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**The Applications of Linear Programming
Models for Planning the Optimal Structure
of foreign trade**

By

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The application of linear programming models for
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Forword

We held a series of discussions on model building for foreign trade planning at the International Economic Relations Centre at the I.N.P. Cairo. As part of it I presented a model for optimizing the regional distribution of exports and imports which should be applied practically for the A.R.E. foreign trade. While this experiment is still going on I will summarize and elaborate on experiences we gained in the German Democratic Republic in building and applying linear programming models for foreign trade planning, which might be of interest for those who are also working in this field.

I am much obliged to all who took part in the discussions, especially the staff of the I.E.R.C., for getting better ideas on problems of model building in developing countries, especially in the A.R.E..

And there is another question to be mentioned. We always consider such models as auxiliary instruments for planning. A model, even highly-sophisticated, is without any use, if the expert or the planner has not elaborated on it in the correct lines, e.g. when determining the objectives and goals, when fixing the conditions which are limiting the economic growth, estimating some inputs etc. Also when using modern tools like models of optimization the role of the planner and the knowledge of the experts are decisively and even growing. Besides we want to stress that the social development and the economy are so manifold and include so many different factors and tendencies which cannot completely be included in models. Thus one has always to check the results coming from the computer and comparing them with the real life before deciding on the future plan.

In the following we will elaborate on models of optimization built in the G.D.R. and applied for planning in the foreign trade sector. Especially we will deal with problems one will be facing in the practical approach.

1. Basic ideas for constructing linear programming models for planning foreign trade.

At the beginning of both either the research work or practical steps in applying models for economic planning we should always rise the question what will be the aim in building the model, what will be the effect of using the model in practical planning.

As regards foreign trade planning, we are aiming at finding the best set of the commodity structure of exports and imports and its regional distribution on areas and countries in order to maximize the foreign exchange receipts and to increase the efficiency of foreign trade. Thus we are searching for optimal plan targets which cover our long-term and short-run objectives and are in conformity with the internal and external conditions. And here it might be useful to apply models as auxiliary instruments for drawing up the foreign trade plan and for determining plan-figures.

There are several programming techniques which have been developed in the 50-ies and 60-ies and which have proved to be useful for economic planning on macro as well as on micro level, such as linear programming, nonlinear programming, dynamic programming. The linear programming technique seems to be the most developed one for economic planning till now and here the socialist countries have practical experiences, too. For constructing linear programming models for foreign trade planning we will refer to the characteristics of this method.

At first there must be a linear objective function of such a type:

$$f(X) = C_j X_j$$

which aims either at a maximization or at a minimization.

At second there is a set of linear constraints

$$\sum_{j=1}^n a_{ij} x_j \leq b_i \quad i = 1, 2, \dots, m$$

which limit the variables of the model, and last not least all variables must be non-negative, i.e.

$$x_i \geq 0 \quad i = 1, 2, \dots, m$$

The decisions on which the planner in the field of foreign trade is elaborating and the frame to which he is often bound seem to be like these characteristics so that applying linear programming models in planning suggests itself.

Let us elaborate on it in details.

Suppose, there is a number of commodities, which shall be exported or imported. The external prices we will get for the exports or have to pay for imports are different from country to country so that we are looking for the best distribution of the exports and imports. But when following such criteria we have to take into consideration some limiting conditions which may exist both internally and on external markets, such as

- (a) limitations in the capacity for producing the exportables,
- (b) the necessity of fulfilling the domestic demand in products which are also exportables,
- (c) usually the external demand for our exports is not unlimited and there may exist upper bounds,

- (d) there may be a maximum supply possibility for commodities we want to import,
- (e) from trade and payment agreements the necessity of following certain balance requirements may arise and we have to follow a fixed trade-policy.

Now we will construct a model which will include these conditions and criteria and the application of which will contribute to the finding of the best solution of the above mentioned task.

When k is the index for export commodities ($k = 1, 2, \dots, n$), g the index for import commodities ($g = 1, 2, \dots, s$) and r the index for countries/areas ($r = 1, 2, \dots, m$) then we are looking for optimal export quantities/values for each commodity to each country ($x_{k,r}$) and for the optimal items of our imports ($y_{g,r}$).

The following table gives insight into the variables (outputs) of the calculation, we are looking for.

Country		1	2	3	m
Commodity						
exportable k	1	x_{11}	x_{12}	x_{13}	x_{1m}
"	2	x_{21}	x_{22}	x_{23}	x_{2m}
"	3	x_{31}	x_{32}	x_{33}	x_{3m}
	
	
	
	n	x_{n1}	x_{n2}	x_{n3}	x_{nm}
importable	1	y_{11}	y_{12}	y_{13}	y_{1m}
	g = 2	y_{21}	y_{22}	y_{23}	y_{2m}
	3	y_{31}	y_{32}	y_{33}	y_{3m}
	
	
	
	s	y_{s1}	y_{s2}	y_{s3}	y_{sm}

And here are the inputs of the model necessary for determining the optimal distribution of the exports and imports.

When e_k stands for the quantity or value of each commodity k, available for export and i_g stands for the quantity/value of commodity g to be imported for covering the internal demand, one can formulate:

$$(1) \quad \sum_{r=1}^m x_{kr} \leq e_k \quad k = 1, 2, \dots, n$$

which means that the export of commodity k to all markets added together can only be equal to the predetermined export capacity e_k or less than it.

$$(2) \quad \sum_{r=1}^m y_{gr} = i_g \quad g = 1, 2, \dots, s$$

The import quantities of commodity g from all markets should be equal to the internal demand for such imports. If i_g is to be understood as the minimum limit for imports of commodity g , one can also formulate:

$$(2 a) \quad \sum_{r=1}^m y_{gr} \geq i_g \quad g = 1, 2, \dots, s$$

Both are limitations from the internal side of the country. On the external markets there may exist a minimum demand (b_{kr}) for our exports, which at least should be covered, and a maximum sale's possibility. Thus one can formulate

$$(3) \quad x_{kr} \leq \bar{b}_{kr} \quad \begin{array}{l} k = 1, 2, \dots, n \\ r = 1, 2, \dots, m \end{array}$$

$$(4) \quad x_{kr} \geq b_{kr} \quad \begin{array}{l} k = 1, 2, \dots, n \\ r = 1, 2, \dots, m \end{array}$$

and for the importables analogously

$$(5) \quad y_{gr} \leq \bar{b}_{gr} \quad \begin{array}{l} g = 1, 2, \dots, s \\ r = 1, 2, \dots, m \end{array}$$

$$(6) \quad y_{gr} \geq b_{gr} \quad \begin{array}{l} g = 1, 2, \dots, s \\ r = 1, 2, \dots, m \end{array}$$

where b_{gr} = minimum import obligations for certain commodities g from certain markets r .

and \bar{b}_{gr} = maximum supply possibility for commodity g from market r.

By formula (4) and (6) we have at the same time included the non-negativity condition for all variables.

And now we have to formulate balance limitations, e.g. in such a way

$$(7) \quad \sum_{k=1}^n p_{kr} x_{kr} - \sum_{g=1}^s d_{gr} y_{gr} = S_r \quad \text{or}$$

$$(7a) \quad \sum_{k=1}^n p_{kr} x_{kr} \geq E_r$$

$$\sum_{g=1}^s d_{gr} y_{gr} \leq I_r \quad r = 1, 2, \dots, m$$

which means that the total value of exports to country r minus the total value of imports from country r should cover a saldo S_r , which can be understood as surplus or deficit (7).

or

One can bind the export value to a certain predetermined amount E_r and the import value to I_r , each on country/area basis (7a).

Of course we will include these limitations only for countries/areas where we are bound to follow such restrictions or where we are aiming at realizing a certain surplus or deficit. In practice the laying down of such limitations and the margins depend on the trade-policy

targets, the type of trade and payment agreements (bilateral or multilateral payment balance equalization, settlement in clearing currency or in terms of convertible currency) and on the concrete situation of the payments in the given period. | Now we have to determine the objective function.

The main criteria for determining the x_{kr} and y_{gr} will be selling the exports to such markets where we get best prices and purchasing the imports from markets, where we pay the lowest prices and this will be formulated as the objective function of the model.

When p_{kr} stands for the export price per unit of commodity k by country r and d_{gr} for the import price we have to pay for commodity g in country r , then the objective function looks as follows:

$$\sum_{k=1}^n \sum_{r=1}^m p_{kr} x_{kr} - \sum_{g=1}^s \sum_{r=1}^m d_{gr} y_{gr} \longrightarrow \max.$$

which aims at a maximization of our foreign exchange receipts and a minimization of the foreign exchange expenditures.

From the linear programming point of view the model is complete now. As regards the practical application for foreign trade planning, the above mentioned model should be considered as the simplest type for optimizing foreign trade. It is a basic model from which a series of specific models for planning foreign trade can be derived. In the following we will elaborate on such models we have applied in the G.D.R., putting special emphasis on problems we had been facing and how to solve them.

2. Models of optimization for the foreign trade sector

2.1 Model for optimizing the regional distribution of exports and imports.

We have stated at the beginning that one aim in foreign trade planning is finding the best regional distribution of exports and imports and determining the optimal regional structure. In the G.D.R. we started applying linear programming models just in this field, as the subject of optimization is limited here and problems which may arise at the beginning of using such a technique can easier be solved.⁽¹⁾

The basic type of the Regional Optimization Model (R.O.M.) can be seen in Appendix 1. It is similar to the model explained before. The 1-st and 2-nd equations, namely

$$(1) \quad \sum_{r=1}^m x_{kr} = e_k \quad k = 1, 2, \dots, n$$

$$(2) \quad \sum_{r=1}^m y_{gr} = i_g \quad g = 1, 2, \dots, s$$

are characteristic ones and make it obviously that the export volume of the commodities (e_k for $k = 1, 2, \dots, n$) and the import volume of the commodities (i_g for $g = 1, 2, \dots, s$) have to be predetermined

(1) Here we made use of earlier studies on this problem carried through by Trzeciakowski (A model of optimization of current directions of policy in foreign trade, Gospodarka Planowa, Warsaw 1960) and Kronsjö (Iterative pricing for planning foreign trade, Economics of Planning, vol. 3 No. 1, 1963).

and are not a subject of optimization here. The R.O.M. is aimed at the optimization of the regional structure of foreign trade with full adherence to national economic and trade policy objectives and the limitations imposed by the concrete internal and external conditions.

There will be an optimal distribution of the exports and imports on regions when the foreign exchange receipts are a maximum and the foreign exchange expenditures are at the lowest level within the bounds previously mentioned. In principle, the objective function

$$\sum_{k=1}^n \sum_{r=1}^m p_{kr} x_{kr} - \sum_{g=1}^s \sum_{r=1}^m d_{gr} y_{gr} \longrightarrow \text{maximum}$$

is fulfilling the above mentioned criteria as the net foreign exchange receipts will be maximized or, to be precise, a possible surplus will be maximized, a deficit minimized. Often we are especially interested in getting some hard currencies and we don't want to maximize bilateral currency receipts. Therefore we can include a preference coefficient in the objective function giving weights to certain markets/countries and maximizing the foreign exchange components in a differentiated manner. The objective function then looks as follows:

$$\sum_{k=1}^n \sum_{r=1}^m t_r p_{kr} x_{kr} - \sum_{g=1}^s \sum_{r=1}^m f_r d_{gr} y_{gr} \longrightarrow \text{maximum}$$

where t_r is a coefficient which stimulates the exports with certain markets and

f_r will slow down imports from certain markets.

One can also include different terms of delivery (fob, cif) and different terms of payment in the objective function. For example when there is a significant difference in the prices on cash basis and on credit terms, we should differentiate in the objective function between cash receipts/payments and payment on credit terms per commodity and country.

From G.D.R. experience applying the R.O.M. has proved to be useful for finding the best regional distribution of our exports and imports and besides for rationalizing the planning procedure.

Up till now there are more than 50 examples of practical application of the R.O.M. We gained good experiences in applying this technique on enterprise level but also on central state level. Sometimes the complete export and import nomenclature of a General Organization or a Foreign Trade Agency had been calculated in its optimal regional distribution. On the other side we also built some special models for exports or for imports only according to the concrete circumstances and tasks of foreign trade planning. When having applied the R.O.M. on macro economic level--either by the State Planning Commission or by the Ministry of Foreign Trade--we were aiming at optimizing the total foreign exchange receipts/expenditures. But here we had been facing some other problems and new difficulties, which mainly resulted from the new dimension (size) of the model. Suppose, there are more than 1.000 commodities to be exported or imported to/from about 50

countries. As it is nearly impossible to build and to solve a model which includes such a number of variables one had to use either aggregated items or selected ones. Both ways are possible and it depends upon the concrete aim of optimizing whether we will choose the first or the second one. But using aggregates means to construct the model on the basis of value units which causes some modifications. For example we had to relate exports and imports on external price basis to the corresponding items on internal price basis and here we introduced the q - coefficients,⁽¹⁾ which act at the same time as profitability indicator and criteria for the optimal regional distribution.

Appendix 2 gives insight into a model for regional foreign trade optimization on value unit basis.

We should add, that the R.O.M. type has been applied in annual, medium-term and long-term planning. When the plan period covers 5 years, e.g., it goes without saying that one can only apply the model for 1 year respectively and not for the whole period all at once. Therefore the model has been built either for the final year of the plan-period or we had to calculate a set of 5 models for each year of the period respectively.

$$(1) \quad q^E = \frac{\text{export prices on external price basis}}{\text{domestic price for exportables}}$$

$$q^I = \frac{\text{domestic price for importables}}{\text{import prices on external price basis}}$$

2.2 Model for optimizing the commodity and regional structure of foreign trade.

As foreign trade planning is always both regional and commodity planning, it is quite naturally that another type of models for optimizing foreign trade has also been developed which can be characterized as Commodity and Regional Optimization Model (CROM). And here are the characteristics of this model.

Again x_{kr} and y_{gr} are the export and import variables of the model. The external side could be built in the same manner as before (ROM). The main differences one will find in the internal side and in the objective function.

Remember: When regional optimizing we take for given the export and import quantities or values (e_k, i_g). Now we want to find the optimal size and structure of e_k and i_g , too, by applying linear programming technique. Therefore we had to relate the exports and imports to the production capacities, the internal demand for consumer goods and intermediate goods, the needs in investment goods etc.

The following equations are covering these requirements in principal:

(1) internal balance equation

imports + production = intermediary consumption + exports
+ final demand

$$\sum_{r=1}^m y_{gr} + X_k = \sum_{j=1}^n a_{kj} X_j + \sum_{r=1}^m x_{kr} + g_k \quad k=1,2,\dots,n$$

where a_{kj} are the technical or input/output coefficients

x_k total production of commodity k

g_k final internal demand (either for consumer or for investment goods).

As the production is limited by the available resources we have further to formulate

$$(2) \quad x_k \leq \bar{K}_k \quad k = 1, 2, \dots, n$$

where \bar{K}_k stands for the available production capacity for commodity k (or the planned capacity for the coming period).

The model can be applied for finding the optimal structure for groups of exportables and importables, for optimizing the production assortment of a General Organization or for a sectorial optimization of the production, export and imports etc.

For better explaining the field or the subject of optimization let us refer to a practical example.

There may exist or will be established a metallurgical complex including the following departments: furnaces, foundry, steel factory and rolling mill.

The input-output relations are the following:

* The furnaces require iron ore, coke and scrap iron for producing pig-iron,

- * The foundry will produce cast-iron on the basis of pig iron and scrap iron,
- * The steel factory produces steel on the basis of pig-iron and scrap iron,
- * The rolling mills need steel in order to produce rolled steel.

The country is depending on importing coke, to a certain extent iron ore and scrap. On the other side, there is not only a domestic demand for pig-iron, cast iron, steel and rolled steel which should be covered, but at the same time these products can be exported.

Now one can rise the following questions:

Which is the optimal structure and size of the metallurgical complex from both the internal and external side? Is it preferable, to import the raw material for the metallurgical complex, which will be established or shall we depend upon the import of manufactured metallurgical products, such as cast iron, steel etc?

Is it better, to export pig-iron and cast iron or to export rolled steel and steel? Or shall we completely exclude such exports and reserve the total output of the metallurgical industry for increasing the production of other industries, like engineering, construction, electronics etc.?

These questions must be answered before the project or the plan get its approval and here the application of a model for finding the best

solution will be useful and necessary. Calculating some variants helps to have a better basis for making the decision.

To be precise: The above characterized project is not only an optimization of the commodity and regional structure of the exports and imports respectively, but at the same time it is an optimization of the production, consumption, and investments of the sector concerned. Here it becomes obviously that planning the foreign trade structure is an integral part of national economic planning and especially related to planning the production, investments and consumption.

Appendix 3 shows a simple example for the C.R.O.M. type which is mainly directed to an optimization of the export and import structure and its regional distribution. The internal side of the model to be applied can be extended and one can include the available labour resources, investment requirements etc.

Finally we will put some remarks concerning the objective function of this model, which looks as follows:

$$\sum_k \sum_r p_{kr} x_{kr} - \sum_k \sum_r p_{kr}^I x_{kr} + \sum_g \sum_r d_{gr}^I y_{gr} - \sum_g \sum_r d_{gr} y_{gr} \longrightarrow \max.$$

where p^I stands for the internal costs or internal price for the exportables and

d^I for the internal price for the importables (or domestic prices of the domestically produced commodities similar or substitutes for imports).

In this form the objective function is directed to a maximization of the foreign trade efficiency which can be understood as the comparison between the receipts/benefits and the costs of foreign trade transactions. The formula includes the direct effects of foreign trade on the economy and here the term profitability usually is used, but also indirect effects on production, investments etc. can be included.

Other criteria for planning the optimal structure of production and foreign trade can be,

- * maximization of the net foreign exchange receipts
- * minimization of the costs of production.

We mainly preferred the first type, optimizing the efficiency and here mainly the profitability of foreign trade, as the foreign exchange receipts in its optimal composition and the minimization of the costs of production can be included here.

There are more than 30 concrete examples of the C.R.O.M. type which in general have shown better economic results than the pure regional

optimization (which is quite understandable as the field of optimization is broader in the C.R.O.M. than in the R.O.M.). The model should be used first of all for medium-term planning, as changes in the commodity structure of production and foreign trade can better be realized over a longer period than in the annual plan.

3 . Experiences in applying linear programming models for planning foreign trade.

3.1 Some practical problems in using the models.

When constructing models of optimization and applying them practically one will be facing some problems which result from both the existing economic and other conditions which have to be included in the model and from the mathematical apparatus to be used. We will elaborate on some of these problems in details and give some ideas how to solve such problems from G.D.R. experience.

The mathematical method applied up till now is the linear programming technique and thus one must take for granted that all coefficients and parameters attached to the variables of the model will behave in a linear way. The real behaviour is different, above all as the external prices are concerned. For example in many cases the export and import prices will behave in a non-linear way according to changes in the export and import volume. On international markets one has usually to grant a certain commission when contracting large quantities so that the external prices will decline per unit of exports.

If the changes are not remarkable one can neglect it when applying linear programming technique. But if it is necessary to take them into due consideration one can handle the problem as follows:

- (1) We make use of the parametric linear programming technique and by this investigate the influence of changes in the coefficients or parameters of the model on the optimal distribution.
- (2) Mainly we applied another possibility. When dividing one commodity into 2, 3 or 4 separate items according to changes in the prices per unit one can include in the model of optimization different prices for the same commodity. For example take for given that onions could be sold for 100 \$ per ton, if we export less than 100 tons. For contracting larger quantities the buyer usually gets a commission of 10%, which will reduce the price to 90 \$ per ton. Thus we should include the following items in the model of optimization,

k = 1 onions to be sold for 100 \$/ton within a limit from 1 till 99 tons

k = 2 onions to be sold for 90 \$/ton when exporting more than 99 tons.

This method is relatively simple but fits the practical requirements satisfactorily. The only problem is the increasing number of commodities to be separately included in the model. By the way we would like to mention that we are also searching for using other programming techniques for foreign trade optimization, such as dynamic programming, which will result in new possibilities for covering non-linear problems.

Another problem which must be taken into consideration is that of the possible or necessary size of the model. Although there is a highly developed computer technique one cannot build gigantic models.

The number of variables and the number of equations to be included in the model is limited, especially from the aspect of collecting all data in a proper way and solving the model within a reasonable time. We gained good experiences in using models which included less than 10.000 equations or non-equations and 1.000 till 2.000 variables. The calculation technique used in finding the solution was the Dantzig Wolfe-decomposition technique, with the help of which a gradual approximation to the optimum could be accomplished by allocation of the entire model to individual parts.

A third problem on which we are still elaborating is that of properly relating partial optimization models to the overall or national economic optimum. From the national economic point of view all foreign trade models must be considered as partial models which should contribute to the increase of the economic efficiency in general and by this to the growth of the National Income. Therefore the partial optimization must be in consistency and in full line with the overall or national targets and limitations and should not follow objectives which are contrary to them.

This problem must be taken into account and be solved when determining the inputs of the partial model, such as e_k , i_k , S_r . These inputs should be derived from the national plan or concept.

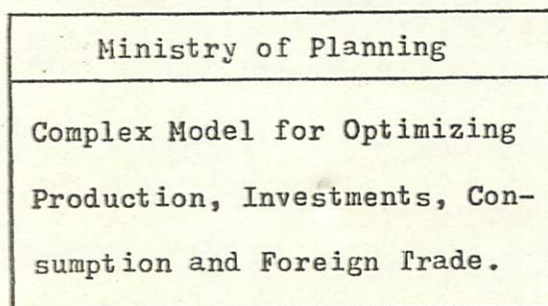
The partial optimization then will contribute to a further de-
tailation and improvement of the national plan.

We are also discussing the project of building a whole system of
models for economic planning.

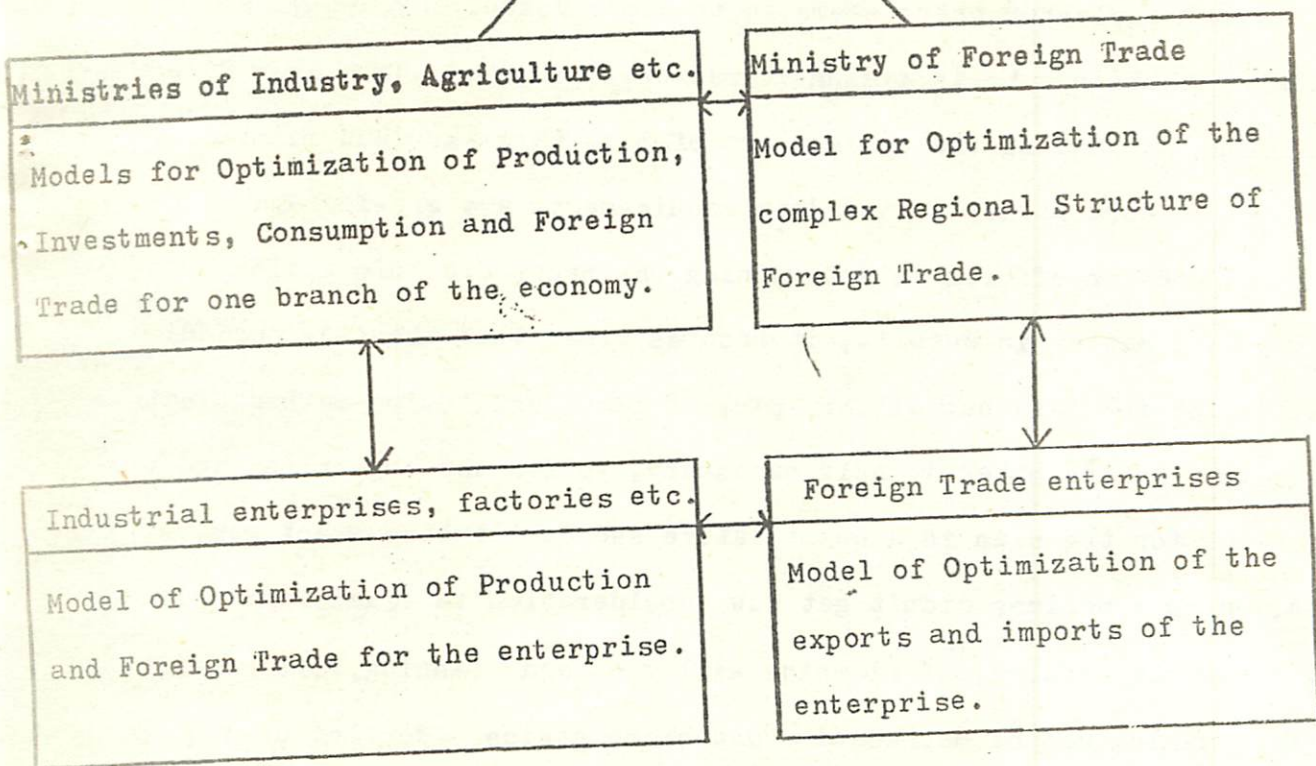
There are 2 aspects according to which one can differentiate mo-
dels of optimization:

- (1) according to the subject of optimization
- (2) according to the level of optimization (macro or mico economic
level).

In the G.D.R. we started building a comprehensive system of models for
economic planning and here foreign trade models are part of it. The fol-
lowing scheme gives a first idea about such a system with special emphasis
on models for foreign trade planning.



to be continued on page 23



3.2 The informational problem

The most crucial problem one will be facing in applying linear programming technique in practice is that of collecting the necessary information (inputs). How to get correct data for the plan period? How to estimate future prices, the demand and the supply possibilities etc.?

From G.D.R. experience the informational problem can only be solved by involving all experts and planners, who are working in the corresponding sector, in the process of applying the model. Besides one must have up-to-date analytical reports, a good market research and should apply statistical methods for calculating future trends and tendencies.

For the planner there seems to be a paradoxon. By applying programming technique he is aiming at rationalizing the whole procedure of planning and finding the optimum set of plan-targets. But first of all he has to work harder as the data requirements are growing and he must spend greater efforts in determining the necessary information. Going into this matter in details, it becomes clear that there is no additional demand for data but all are part of planning. Using mathematical methods in planning makes it only necessary, to fix the objectives and the limitations for the plan in a quantitative and qualitative exact way, a matter, which sometimes didn't get due consideration in traditional planning. Thus the accuracy of planning will grow and planning will benefit at all by making use of mathematics and econometrics. Now let us discuss some ideas how to get the necessary inputs.

As regards the external side of the R.O.M. and C.R.O.M. one has to predetermine the following inputs:

prices	(p_{kr}, d_{kr})
upper bounds	$(\bar{b}_{kr}, \bar{b}_{gr})$
lower bounds	(b_{-kr}, b_{-gr})
balance limitations	(S_r, E_r, I_r)

Balance limitations are mainly related to trade policy aspects and the existing trade and payment agreements so that it is first of all a matter of trade policy making, to determine S_r, E_r, I_r . These inputs should be fixed at the end of collecting all data and if it is an optimization on

micro economic level one should be generous. The lower bounds usually can be derived from trade and payment agreements, contracts, concluded and by analysing the past development.

In medium-term and long-term planning the lower bounds tend to be 0 if there are no contracts and long-term agreements. Thus the field of optimization will become larger.⁽¹⁾

The informational problem on the external side of the model is mainly concentrated on determining the external prices and the upper bounds for exports and imports. And here we need a good market research on prices and on supply and demand possibilities.

In the G.D.R. we make use of the experiences of all organs which are engaged in foreign trade and those are: the export and import companies, the producers of exports, the industrial consumer of imports, the Ministry of Foreign Trade, research institutions in the field of foreign trade, the Foreign Trade Representations abroad. All information coming from these organs and besides from international statistics and projections should be collected and analysed. When estimating figures for the plan period one should also apply statistical methods like trend calculations, regression calculation etc.

(1) Of course, we will also include lower bounds for exports and imports in models for perspective planning, e.g. for ensuring minimum exports of some commodities to some markets.

For making the optimization more realistic one can calculate some variants, varying the external prices and/or the upper bounds for our exports and imports. Deciding on the figures to be fixed in the plan then means choosing between some variants which have been calculated exactly and laying down the policy to be followed within this frame.

As regards the internal side the R.O.M. and C.R.O.M. include the following inputs:

the internal demand	E_k
production capacities	K_k
export and import volumes	e_k, i_k
internal prices or costs of	p_k, p_k^I
production	d_g, d_g^I
technical coefficients	a_{kj}

Part of these data can be taken from other sources, such as balance reports and preliminary balances for the plan period. First of all we have in mind commodity balances, from which one can get information on the domestic demand and - in case of applying the R.O.M. - on the export and import volumes. The technical coefficients a_{kj} should be obtained from input-output balances and the production capacity has to be calculated on the basis of the actual output, taking into consideration new capacities which will result from both further improvements in the efficiency of production and new investments. Prices and costs of production for future

periods should be calculated according to the internal price principles and actual trends in the development of costs. The model can be based in one variant on constant prices internally. For making projections or estimating the future development it might also be useful to apply statistical trend calculations, elasticity calculations etc.

As the constraints are limiting the field of optimization one should be realistic but not too cautiously, when determining the inputs. For example: When applying the R.O.M. it is better if there is a comparatively large difference between the maximum sales possibilities and the predetermined export volume. Thus the possibility of varying the exports per country is growing and the objective function will result in a better distribution. On the other side, it goes without saying that the constraints

$$(1) \quad b_{kr} \leq x_{kr} \leq \bar{b}_{kr}$$

$$(2) \quad b_{gr} \leq y_{gr} \leq \bar{b}_{gr}$$

have to ensure that the optimal solution is a realistic one as seen from the angle of the real purchase and sales possibility per commodity and country. There will be a greater flexibility in determining the inputs when we are applying such models for medium-term and long-term planning. The calculation should be understood then as a projection on which basis the plan could be drawn up.

From G.D.R. experience it needs a lot of time and one has to spend great efforts especially at the beginning of applying models in practical.

planning before there will be a considerable benefit. But here we should be patient and profound in studying all problems which may arise as we will benefit from these efforts in the long-run. After having gained first experiences one can better proceed. Finally the programming technique will lead to a rationalization of planning and to plan-figures, which are not only consistent but likely optimal ones.

There is another experience we gained: One should not rise the question of replacing traditional tools like balances by modern tools like models of optimization. We are following the concept of combining the traditional and the modern tools in practical planning and here we got good results. The R.O.M. and C.R.O.M. nowadays are elaborated in all details and are subject of practical application by various organs and on different levels together with using the other tools for foreign trade planning, such as balances, efficiency calculations etc.

3.3 Steps to be undertaken when applying linear programming models in practice

On the basis of practical experiences we gained and in accordance with the principles applied in operations research in general one can formulate a set of different steps to be undertaken when applying such models in foreign trade planning. And these are the main steps:

1-st. step : One should start with a profound analysis on the subject of optimization and investigate the main elements, the objectives and goals for the plan-period.

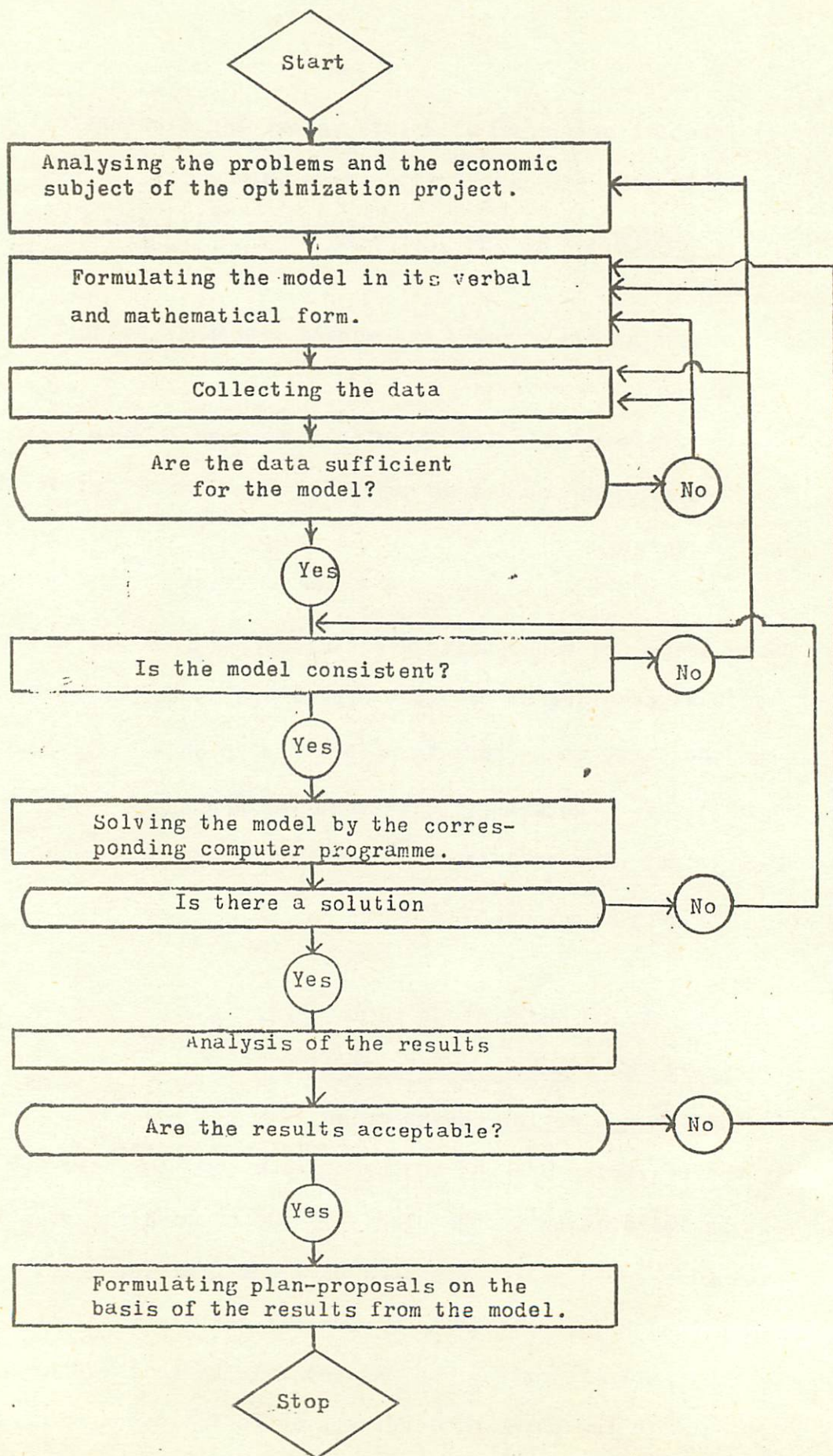
- 2-nd step : Formulating the model in its verbal and mathematical form.
- 3-rd step : Collecting and checking the necessary data.
- 4-th step : Preparing the computer programme and solving the model by the computer.
- 5-th step : Analysing the results and making proposals either for the plan,
or for another calculation on the computer when changes in the constraints and/or objective function are preferable or necessary.

These steps are consecutive steps resulting from the procedure of planning and the technique to be used. There are close correlations and often it is necessary, to go back to a former step when dealing with the collection of data, elaborating on the computer programme etc. The following scheme might give an idea on the procedure and the possible correlations. (page 30 , please)

Now let us elaborate on these steps in details.

(a) Analysing the economic problem and
fixing the subject of optimization

At the beginning one has to investigate the long-term and short-run objectives for drawing up the plan, the factors, which are stimulating the economic growth and at the same time the limiting conditions. Which are bottlenecks, shortcomings at present and for the future period? As regards foreign trade one has to fix the role of this sector in the economic policy and in the structural concept to be followed. Summarizing



all these aspects one has finally to decide whether it is useful or not to apply linear programming technique and here the concrete task for the optimization has to be determined. Is it useful to apply a model for the regional optimization or for optimizing the regional and commodity structure, should we calculate the exports and imports separately or in its relation to production, consumption etc.

(b) Formulating the model

On the basis of a profound analysis and following clear outlines for the optimization one can logically proceed in constructing the model to be applied in its verbal and mathematical form. As it is necessary to formulate the objectives and the constraints in an exact way often one has to check again and to reconsider the analysis and its outcome.

When formulating the mathematical model it becomes quite clear which are the variables of the model and which are the necessary inputs for the calculation. Here or even in the first step one has also to lay down the concrete nomenclature for the optimization by answering the question which commodities and which countries should be included and which could be neglected or omitted. This mainly depends upon the aim of the optimization.

To give an example.

The Ministry of Foreign Trade is aiming at finding the optimal distribution of the exports and imports on regions. Thus the regional nomenclature

of the model should be very detailed including all or nearly all countries which are of importance for our exports and imports. The commodity nomenclature can be a selected or an aggregated one including main commodities or groups of commodities.

But if we are aiming at an optimization of the commodity structure of exports and imports, we should handle the matter vice versa. The commodity nomenclature should be detailed and the regional nomenclature a selected or aggregated one.

(c)' Collecting the data

From the model in its mathematical form one can directly derive a list of data, which we need for calculating the optimal structure of the problem concerned. The data must be defined very precise. For example we have not only to fix that we need external prices for commodities by countries. Besides the terms of delivery (fob, cif or any other basis) and the terms of payment must be taken into consideration and here a unified price basis must be found to enable a correct optimization.

After having collected the data and before starting the computation itself one has to check whether the data are consistent and acceptable. We developed a special programme for the check-up which will be calculated by the computer.

The check-up includes a series of control calculations, such as (for the R.O.M. e.g.)

- * The upper bounds per commodity must be larger than the export quantity/value

$$\left(\sum_r \bar{b}_{kr} \geq e_k \right)$$

- * the upper bounds per commodity must be larger than the import quantity

$$\left(\sum_i \bar{b}_{gr} \geq i_k \right)$$

- * the lower bounds per commodity must be less than the export quantities

$$\left(\sum_r b_{kr} \leq e_k \right)$$

or the import quantities

$$\left(\sum_r b_{gr} \leq i_k \right)$$

- * the lower and upper bounds must be within the frame fixed by the balance limitations

e.g.

$$\sum_k p_{kr} \bar{b}_{kr} \geq E_r \quad (E_r \text{ and } I_r \text{ here are to be understood as lower balance limitations})$$

$$\sum_g d_{gr} \bar{b}_{gr} \geq I_r$$

Only if these calculations prove to be successful one can actually start the optimization. Otherwise the data include some contradictions which cannot be overcome by the computer and one will not get a result. In such a case we have to reconsider the inputs for making necessary changes either in the inputs of the optimization or in the model itself.

(d) Analysing the results of the optimization
and preparing the necessary decisions.

The results which we will get by the computer cannot be considered automatically as the final decision as one has to check once more whether the outputs are covering the predetermined objectives and are within the limits existing internally and on external markets.

If we succeeded in including all market and other limitations correctly in the model no problems may arise. But often we were bound to make some simplifications, aggregations or even it was hard to fix correct data. Therefore it is necessary to check the outcome of the optimization and to decide upon:

- a) whether the results could be accepted and taken as basis for fixing the corresponding items in the plan or
- b) whether it is necessary to recalculate the model according to necessary changes in some equations or in the inputs.

As it is relatively easy to calculate several variants when the basic type of the model is available, we should aim at several calculations for making the picture clearer and thus having a better basis for the necessary decisions.

The model can also be applied continuously, from time to time. When there are considerable changes in the inputs of the model - which must be properly followed - up-it is possible to use the model we have previously prepared, again and to calculate a new set of optimal figures based on up - to-date information.

4 . Further research work on model building

In the further research we are following 2 lines:

- (a) Using linear programming technique for other problems of planning and decision making.

For example we are elaborating on models for optimizing the economic relations between the member countries of the C.M.E.A. We developed a model for finding the optimal structure of exports and imports inside and outside the CMEA and a model for finding the best specialization and cooperation on sectoral level between the socialist countries. Besides we are searching for linear programming models especially for medium-term and long-term planning (-multi-stage-models).

- (b) We are trying to make use of other programming techniques, such as dynamic programming, for foreign trade planning. Thus we succeeded recently in building and solving a dynamic model of optimizing the exports and imports.⁽¹⁾ Here we were able to include both non-linear problems and several years of the plan separately. This makes the model useful first of all for 5-or 10-year planning. We are also searching on further improvements of the linear programming technique (improving the computation technique, building larger models, which can be solved, making use of the dual variables and the dual programme etc.)

(1) For this technique the Bellman - algorithm has been the basis.

Appendix 1

Model for Optimizing the Regional Structure of Exports and Imports

Objective function:

$$\sum_{k=1}^n \sum_{r=1}^m p_{kr} x_{kr} - \sum_{g=1}^s \sum_{r=1}^m d_{gr} y_{gr} \longrightarrow \max.$$

Constraints:

$$(1) \quad \sum_{r=1}^m x_{kr} = e_k \quad k = 1, 2, \dots, n$$

$$(2) \quad \sum_{r=1}^m y_{gr} = i_g \quad g = 1, 2, \dots, s$$

$$(3) \quad x_{kr} \leq \bar{b}_{kr} \quad k = 1, 2, \dots, n \\ r = 1, 2, \dots, m$$

$$(4) \quad x_{kr} \geq \underline{b}_{kr} \quad k = 1, 2, \dots, n \\ r = 1, 2, \dots, m$$

$$(5) \quad y_{gr} \leq \bar{b}_{gr} \quad g = 1, 2, \dots, s \\ r = 1, 2, \dots, m$$

$$(6) \quad y_{gr} \geq \underline{b}_{gr} \quad g = 1, 2, \dots, s \\ r = 1, 2, \dots, m$$

$$(7) \quad \sum_{k=1}^n p_{kr} x_{kr} - \sum_{g=1}^s d_{gr} y_{gr} = S_r \quad r = 1, 2, \dots, m$$

Appendix 2

Model for Optimizing the Regional Distribution of Exports and Imports on Value Basis

Objective function

$$\sum_k \sum_r x_{kr} - \sum_g \sum_r y_{gr} \rightarrow \max.$$

Constraints

$$(1) \quad \sum_{r=1}^m \frac{1}{q_{kr}^I} x_{kr} = E_k \quad k = 1, 2, \dots, n$$

$$(2) \quad \sum_{r=1}^m q_{gr}^I y_{gr} = I_g \quad g = 1, 2, \dots, s$$

$$(3) \quad x_{kr} \geq B_{kr}$$

$$(4) \quad x_{kr} \leq \bar{B}_{kr}$$

$$(5) \quad y_{gr} \geq B_{gr}$$

$$(6) \quad y_{gr} \leq \bar{B}_{gr}$$

$$(7) \quad \sum_{k=1}^n x_{kr} - \sum_{g=1}^s y_{gr} = S_r$$

Hint:

The symbols are of the same meaning like before (Appendix 1).

By capital letters we want to express the value unit-basis on which this model is built.

Appendix 3

Model for Optimizing the Commodity and Regional Structure of Foreign

Trade

Objective function

$$\sum_{k=1}^n \sum_{r=1}^m p_{kr} x_{kr} - \sum_{k=1}^n \sum_{r=1}^m p_{kr}^I x_{kr} + \sum_{g=1}^s \sum_{r=1}^m d_{gr}^I y_{gr} - \sum_{g=1}^s \sum_{r=1}^m d_{gr} y_{gr} \rightarrow \max.$$

Constraints

$$(1) \quad \sum_{r=1}^m y_{gr} + x_k = \sum_{j=1}^n a_{kj} x_j + \sum_{r=1}^m x_{kr} + \varepsilon_k$$

$$(2) \quad x_k \leq \bar{k}_k$$

$$(3) \quad x_{kr} \leq \bar{b}_{kr}$$

$$(4) \quad x_{kr} \geq \underline{b}_{kr}$$

$$(5) \quad y_{gr} \geq \underline{b}_{gr}$$

$$(6) \quad y_{gr} \leq \bar{b}_{gr}$$

$$(7) \quad \sum_{k=1}^n p_{kr} x_{kr} - \sum_{k=1}^n d_{gr} y_{gr} = S_r$$

Appendix 4

Numerical example for building a linear model for planning foreign trade: The case of optimal distributing export commodities on different countries.

Suppose, there are 4 commodities, A, B, C and D, which can be exported to 3 different markets (countries). The total quantities available for export amount to 100 units of commodity A, 20 unit of B, 200 units of C and 500 units of D. The demand in the foreign markets for a given structure of prices is the following:

for commodity A on market 1 = 50 units

 on market 2 = 100 "

 on market 3 50 "

 total demand= 200 "

for commodity B on market 1 = 10 "

 on market 2 = 20 "

 on market 3 = 20 "

 total demand= 50 "

for commodity C on market 1 = no demand

 on market 2 = 150 units

 on market 3 = 200 "

 total demand= 350 "

for commodity D on market 1 = 200 "

 on market 2 = 400 "

 on market 3 = 200 "

 total demand= 800 "

As easily can be seen the total demand for each commodity is larger than the quantities available for export. Thus one should try to find the best distribution of the exports to the markets, which mainly will depend upon the prices we will get on different markets/countries. Therefore next we need exact information about the export prices. The external prices we will get are included in table 1.

Table 1

		Country 1	Country 2	Country 3
Commodity	A	10	12	15
"	B	50	40	40
"	C	10	20	15
"	D	15	10	12

The prices are to be understood as fob-prices per unit of export. If we are contracting the exports on terms which are different from the fob-basis (e.g. on cif or c.f.f. - basis), we have to re-calculate these prices for getting a unified price basis.

It can be seen from table 1, that the external prices for each commodity are different from country to country, thus we should try to follow the criteria of maximizing the foreign exchange receipts when distributing the export quantities on different markets.

But there are 2 limiting conditions:

- (1) We are bound to export at least the following quantities to the markets concerned (because of long-term contracts, items included in trade agreements etc.):

10 units of A to country 2

5 units of B to country 1

5 units of B to country 3

50 units of C to country 2

100 units of C to country 3

100 units of D to country 1

50 units of D to country 2

- (2) The foreign trade relations between us and country 1 and 2 are based on bilateral agreements, from which balance limitations will result. Our total exports to country 2 should be equal or less than 2.600 foreign exchange units, whereas there is a minimum value of exports to country 1 of 2.000 foreign exchange units. For country 3 there are no balance limitations.

If there are no other limiting conditions and preferences to be included in the programme we can start building the model now.

At first we have to determine, which are the unknowns (outputs of the calculation). We are aiming at finding the optimal export quantities of commodity A, B, C, D for each country. The variable x_{kr} which stands

for the export quantities, runs from $k = 1$ till 4 and here k is the index for commodities, and from $r = 1$ till 3, where r is the index for countries.

Table 2 includes all variables which can be chosen

		Country 1	Country 2	Country 3
Commodity A		x_{11}	x_{12}	x_{13}
"	B	x_{21}	x_{22}	x_{23}
"	C	x_{31}	x_{32}	x_{33}
x	D	x_{41}	x_{42}	x_{43}

For bringing the problem in its mathematical form we have to introduce the following symbols.

$p_{k,r}$ stands for the foreign exchange prices we get for exports of commodity k to country r ,

$b_{k,r}$ stands for limitations in the demand side when exporting product k to country r , and here $b_{k,r}$ are lower limits and $\bar{b}_{k,r}$ upper bounds.

E_r is a balance limitation for the total exports to country r

e_k stands for the total volume of commodity k available for export.

Now we are able to formulate the mathematical model, which looks as follow:

At first, the model in general

Objective function:

$$\sum_{k=1}^4 \sum_{r=1}^3 p_{kr} x_{kr} \rightarrow \text{maximum!}$$

Constraints:

$$(1) \sum_{r=1}^3 x_{kr} = e_k$$

$$(2) x_{kr} \leq \bar{b}_{kr}$$

$$(3) x_{kr} \geq \underline{b}_{kr}$$

$$(4) \bar{E}_r \leq \sum_{k=1}^4 p_{kr} x_{kr} \leq \bar{E}_r$$

$$(5) x_{kr} \geq 0$$

And here is the model in its concrete form, including the data. we have collected or predetermined before.

Objective function:

Maximization of the foreign exchange receipts

$$10 x_{11} + 12 x_{12} + 15 x_{13} + 50 x_{21} + 40 x_{22} + 40 x_{23} +$$

$$10 x_{31} + 20 x_{32} + 15 x_{33} + 15 x_{41} + 10 x_{42} + 12 x_{43} \rightarrow \text{max.}!$$

Constraints

(1) Limitations in the quantities, available for export.

$$x_{11} + x_{12} + x_{13} = 100$$

$$x_{21} + x_{22} + x_{23} = 20$$

$$x_{31} + x_{32} + x_{33} = 200$$

$$x_{41} + x_{42} + x_{43} = 500$$

(2) Upper bounds on foreign markets for our exports.

$$x_{11} \leq 50$$

$$x_{12} \leq 100$$

$$x_{13} \leq 50$$

$$x_{21} \leq 10$$

$$x_{22} \leq 20$$

$$x_{23} \leq 20$$

$$x_{31} = 0$$

$$x_{32} \leq 150$$

$$x_{33} \leq 200$$

$$x_{41} \leq 200$$

$$x_{42} \leq 400$$

$$x_{43} \leq 200$$

(3) Lower bounds for our exports on foreign markets.

$$x_{11} \geq 0$$

$$x_{12} \geq 10$$

$$x_{13} \geq 0$$

$$x_{21} \geq 5$$

$$x_{22} \geq 0$$

$$x_{23} \geq 5$$

$$x_{31} \geq 0$$

$$x_{32} \geq 50$$

$$x_{33} \geq 100$$

$$x_{41} \geq 100$$

$$x_{42} \geq 50$$

$$x_{43} \geq 0$$

(4) Balance limitations for 2 countries

$$10 x_{11} + 50 x_{21} + 10 x_{31} + 15 x_{41} \geq 2.000$$

$$12 x_{12} + 40 x_{22} + 20 x_{23} + 10 x_{24} \leq 2.600$$

The model can be solved by applying the simplex algorithm.

And here are the results. Table 3 gives insight into the outputs of the optimization, i.e. the quantities to be exported from each commodity to country 1, 2 and 3.

		Country 1	Country 2	Country 3
Commodity A		0	50	50
"	B	10	0	10
"	C	0	50	150
"	D	200	100	200

The export value with country 1 amounts to

3.500 foreign exchange units

with country 2 to 2.600 " " "

with country 3 to 5.800 " " "

Thus the objective function amounts to 11.900 foreign exchange units.

Last but not least we have to check whether all constraints are followed which easily can be seen and whether the solution can be confirmed from the practical economic, political and other points of view. If this will be the case/^{the} solution is the optimal one and should be realized in practice.

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