat{\underline{y}}$, corresponding to $\hat{\underline{z}}^{**}$ such that:

$$L^s . \hat{y}_{rt} = \hat{y}_{rt}$$

then (9) becomes:

$$\underline{\underline{B}}^*_{(1)} . \hat{\underline{y}}' + \underline{\underline{C}}^{**} . \hat{\underline{z}}^{**'} = \underline{0}'$$

from which

$$\hat{\underline{y}}' = - \underline{\underline{B}}^{*-1}_{(1)} . \underline{\underline{C}}^{**} . \hat{\underline{z}}^{**'} \qquad (13)$$

Hence (12) can be rewritten as:

$$\underline{y}' = (\sum_{j=1}^{n} d_j L^j) . \underline{y}' + (1 - \sum_{j=1}^{n} d_j) . \hat{\underline{y}}' + \underline{w}' \qquad (14)$$

Writing:

$$\tilde{\underline{y}} = \underline{y} - \hat{\underline{y}}$$

then $\tilde{\underline{y}}$ are deviations from the equilibrium values, and (14) can be rewritten as follows (noticing that $\sum d_j . \hat{\underline{y}} = \sum d_j L^j \hat{\underline{y}}$):

$$\tilde{\underline{y}}' = (\sum_{j=1}^{n} d_j L^j) . \tilde{\underline{y}}' + \underline{w}' \qquad (15)$$

Ignoring $\underline{w}$, the resulting set of equations is found to be:

$$\frac{1}{b_o} . B^* . \tilde{\underline{y}}' = 0$$

i.e.,

$$B^* . \tilde{y}_r = 0 \qquad (r = 1, \ldots, J) \qquad (16)$$

This means that the deviations of each endogenous variable, follow the same linear difference equation of the n-th order. To obtain a solution, we have to calculate the n roots λ_j (assumed for simplicity to be different) of:

The determinental equation: $B^* = 0$ (17)

Writing:

$$m_j = 1/\lambda_j$$

the general solution is found to be

$$y_{rt} = \sum_{j=1}^{n} K_{rj} . m_j^t \qquad (18)$$

where the K_{rj} are constants obtained by means of n initial conditions.
Thus:

$$y_{rt} = \hat{y}_r + \sum_{j=1}^{n} K_{rj} . m_j^t + w_{rt} \quad (r = 1, \ldots J) \qquad (19)$$

This set of J solutions gives the resolved form, with $\hat{y}_r$ calculated from (13). The importance of this form for the study of growth is quite obvious.

VI - ANALYTICAL MODELS OF ACTION:

By means of interaction models we can answer many questions as regards the values of the endagenous variables, given certain values of the exogenous variables and - in dynamic cases - past values of the endogenous variables. We can also study the effects of changes in these latter variables on the values of the endogenous variables. In all such cases we assume that the various economic units behave as they do, and any change in their behaviour can be in many cases reflacted in changes in the values of the parameters, which can be introduced in the model, and its effects investigated.

In action models we reformulate the question so that it becomes: Given certain outside conditions, how are the economic units going to determine their behaviour? Consequently, how are they going to change their decisions according to changes in those outside conditions?

Now, in any given economy, there are, in principle, a very large number of decision-makers. The first step towards a meaningful study of the decision-making process, is to subdivide this number into a smallar number of groups, each group containing a certain number of homogeneous units, in the sense that they are units satisfying certain conditions of similarity under which they are acting, e.g., consumers, producers in a market of a given nature, etc.. We start by defining a representative unit of each group, which might be a negligible part of the group, or it might be as important as to cover the whole group in the extreme: e.g., the competitive producer vs. the monopolist. Three cases can be distinguished:

Case A: The representative unit has no appreciable effect on any other unit:

In this case we assume that the decision-maker has no appreciable effect on any other unit, whether in his group , or in any other group. Assuming that he reailizes this fact he would be simply <u>reacting passively</u> to the results of their actions. Rationality would require him to limit his actions to those variables which are virtually under his control, without any presumption that he can affect their behaviour. There will be no need to introduce explicitly in the model the manner in which other units reach their decisions with respect to variables affecting his decisions; it suffices only to introduce the values of those variables as decided upon by them.

Thus the model in this case would be simply a <u>model of pure reaction</u>; and if this attitude prevails among all similar decision-makers, <u>harmony</u> among decisions can prevail, and a clear-cut pattern of behaviour becomes apparent.

<u>Case B</u> - <u>The representative unit has distinct effects on others but does not try to exceed it.</u>

In this case the decisions taken by the unit under study would lead others to react in a certain known way. Nevertheless, the decision-maker declines to exercise any direct control over the values of the variables which are virtually under the control of other decision-makers. Even if the representative unit does not know for sure the exact way by which others formulate their decisions, we can, for analytical purposes, introduce it explicitly in the model to account for the total amount of information required by the unit to determine its actions. Thus the model becomes one of <u>action and reaction</u>.

The need to account for the reaction of other units leads to two cases:

1. There is some other model which describes the behaviour of the other units in terms of a single-valued function (1), which

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(1) See next page.

is a function of, among athers, the variables under the control of the representative unit itself. In this case we have to obtain from the other model the relevant function,and confine our model to the study of the behaviour of the representative unit; and there is no need to widen the scope of either model so as to investigate simultaneously the behaviour of all units. Further the account taken of the behaviour of others, ensures that the decisions to be taken by our unit will be in harmony with those of others. This means that the model is one of harmonic uni-action, exactly as in case A above. The model of a monopolist who realises that he cannot suppress completely the demand function of consumers, where this demand function assumes the normal shape, is one example of this type. There is no need to formulate a model which attempts at giving an explanation for behaviour of both the monopolist and the consumers at the same time.

2. The other decision makers can follow more than one pattern of behaviour, either because there is more than one function which give them possibilities of alternative decisions in any given situation, or because that although there is a single function, it is multi-valued(1) In such cases, the model has to study all units at the same time. i.e., it becomes one of multi-action. Two possibilities arise:

a) The decisions taken by the different parties can be consistent, thus yielding a unique solution, and the model is therefore one of harmonic multi-action.

b) The various decisions are inconsistent which leads to contradictory results. The fact that the interest of the various decision-makers are in conflict, does not by itself imply inconsistency. But if this latter exists, the decisions taken by one unit will be contradicting their own aims. Hence we have

(1) It need not be a continuous function: The kinked demand curve is an example.

models of <u>contradicting multi-action</u> (conflict being another side of the story). This is the genesis of the theory of games. The main concern of the model-builder is to reconstruct situations in which harmonisation is regained among decisions, which requires the intoruction of extra information and additional rules. Clearly there is a high probability that harmonisation might be achieved in more then one way.

<u>Case C - The representative unit has distinct effects on others and tries to exceed them.</u> In this case the decision-maker assumes for himself a wider power than he actually possesses in determining the behaviour of others. We can distinguish again two cases:

a) The others yield to this power, which means that the variables which are assumed by the decision-maker to be under his control become eventually so. The model would be then of harmonic multi-action, e.g., the model of a leader and a follower in duopoly.

b) The others do not yield to him, which means that harmony will be lacking, and we have again a case of contradictory multi-action model. Such would be the case of two duopolists each of them trying to lead the market. In mathematical terms, the model is <u>over-determined.</u> To bring harmony back, certain extra conditions have to be added, leading to a number of alternative models each advocating a certain change in behaviour.

VII - <u>MODELS OF HARMONIC ACTION:</u>

Let us assume that a model is to be constructed with the purpose of explaining the behaviour of a single decision-maker who is working in harmony with all others. The first step is to construct the <u>basic form</u> of the model, which corresponds to the structural form in interaction models, though its constitution is somewhat different. The purpose of this form is to state the problem in which decision is required, and the information on which the decision is to be based. The nature of the decision itself is then obtained as solution of this form. Let us consider the steps involved in constructing the model and solving it.

First of all, an <u>ophelimity index</u> is postulated, and the problem is stated as an attempt to optimize its value over a given period of time (by making it a maximum, or a minimum). This index might be an economic variable (profits, costs, etc ...), or it might be an indicator of certain psychological, sociological or political desires (consumer's preferences, welfare,...). The economist is advised not to question the advisability of such index; it is a working assumption, to be taken as a point of departure.

The second step is to build up a set of <u>structural relationships</u>, relating this index to certain economic magnitudes which express the alternative tools available to the decision unit for use in the process of optimization. Both these tools and the index itself are considered as endogenous variables; i.e., the model is responsible for explaining the values they should attain under given conditions.

It is clear that, if the decision-maker is free to give the above-mentioned tools any values at will, the problem would be trivial. In reality, such choices are limited by means of certain <u>restricting conditions</u>. Two types of these restrictions can be distinguished:

1. <u>Limiting conditions</u>, usually taking the shape of equations to be satisfied at the optimum; e.g., the equality of the consumer's total expenditure to his income; the condition imposed on the actions of the producer through his production function.

2. <u>Boundary conditions</u>, usually taking the shape of inequalities, imposing a lower or upper bound on the values of certain variables or functions there-of, reflecting the scarcities of certain resources, the needs to meet certain minimal needs, or the non-negativity condition common to most economic variables. In general the boundary conditions can be reformulated into limiting conditions

by introducing further slack variables which would be added to the list of endogenous variables, with the boundariss rewritten in the shape of further non-negativity conditions.

In elaborating these structural conditions and restrictions, variables outside the control of the decision-maker are introduced. One group of such variables, would be the values of the restrictions themselves. The remaining ones are those which are under the direct control of other decision-makers, or those which are determined through some model of interaction, to which our decision-maker might be a member. All those variables which are not directly or indirectly affected by the decision-maker under study, would be considered as exogenous. Others which are affected by his own behaviour, but are not among his direct tools, nor among his targets, should be considered as endogenous, hence requiring corresponding equations to explain the values they would attain.

This classification of variables into endogenous and exogenous is still useful and meaningful. However, for purposes of decision-making models, it is more convenient to adopt an alternative classification of variables, suggested by Tinbergen.[1] We are suggesting here that Tinbergen's classification is of great value for purposes of analytical action models. In this classification, the group of endogenous variables are subdivided into three subclasses, the fourth subclass corresponding to the whole group of exogenous variables:

1. Target variables: w_i ($i = 1, \ldots, M$). They are those endogenous variables whose values are to be optimized. There might be only one such variable, which might be an economic variable (such as profits), psychological (such as utility), or sociological (such as welfare). In other cases, there might be more than one target variable; e.g., a series of future incomes, or a group containing national income, the level of employment, and the foreign balance.

(1) J. Tinbergen: On the Theory of Economic Policy; Ch. 2.

The ophelimity function would be then an axpression for the ophelimity index in terms of the target variables. Thus the index itself is not included in that set; it can be eliminated through the ophelimity function, (see next section).

2. Instruments or tools: u_i (i=1,....,M). They are variables belonging to the endogenous group, and used by the decision-maker to achieve the required optimum.

3. Irrelevant variables: v_i (i =1,....,L). The remaining endogenous variables which belong to neither of the previous two sets, but which are introduced into the model through the structural equations, as functions of other endogenous variables, are considered as irrelevant not to the model itself (in fact they are quite essential to it), but for the process of optimization. Their values would be determined simultaneously with other endogenous variables, and failure to take them into consideration, would make the model incapable of giving a proper solution for the problem.

4. The data: z_i (i=1,....,K). These are all exogenous variables, and values of the restrictions, considered as given in the problem.

VIII - THE BASIC FORM:

Let us denote the ophelimity index by ϕ. In its primary form it can be expressed as a function of the M target variables, w_i:

$$\phi = \phi(w_1, w_2, \ldots, w_M) \qquad (20)$$

The problem is to set the values of the w's so that ϕ attains an optimum value. If these values are directly under the control of the decision-maker, then they will be instruments of action. Since this is not usually the case, we have to introduce the instruments u_i and this is done through the statement of structural equations. If some of these equations involve endogenous variables other than the w's or the u's, we have to add extra structural equations enough to relate those variables to the latter ones. These extra variables are the irrelevant variables v_i, which means that:

the number S of structural equations = The number of target and irrelevant variables: M + L. (= J-N, see below) These equations bring in some (G≤K) exogenous variables as well as all, N, instrumental variables. Let us denote all endogenous variables by y_i $(i=1,\ldots,J=M+N+L)$:

$$\underline{y} = (\underline{w}\ \underline{u}\ \underline{v}) \qquad (21)$$

The structural equations can be expressed as follows:

$$g_i(y_1,\ldots,y_J;\ z_1,\ldots,z_G) = 0 \qquad (i=1,\ldots,S = J-N) \qquad (22)$$

These are to be followed by the two sets of restrictions:

a) The R limiting conditions affecting some or all endogenous variables. They might include extra exogenous variables, bringing up their number to G':

$$f_j(y_1,\ldots,y_J;z_1,\ldots,z_{G'}) = 0 \qquad (j=1,\ldots,R) \qquad (23)$$

b) E boundary conditions defining lower or upper bounds for some or all endogenous variables $(E \leqslant 2J)$. A formal description of such boundaries can be stated as follows:

$$y_i \geqslant z_{G'+i} \qquad y_i \leqslant z_{G'+J+i} \qquad (i=1,\ldots,J) \qquad (24)$$

where some of the $z_{G'+i}$ might be $-\infty$, while some of the $z_{G'+J+i}$ might be $+\infty$ (in which cases we omit the equality sign).

Thus the basic form of the model is:

It is required to optimize ϕ; where

$$\phi = \phi(w_1,\ldots,w_M)$$

and the w's satisfy the S structural equations:

$$g_i(y_1,\ldots,y_J;\ z_1,\ldots,z_G) = 0 \qquad (i=1,\ldots,S)$$

subject to R+E restrictions:

$$f_j(y_1,\ldots,y_J;z_1,\ldots,z_{G'}) = 0 \qquad (j=1,\ldots,R)$$

$$y_i \geqslant z_{G'+i} \qquad y_i \leqslant z_{G'+J+i} \qquad (i=1,\ldots,J)$$

IX - THE STANDARD FORM:

The basic form gives the statement of the economic problem leading to decision making. To facilitate mathematical solution, another form, called the standard form, is found to be more convenient. Using the S (=M+L) structural equations (22), we can eliminate S endogenous variables, e.g., the M target variables, and the L irrelevant variables. This leads to the expression of all endogenous variables as functions $\underline{y}$ of the instruments, $\underline{u}$:

$$y_i = y_i(u_1,\ldots, u_N) \qquad (i=1,\ldots;J) \qquad (25)$$

The ophelimity function becomes:

$$\phi = \Phi(u_1,\ldots,u_N; z_1,\ldots,z_G) \qquad (26)$$

To dispense completely with the structural equations we have to use them to eliminate the target and irrelevant variables from the restrictions. Thus all restrictions become functions of the instruments only:

$$F_i(u_1,\ldots,u_N; z_1,\ldots,z_{G'}) = 0 \quad (i=1,\ldots,R) \qquad (27)$$

$$\left.\begin{array}{l} y_j(u_1,\ldots,u_N) \geq z_{G'+j} \\ y_j(u_1,\ldots,u_N) \leq z_{G'+J+j} \end{array}\right\} \quad (j=1,\ldots,J) \qquad (28)$$

Now some of the boundary inequalities impose non-negativity (or non-positivity, easily reversed by change of sign) conditions on single instruments. The rest (n in number say) are conditions either on single instruments involving finite non-zero bounds, or on other endogenous variables, involving finite (zero or not) bounds. (We ignore infinite bounds). We can easily restate these inequalities in the shape of equations by introducing slack variables, each of them equal to the difference between the two sides of an inequality, such that they also satisfy non-negativity conditions. This means that the n original inequalities will be restated as n equations annexed with a new set of n inequalities, taking the shape of non-negativity conditions on the n slack variables. The number of boundary conditions will remain E, all of them non-negativity conditions, while the number of limiting conditions becomes $R' = R + n$. At the same time we consider the new variables as slack instruments, which brings the total number of instruments to $N' = N + n$.

Thus the model can be restated as follows:

It is required to optimize ϕ ; where,

$$\phi = \Phi(u_1, \ldots, u_{N'}; z_1, \ldots, z_K) \qquad (29)$$

subject to the R' limiting conditions:

$$F_i(u_1, \ldots, u_{N'}; z_1, \ldots, z_K) = 0 \quad (i=1, \ldots, R') \qquad (30)$$

and E boundary conditions:

$$u_j \geqslant 0 \quad (j=1, \ldots, E) \qquad (31)$$

In this form, all variables have been unequivocally introduced, although in reality they might have zero coefficients (e.g., the n slack instruments in the ophelimity function). It is clear that any model can be always expressed in this form; hence its name.

X - SOLUTION OF THE MODEL : THE DECISIONAL FORM:

Starting from the standard form, we have to find some way for determining the values of the instruments required to attain the optimum targets. Once their values are determined, they can be substituted in equations (25) to obtain the values of the target variables, and the irrelevant variables. Two possibilities arise:

1. The exogenous variables are assigned specific values. The solution can give directly the required values of the instruments, and hence of all other endogenous variables.
2. The solution takes the form of a set of functional relationships which express the instruments in terms of the exogenous variables. This case subsumes the previous one as a special case, since we can always substitute for the exogenous

variables.any given values to obtain the values of the instruments. However, this case is not always available, as might be noticed from problems of linear programming, and we have to use the previous way, changing the values of the exogenous variables and studying the effects of the changes on the decisions to be taken.

To obtain the solution, we have to specify the economic approach on which the statement of the problem was based. Two main approaches can be distinguished each of them entailing a different mathematical technique of solutions. They are:

a) marginal analysis, and

b) activity analysis.

Consider first the classical <u>marginal analysis</u> approach. The following conditions have to be satisfied:

1. The boundary conditions are seldom stated explicitly. Non-negativity is usually assumed to hold for the relevant range of study. This leaves $N' = N$, and $E = 0$.
2. The number of limiting conditions is less than that of instruments: $R < N$. Thus the problem becomes:

$$\text{to optimize:} \quad \phi = \Phi(u_1,\ldots,u_N;\ z_1,\ldots,z_K) \qquad (32)$$

subject to the R restrictions:

$$F_i(u_1,\ldots,u_N;\ z_1,\ldots,z_K) \qquad (i=1,\ldots,R<N) \qquad (33)$$

3. All these functions possess continuous derivatives up to the second order at least.

To solve this problem, two alternative approaches are available:

i) The first is to use the R equations in eliminating R instruments, thus reducing the problem as one of optimizing the ophelimity index as an unrestricted function in:

$$F = N - R$$

instruments. The choice of the specific F variables is arbitrary, and F is the <u>actual number of degrees of freedom</u>. The decision-maker is free to determine the values of F instruments, the <u>free</u>

instruments. Once this is done, the remaining R instruments follow by necessity from the R restricting equations; which means that there are R restricted instruments.

To obtain a "solution" we have to introduce F new equations, such that their solution would determine the values of the F free instruments, which lead to the optimum value of ϕ. If the solution gives more than one set of values for those instruments, a test should be available to offer a criterion by which the appropriate set is to be chosen.

Suppose that we have decided to solve the R restrictions (33) for the last R instruments:

$$u_{F+i} = d_i(u_1,\dots,u_N;\ z_1,\dots,z_K) \qquad (i=1,\dots,R) \qquad (34)$$

Substituting these values in (32), we obtain the unrestricted ophelimity function:

$$\psi = \psi(u_1,\dots,u_F;\ z_1,\dots,z_K) \qquad (35)$$

The problem is then: to obtain the optimum value of (35), with no additional restrictions. To do this we calculate the first-order partial derivatives:

$$\psi_i = \frac{\partial \psi}{\partial u_i} \qquad (i=1,\dots,F) \qquad (36)$$

Equating these derivatives to zero, we obtain the F equations:

$$\psi_i(\hat{u}_1,\dots,\hat{u}_F;\ z_1,\dots,z_K) = 0 \qquad (i=1,\dots,F) \qquad (37)$$

These equations, together with (34) are sufficient to determine the N instruments. Substituting their values in the S = M + L structural equations, (22), we can solve for the values of the M target variables, w, and the L irrelevant variables, v. The value of the optimum ophelimity index is that obtained by substituting in (20), or its equivalent (35). Hence the decisional form is:

$$\psi_i(\hat{u}_1,\dots,\hat{u}_F;\ z_1,\dots,z_K) = 0 \qquad (i=1,\dots,F)$$

or their solutions: $\hat{u}_i = u_i(z_1,\dots,z_K)$ $\qquad (i=1,\dots,F) \qquad (38)$

equations (34) $\hat{u}_{F+i} = d(\hat{u}_1,\ldots,\hat{u}_F; z_1,\ldots,z_K)$ $(i=1,\ldots,R)$ (or 33)
& equations (22) $g_i(\hat{y}_1,\ldots,\hat{y}_J; z_1,\ldots,z_K) = 0$ $(i=1,\ldots,S)$

Hence, the optimum: $\hat{\Psi} = \Psi(\hat{w}_1,\ldots,\hat{w}_M)$

These solutions determine the values of all $J = N + M + L = F+R+S$, endogenous variables as functions of the exogenous variables $z_1,\ldots,z_K$. For any given set of values of the z's, we can obtain the corresponding set of values of the y's. The optimum set of values is selected and tested according to second-order conditions, of a maximum or a minimum. The implications of such conditions can be formulated in terms of certain conditions to be satisfied by the functions determining the solutions for the endogenous variables, and by the ophelimity function itself.

ii) An alternative method of solution runs as follows: We introduce a set of <u>pseudo instruments</u> equal in number to the restrictions, say $\lambda_1,\ldots,\lambda_R$. The complete set of instruments contains $N + R = F + 2R$ instruments: $(u_1,\ldots,u_F,\ldots,u_N; \lambda_1,\ldots,\lambda_R)$. The ophelimity function is rewritten as follows, using (26) and (33):

$$\Psi = \Phi(u_1,\ldots,u_N;z_1,\ldots,z_K) - \sum_{i=1}^{R} \lambda_i F_i(u_1,\ldots,u_N;z_1,\ldots,z_K) \tag{39}$$

This new function accounts for all restrictions, hence it is itself unrestricted, and its <u>pseudo number of degrees of freedom</u> is $F' = N + R = F + 2R$. Solution of this problem requires the construction of an equal number of restrictions.

Let us differentiate partially with respect to the u_j, and put:

$$\Phi_j = \frac{\partial \Phi}{\partial u_j}, \qquad F_{ij} = \frac{\partial F_i}{\partial u_j}$$

Further we differentiate partially with respect to the pseudo instruments:

$$\frac{\partial \Psi}{\partial \lambda_i} = - F_i$$

thus obtaining F' partial derivatives. Equating each of them to 0, we obtain the decisional form as follows:

$$\Phi_j(\hat{u}_1,\dots,\hat{u}_N;\ z_1,\dots,z_K) = \sum_{i=1}^{R} \lambda_i F_{ij}(\hat{u}_1,\dots,\hat{u}_N;\ z_1,\dots,z_K) \qquad (j=1..N) \quad (40)$$

$$F_i(\hat{u}_1,\dots,\hat{u}_N;\ z_1,\dots,z_K) = 0 \qquad (i=1,..,R) \quad (33)$$

Solution of these F' equations gives the values of the N actual instruments u_j, and of the R pseudo instruments λ_i. Substitution in (39) gives the optimum ophelimity:

$$\hat{\Psi} = \Phi(\hat{u}_1,\dots,\hat{u}_N;\ z_1,\dots,z_K) - 0$$

As before the values of the target and irrelevant variables can be obtained by substitution in (22).

It can be shown that the solutions obtained in this way are the same as those obtained under (i) before. It is clear that the solution would depend on the shape of ϕ and its parameters. Unless a unique form of ϕ is given, no single set of functional relationships can represent the actual policy to be adopted by the decision-maker. However, as mentioned before, meaningfulness of the solution from an economic point of view would help to ascertain certain conditions to be fulfilled by ϕ. Notice that no non-economic considerations (ethical, moral or whatsoever) are implied by such conditions.

The above approach is not the only alternative; methods of activity analysis, have been recently developed to deal with a family of decision-making problem under somewhat different conditions. The conditions stated by the standard form (29)-(31) are assumed to hold. The u's are certain alternative activities, and they represent a finite number of choices open to the decision-maker, among which he has to make his final choices. As before, we assume that the number of exact limiting conditions, R, is less than N, the number of instruments (or activities).

This leaves the number of degrees of freedom as before equal to;

$$F = N - R$$

In the process of converting boundary conditions other than the non-negativity conditions on single instruments, both N and R are increased by n', which leaves the number of degrees of freadom unaffected:

$$N' - R' = N - R = F$$

It remains only to decide on the appropriate mathematical technique to be used in determining the optimum.

If equations (29) & (30) are all linear the mathematical technique would be that of <u>linear programming</u>. The standard form is then:

$$\text{to optimize: } \Phi = \sum_{i=1}^{N'} f_i u_i + f_o \qquad (41)$$

subject to R' limiting conditions:

$$\sum_{j=1}^{N'} a_{ij} u_j = a_{io} \qquad (i=1,\ldots,R') \qquad (42)$$

& N' boundary (non-negativity) conditions:

$$u_j \geq 0 \qquad (j=1,\ldots,N') \qquad (43)$$

where: f_o & a_{io} are linear combinations of $z_1,\ldots,z_K$, the data in the problem. Given the values of all f_i and a_{ij} we can apply the technique of linear programming to obtain the values of the u's. Solutions of the model are available only <u>numerically</u>.

XI - <u>EXAMPLES</u>:

<u>Example (1)</u>: The static model of consumer's behaviour takes the following basic form: To optimize $\Phi = U$ (U denoting preference or utility). Structural equations contain the equation defining the utility index, U, which is the single target variable:

$$U = U(X_1, \ldots, X_N)$$

where X_i is the quantity consumed from commodity i, the x's being the instruments. The limiting condition is the income-expenditure equstion; sometimes called the budget equation;

$$\sum_{i=1}^{N} P_i X_i = Z$$

where P_i are the given commodity prices (being the market prices), and Z is the consumer's income, all of them considered exogenous to the model.

The standard form is:

to maximize $\Phi = U(X_1, \ldots, X_N)$

subject to $\sum_i^N P_i X_i = Z$

This can be rewritten alternatively as:

to maximize $\Psi = U(X_1, \ldots, X_N) - \lambda(\sum_i p_i x_i - Z)$

The decisional form is:

$$\frac{\partial}{\partial x_i} U(X_1, \ldots, X_N) - \lambda p_i = 0 \quad (i=1, \ldots, N)$$

$$\sum p_i X_i - Z = 0$$

If explicit expressions are available, this latter form can be rewritten as follows:

$$\hat{X}_i = d_i(p_1, \ldots, p_N; Z) \qquad (i=1, \ldots, N)$$

$$\hat{\lambda} = \lambda(p_1, \ldots, p_N; Z)$$

The functions d_i are the demand functions of the consumer, while λ is the marginal utility of expenditure "at equilibrium". From the decisional point of view λ is a pseudo instrument, hence irrelevent. The determination of any (N-1) commodities determines the consumption of the N-th, through the income-expenditure aquation. Hence the number of degrees of freedom is (R being = 1):

$$F = N - R = N - 1 \quad \text{or,} \quad F = (N+R) - 2R = N - 1$$

Example (2): A more elaborate model is provided by the study of the static equilibrium of a monopolist, producing a single output X, using n variable inputs F_i, with given prices q_i, and facing a market with a demand function d relating the price p of the output X to its level and to other - exogenous - variables (e.g., prices of other goods; consumers' income, etc ...), β_j (j=1,...,m). The producer is producing according to a production function f relating his output to the N variable inputs F_i and to other n fixed (hence exogenous) inputs α_j. Denoting his profits by π, total revenue by R, total costs by C, we can write the model in its basic form as follows:

$$\text{to maximize} \quad \phi = \pi$$

with the structural equations:

$$\pi = R - C$$
$$R = P.X$$
$$P = d(X; \beta_1, \ldots, \beta_m)$$
$$C = \sum_i^N q_i \; F_i + \alpha_0$$

(where α_0 = total fixed costs, and might be written as: $\alpha_0 = \alpha(\alpha_1, \ldots \alpha_n)$. Notice that such an expression does not count as a restriction on the model, since all its members are exogenous variables). The optimum is subject to the limiting condition:

$$X = f(F_1, \ldots , F_n; \alpha_1, \ldots, \alpha_r)$$

In this formulation we have:

$M = 1$ target variable (π)

$L = 3$ irrelevant variables (R, P, C)

$N = n+1$ instruments ($F_1, \ldots, F_n$, X)

$K = m+n+r+1$ exogenous variables ($\beta_1, \ldots, \beta_m; q_1, \ldots, q_n; \alpha_1, \ldots, \alpha_r; \alpha_o$)

$S = M + L = 4$ structural equations

$R = 1$ limiting conditions

$F = N-R = n+1-1=n$ degrees of freedom.

It is clear that one can consider both P and X as irrelevant variables, if we choose to limit the instruments to the actually free ones, say, the n variable inputs. Once they are determined both output X and price P are determined through the production function and the damand function. Although X is an instrument, it is a restricted one. Alternatively, We can consider X plus (n-1) variable factors as free instruments, the n-th being restricted by the production function. On the other hand, although revenue R, and hence price P are essential in determining the level of profits, they are irrelavant to the decisional problem since they are not completely under the control of the producer. It might be observed here that there is nothing which prevents the producer from using p as his instrument, but this would deprive him from any control on his "saleabl" output, and X becomes irrelevant. The model can be reformulated to represent this case. The structural equation become:

$$\pi = R - C$$
$$R = P.X$$
$$X = f(F_1, \ldots, F_n; \alpha_1, \ldots, \alpha_r)$$
$$C = \sum_{i=1}^{n} q_i F_i + \alpha_o$$

and the limiting condition would be

$$P = d(X; \beta_1, \ldots, \beta_m)$$

This formulation of the model is less familiar; but it would yield the same solution.

Returning to the former basic form, we use the structural equations to eliminate the target and irrelevant variables, keeping only the instrumental variables. Thus

$$R = P.X = X.d(X; \beta_1, \ldots, \beta_m)$$

Substituting in the expression for Π, we obtain the standard form:

$$\text{to maximize: } \phi = d(X; \beta_1, \ldots, \beta_m).X - \sum_{i=1}^{n} q_i F_i - \alpha_0$$

under the limiting condition;

$$X = f(F_1, \ldots, F_n; \alpha_1, \ldots, \alpha_r)$$

Or, introducing the Lagrange multiplier, λ,

$$\text{to maximize: } \Psi = d(X; \beta_1, \ldots, \beta_m).X - \sum_{i=1}^{n} q_i F_i - \alpha_0 + \lambda\left[X - f(F_1, \ldots, F_n; \alpha_1, \ldots, \alpha_r)\right]$$

with no restrictions. In this formulation the pseudo number of degrees of freedom is $F' = n+2$

If the producer is working under competitive conditions, p has to be considered exogenous, hence we drop it from the list of irrelevant variables and in the meantime drop the market demand function from the structral equations. The rest of the model remains the same. The solution of the system determines both the production and input policies of the producer. The input policy is in fact the demand function of the producer for the n variable inputs. It is clear that these demand functions can be written in terms of the parameters of the production function, the prices of the variable inputs and that of the output, as well as the parameters of the demand function. The textbook treatment would not go that much much; the level of the output itself would appear as one of the variables explaining the demand for inputs, which means that the process of substitution does not go to its far and. Another feature of the model is that it contains a demand function which itself is a result of the solution of another action model.

XII - MODELS OF MULTI-ACTION:

The main difference of these models lies in the fact that more than one unit of decision-making have to be considered in the model in an essential way. This means that there will be an equivalent number of ophelimity functions to be optimized at the same time. Let us assume that there are two decision-makers, A and B with well-defined ophelimity functions ϕ_a and ϕ_b. Since the actions of each decision-maker affect the behaviour of the other the two processes of optimization have to be carried out simultaneously. Certain instruments of A should be taken into consideration in determining the optimum for B, and vice versa. The following cases can be distinguished (see sec. VI above):

1. Each decision-maker considers the set of instruments under the command of the other decision-maker as exogenous to his problem of optimization. A solution can be obtained for each decision-maker independently, by considering the other's instruments as exogenous variables. This means that the instruments of each decision-maker are obtained as functions of - among others - the instruments of the other decision-maker. These functions can be considered as reaction functions, expressing the way in which each decision-maker behaves in the face of any decisions taken by the other.

   To reach a final solution, we have to study the results of the interaction arising from these formulae of reaction. This means that we have to consider some sort of an interaction model, in which the two sets of instruments are considered as endogenous and determine their values in terms of the really exogenous variables in the model. The simultaneous solutions of the reaction functions will determine the policies of both decision-makers in such a way that the decisions they take would be consistent, hence harmony would prevail. The Cournot equilibrium in duopoly is an example.

2. If A considers B's instruments as exogenous, while B considers A's instruments as irrelevant, and expresses them as functions of the variables relevant to his process of optimization, the situation becomes different. If, and only if, this function is exactly the one obtained from the solution of A's optimizing process, the results would be consistent, and a unique solution is obtainable. Again harmony of action is guaranteed. An example, is the case of a leader and a follower in duopoly.

3. If B (or A, or both) assumes that some of A's (or B's) instruments are related to his own instruments in a form which differs from the actual one arising from the actual process of optimization, the model becomes inconsistent. Unless the postulated function is reconciled to the actual one, the optimum sought on its basis will not be achieved, and the action becomes contradictory. Now this reconcilation opens an infinite number of possibilities, and any solution to the model has to be built on certain assumptions as to the factors determining reconciliation. The fact that game theory fails to give a unique set of answers in games involving more than two persons, is an indication of the type of problem involved. This is usually taken as an indication of the limited power of the theory itself, at least as far as economic problems are concerned. However, this is an unfair conclusion.

If we start by assuming harmony of action; we will get it !! But the fact that the model shows harmony is not a proof that it is realistic; in fact it may be an indication to the contrary. To illustrate the point let us consider the beautiful abstraction dominating economic thinking, always putting a happy conclusion to the story in the shape of the existence of a static equilibrium. One might go further and prove the stability of this static equilibrium, either by static or dynamic methods of analysis. It is usually assumed that since harmony is thus established, and since the static situation is a limiting case to the actual dynamic reality, harmony can be postulated in actual life. It is true that for each dynamic model, one can build a static limit, but the fact remains that an infinite number of dynamic models have the same static limit. But they need not all of them possess the same properties, and the study of conditions of stability in each dynamic case might lead to quite divergent results. Hence we have to review all static conclusions, and obtain more realistic expositions through the specification of stability conditions in the dynamic case.

Our main conclusion is that useful as they are, familiar models of economic analysis avoid an important aspect of real life by showing a great bias towards short-cuts for harmony. More can be learned through models of contradictory action than through models of harmony. They might be more difficult, and they might require resort to other fields of social science, but that is the price which economics has to pay in order to become more realistic. I am simply stating the problem, but I am not ready to offer any concrete solutions; not at this stage.

XIII - <u>CONDITIONAL ACTION MODELS</u>:

Not every model in economic theory purports to carry a given policy problem to its far end. A deeper insight in many aspects of the policy making process leads to the study of what we suggest to call "conditional action" models. Let us illustrate by an example familiar to students of the theory of production.

As shown in Sec. (XI) above, a complete solution of the producer's problem, requires a statement of two policies; the input-policy, and the output-policy. In order to study the various aspects of the input-policy we usually assume that the level of output has been somehow determined. Given the level of output, the problem is to find that combination of inputs which maximizes profits; but this means that revenue would be constant, hence the problem is equivalent to the search for minimum costs. In this sense, the suggested policy is conditional and the maximum attained is not an absolute one. However, there might arise cases when such as approach can lead to a well-defined policy of action.

Consider for example the case where the producer adopts the policy of a fixed share in the market. The level of output would be determined through exogenous factors; the only parameter at the disposal of the producer is the actual share which he decides to hold. If this is determined through past experience, then it is also predetermined. Hence the only policy open to him is to change his costs so that they attain their minimum according to the given level of output. But if the level of output is still to be determined, the conditional policy would be only meaningful if the two policies can be separated in such a way that the determination of one can be achieved independently of the way in which the other is determined. If this is the case, we can determine the absolute maximum in two steps rather than one.(1) Determining the optimal input policy, we can proceed to determine the optimal level of output, moving all the time on the optimal surface in the input space. We can also change the order: we can start by determining the best output policy corresponding to given costs, viz., the policy which maximizes profits through maximisation of revenue, given total costs. Later we can look for the optimal input policy by allowing costs to change, and owing to the independence property assumed before the result will be the same.

This discussion is quite relevant to policy problems connected with "fixed targets". It is clear that discussions under the assumptions of fixed targets would be only conclusive if the fixation is final and represents certain well-defined desires. Otherwise, this approach should be considered as an attempt to simulate certain hypothetical situations, in order to study their implications in some detail. A final policy is determined only after a completition of the model, by allowing for the other part (or parts) of

(1) Thus we follow the well-known method of "stepwise maximization".

conditional actions. This problem will be treated in more detail in a later occasion where we deal with models of public economic policy.

XIV - SUMMARY AND CONCLUSIONS:

In an attempt to approach the problem of constructing analytical models in a systematic way, the following classification is found useful, especially if we have policy models in mind: Models of action & Models of interaction.

A. Models of action: quite familiar in economic analysis, have been found to lack formal discussion in the literature. An attempt was made in this direction; leading to the following setup:

1. Variables: Four categories are distinguished (Tinbergen's):
   Target variables, w_i $(i = 1, \ldots, M)$
   Instrument variables, u_i $(i = 1, \ldots, N)$
   Irrelevant variables, w_i $(i = 1, \ldots, L)$
   Data or exogenous variables, z_i $(i=1, \ldots, K)$
2. Forms: Three forms are distinguished:
   a) The basic form, summarising the economic formulation of the problem, containing
   An ophelimity index, ϕ, to be optimized.
   An ophelimity function, expressed in terms of target variables.
   Structural relationships expressing both target and irrelevant variables in terms of instruments and data. Their number $S = L + M$.
   Two types of restriction:
   R limiting conditions in the shape of equations involving target variables and/or instruments; E boundary conditions in the shape of lower and upper bounds on some or all endogenous variables.

b) The standard form, which prepares the economic problem for direct mathematical manipulation, by means of standard techniques. Two main approaches in economic forms lead to different mathematical techniques; marginal analysis leading to the application of ordinary calculus, and activity analysis leading to the application of programming techniques. In this form all endogenous variables except instruments are eliminated from the model by means of the structural relationships. All boundary conditions are transformed into non-negativity conditions, with the appropriate change of limiting conditions.

c) The decisional form, giving the mathematical solution of the optimization problem, as an expression of the instruments in terms of the exogenous variables. In some cases the numerical values of the data have to be given beforehand, and numerical methods of solution have to be adopted.

3. Number of decisional units:
Modles of uni-action, involving a single decisional-maker, considered as representative of certain homogenous group.
Models of multi-action; involving more than one decision-maker in an essential way

4. Harmony of action:
Models of harmonic action, where the behaviour of the unit under study is reconcilable to the behaviour of other units in the same or other groups.
Models of contradictory action, where one of the units under study (at least) develops an attitude towards assigning unrealistic values to some uncontrollable variables

5. Modles of conditional action are sometimes constructed to help in the analysis of certain theoretical problems. Certain target variables are given predetermined values, and the

optimization problem is reformulated in tersm of the remaining target variables, thus defining a relative optimum.

6. The number of degrees of freedom is given some attention. The total number of instruments is reduced by means of the limiting conditions, leading to unrestricted instruments, hence the number of degrees of freedom. Pseudo degrees of freedom are envolved when we adopt the procedure of introducing pseudo instruments for mathamtical purposes.

B. Models of interaction: summarise the results of interaction among a number of decision -makers whose behaviour is determined by means of action models. The setup runs as follows:

1. Variables:
Jointly dependent variables, viz; current values of endogenous variables; Predetermined variables, including exogenous variables and past values of endogenous variables.

2. Structural equations:Two categories of three types each are distiguished:Identities;including definitional, balance, and equilibrium conditions. Stochastic equations; including; institutional, technical and behaviouristic equations.
Other types of equations might be added, belonging to the second group; e.g., demographic equations.

3. The structural form: includes a number of equations equal in number to the number of jointly dependent variables. It has as components the above structural equations.

4. Solutions of the model: Three types of solutions are distinguished: The reduced form, expressing each jointly dependent variable as a function of the predetermined variables.

The separated form, expressing each jointly dependent variable in terms of its own lagged variables, and the exogenous variables only.

The resolved form, giving the value of each jointly dependent variable in terms of time and exogenous variables.

In the treatment of the action models two problems became apparent:

1. The problem of harmony seems to need further careful investigation. Most economic models seam to assume some sort of passive harmony, a fact which has still to be established.
2. The problem of aggregation, often dispensed with by means of some simplified approach has to be studied in another light. In particular, the problem of passge from the representative unit to the behaviour of the whole group, and the properties of the preference function for the whole group have to be given explicit consideration. If for example, the member of a given group tries to act in some sort of a collective manner, the individualistic approach and the theorems of maximum welfare derived from it have to be reviewed.

These problems are only indicated here for further investigation. The above exposition as well as the solution of these latter problems are found to be a necessary background for a systematic treatment of policy-making models.