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The Projection of Total Expenditure of Private Households in the ARE Till 1980

By

Dr. Hans Waschkau

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SALAH SALEM St.-NASR CITY

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1 . Preliminaries

National planning as it is taken in socialist countries aims exclusively at rising the standard of living of the people. All planning activities and in general all economic activities are being done for the benefit of the country and its inhabitants. Accepting this starting point the planning of the people's welfare ought to be highlighted to national planning. Following from that planning of individual consumption has to be considered more or less as the first step of national planning i.e. the needs and the demand of the individuals are the nucleus of national planning.

The consumption (we are only considering individual consumption) is mainly based on the income for the individuals, hence income planning will be one of the important parts of national planning. Up till now there are no official information (statistical figures) available about individual income in the past thus planners will get into difficulties if they want to deal with income planning. To overcome this present difficulty planners might made use of information about total expenditure of households as well as of individuals which share about 90% of the disposable income in the ARE. Experiences have been made that the total expenditure represent an acceptable and reasonable explanatory variable for consumption. That is why we were dealing with the analysis and the projection of total expenditure and remembering the starting point the projection (including the analysis) of total expenditure ought not to be an unimportant subject of national planning.

2 . Statistical and other information used in the projection

In this section we are going to list information used in the projection supplemented by some comments.

(i) The following figures were estimated by the Ministry of Planning (Follow-up Reports).

Year	Total expenditures			
	Fixed Prices[1)]		Current prices	
	Absolut	Rate of growth %	Absolute	Rate of growth %
1964/65	1462.9	-	1462.9	-
1965/66	1503.8	2.8	1583.3	8.2
1966/67	1505.7	0.1	1632.7	3.1
1967/68	1564.2	3.9	1762.5	8.0
1968/69	1611.0	3.0	1807.1	2.5
1969/70	1702.8	5.7	1939.6	7.3

1) Fixed prices of 1964/65.

Table (1)

Figures related to the periods before 1964/65 had been calculated by a different method than those related to the periods given in table 1, hence we considered figures before 1964/65 as useless ones for our purposes.

(i i) Table 2 gives the rates of growth of population based on the research "Estimates of population in the ARE*

Period	Rates of population growth %		
	Total	Urban	Rural
1960/65	2.66	4.09	1.78
1965/70	2.78	4.09	1.83
1970/75	3.10	4.40	2.02
1975/80	3.26	4.55	2.02

Table (2)

(iii) On an average the annual rate of growth of total expenditure for 1970 till 1980 is supposed to be about 5%, which is backed by figures given in the "Programme of National Action" (July 1971). According to this programme** the annual average rate of growth of individual income per capita should be nearly 2%. Total expenditure spent by the individuals share the biggest part of the individual income, hence we adopted the two-percent rate also for total expenditure per capita. Finally, the following rates can be determined.

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* INP memo., No. 642, series 25 (arabic), 1971.

** Published by the Ministry of Information (engl.) 1971, p. 41.

Period	Rates of growth of		
	Population	Per capita expenditure	total expenditure
1970/75	3.10	2.00	5.16
1975/80	3.26	2.00	5.33

Table (3)

Figures in the last column were calculated by the equation:

$$r_e = (1 + r_p)(1 + r_c) - 1 \qquad (1)$$

r_e = rate of growth of total expenditure, r_p = rate of growth of population, r_c = rate of growth of per capita expenditure. At the same time the 5% rate we accepted for total expenditure from 1970 till 1980 shows that it brings about an increase of per capita expenditure for both urban and rural population because the latters' rates of growth are less than 5% (see table 2).

(iv) By the aid of the equation (Z_t = expenditure)

$$Z_{79/80} = 1{,}940 \, . \, 1.05^{10} \qquad (2)$$

one gets <u>3,160 mio L.E.</u> for 1979/80. The first figure on the right hand side of (2) comes from table 1 (last row).

(v) The following information given in tables 4 and 5 are known from the family budget surveys 1958/59 and 1964/65

Year	Percentages of expenditure spent by	
	Urban population	Rural population
1958/59	51.32	48.68
1964/65	52.93	47.07

Table (4)

Year	Per capita annual expenditure spent by			
	Urban population		Rural population	
	abs.	rate of growth	abs.	rate of growth
1958/59	48.84	5.61	27.21	6.82
1964/65	67.77	-	40.43	-

Table (5)

There are two points coming from the last tables which should be emphasized. With respect to the above-mentioned period it can be stated:

* The share of expenditure spent by the urban population was going up whereas that for the rural poulation was going down. In addition to that the former share is greater than the latter one.

* The per capita expenditure of the urban population are greater than those of the rural population but the annual rate of growth of the urban population is less than that for rural population.

3 . <u>The procedure of the projection</u>

The projection does not aim only at forecasting the expenditure for the whole population but should give the chance to split the expenditure according to urban and rural population. That is why one had to deal with two problems.

3.1 <u>Projection of total expenditure for the whole population</u>

The general function the projection is based on is an exponential one

$$Z_t = \alpha_o e^{\alpha_1 t + \alpha_2 t^2} \qquad (3)$$

having variable rates of growth

$$r_Z (t) = \frac{dZ_t}{Z_t dt} = \alpha_1 + 2\alpha_2 t \qquad (4)$$

Z_t denotes total expenditure with respect to period t, $r_Z(t)$ means the rate of growth (being a function of t) from period t to (t + 1), α_o, α_1 and α_2 stand for constant parameters, e is the basis of natural logarithms and t takes the values 1, 2, 3, ... Equ. (4) gives the growth from one year to the next one, but often it is necessary to have the rate of growth from period t to period (t + i) having $i \neq 1$. The rate between these two periods is given by:

$$r_{t,t+i} = \sqrt[i]{e^{\alpha_1 i + \alpha_2 (2 it + i^2)}} - 1 \qquad (5)$$

This equ. follows from equ. 4 because of

$$d\ln Z_t = (\alpha_1 + 2\alpha_2 t)\, dt$$

and

$$\int_t^{t+i} d\ln Z_t = \alpha_1 \int_t^{t+i} d\tau + 2\alpha_2 \int_t^{t+i} \tau\, d\tau$$

One gets

$$\frac{Z_{t+i}}{Z_t} = e^{\alpha_1 i + \alpha_2 (2 it + i^2)}$$

and finally equ. (5)

The total expenditure are supposed to grow continously over time due to population growth and increasing demands of the people. Therefore we have the following restrictions concerning equ. (4).

(i) $0 < r_Z(t) < +\infty$

(i i) α_1 and α_2 must not be negative at the same time.

(iii) If $\alpha_1 < 0$, hence

$$|\alpha_1| < 2\alpha_2 t.$$

If $\alpha_2 < 0$, hence

$$|\alpha_2| < \frac{\alpha_1}{2t}$$

The trend function (3) was estimated according to two different concepts.

First concept:

The information we used were given by total expenditure at current prices (table 1) supplemented by the figure 3,160 mio L.E. for 1979/80 mentioned in point (iv) of the previous section. By the aid of least squares method we got (t = 1 for 1964/65).

$$Z_t = 1,395.65\ e^{0.0562\ t\ -\ 0.0003\ t^2} \tag{6}$$

and

$$r_Z(t) = 0.0562 - 0.0006\ t \tag{7}$$

showing decreasing rates of growth (negative slope in equ. 7).

Table 6 gives the estimates (projections) of total expenditure and the annual rates of growth from 1964/65 till 1979/80. The figure for 1970/71 estimated by the Ministry of Planning which we could not allow for in the calculations is given as 2083.7 mio L.E., this means an error of 2.34% (related to our estimate) which might be acceptable. But on the other hand it may not be excluded that the difference is partly caused by an actual higher price rise than that involved in our estimations.

Due to this concept we have the following average annual rates of growth

Period	Rates of growth
1969/70-74/75	5.20
1974/75-79/80	4.87
1969/70-79/80	5.04

Year	Total expenditure mio L. E.	Rates of growth %
1964/65	1,475.9 (1,462.9)	5.56
65/66	1,559.7 (1,583.3)	5.50
66/67	1,647.2 (1,632.7)	5.44
67/78	1,738.5 (1,762.5)	5.38
68/69	1,833.7 (1,807.1)	5.32
69/70	1,932.9 (1,939.6)	5.26
70/71	2,036.1	5.20
71/72	2,143.5	5.14
72/73	2,255.1	5.08
73/74	2,370.9	5.02
74/75	2,491.1	4.96
75/76	2,615.7	4.90
76/77	2,744.8	4.84
77/78	2,878.4	4.78
78/79	3,016.6	4.72
79/80	3,159.3 (3,160.0)	4.66

Table (6)*

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* Figures in brackets represent the information we used for estimating function (6).

Second concept:

On the contrary to the first concept which started in the year 1964/65 the second one has its origin in year 1969/70. We used 3 information:

(1) The total expenditure at current prices for 1969/70 (table 1), 1,940 mio L.E.

(2) The total expenditure at current prices for 1979/80 (point iv of the previous section) 3,160 mio L.E.

(3) It has been assumed that the rate of growth of total expenditure with respect to the first half till 1975, is less than that with regard to the second half from 1974/75 till 1979/80, but on an average this rate should be about 5% for the whole period from 1969/70 till 1979/80.

A part from that the rate of growth of expenditure till 1974/75 is supposed to guarantee an increase of per capita expenditure of both urban and rural population. The highest rate of growth of population during this period is 4.40%, namely for urban population (see table 2). Subsequently, the rate of growth up to 1974/75 must be greater than 4.40% and less than 5%. We fixed the mean of the two bounds 4.7% as the rate of growth till 1974/75.*

* Having this rate of 4.7% that one for the period from 1974/75 till 1979/80 must be about 5.3% in order to get the average of 5% for the whole period.

Using this rate and the figure given in point 1. We get a figure for 1974/75 by means of the equation.

$$Z_{74/75} = 1,940 . 1.047^5 = 2,450$$

Eventually, three figures were at our disposal:

Year	Total expenditure mio L.E.
1969/70	1,940
1974/75	2,450
1979/80	3,160

Function (3) was estimated by means of a simultanous equation system.*

$$\log Z_t = \log \alpha_0 + (\alpha_1 \tau + \alpha_2 \tau^2) \log e$$

$$\log Z_{t'} = \log \alpha_0 + (\alpha_1 \tau' + \alpha_2 \tau'^2) \log e \qquad (8)$$

$$\log Z_{t''} = \log \alpha_0 + (\alpha_1 \tau'' + \alpha_2 \tau''^2) \log e$$

where τ, τ' and τ'' represent the years 1969/70, 1974/75 and 1979/80. As it is known from the trend calculus we put

$$\tau = 0 \qquad \tau' = 5 \qquad \tau'' = 10$$

Now, we have for (8)

$$Z_0 = \alpha_0 = 1,940$$

$$\log Z_5 = \log \alpha_0 + (5 \alpha_1 + 25 \alpha_2) \log e$$

$$\log Z_{10} = \log \alpha_0 + (10 \alpha_1 + 100 \alpha_2) \log e$$

$$(Z_5 = 2,450, \ Z_{10} = 3,160)$$

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* The decadal logarithms have been used.

The final results are

$$Z_t = 1,940\ e^{0.04458\ t\ +\ 0.00042\ t^2} \qquad (9)$$

$$r_Z(t) = 0.04458\ t + 0.00084\ t \qquad (10)$$

indicating that the rates of growth increase over time (positive slope in equ. 10).

Table 7 shows the projections of total expenditure and the annual rates of growth from 1969/70 till 1979/80.

Year	Total expnditure mio L.E.	Rates of growth %
1969/70	1,940.0	4.46
1970/71	2,029.1	4.54
1971/72	2,124.4	4.63
1972/73	2,225.8	4.71
1973/74	2,334.1	4.79
1974/75	2,449.6	4.88
1975/76	2,573.0	4.96
1976/77	2,704.8	5.02
1977/78	2,845.8	5.13
1978/79	2,996.6	5.21
1979/80	3,158.0	5.30

Table (7)

Remembering the figure 2,083.7 mio. L.E. estimated by the Ministry of Planning for 1970/71 there is a deviation of 2.69% (related to our estimate). Based on the figures in table 7 the average annual rates of growth are as follows:

Period	Rates of growth %
1969/70-74/75	4.77
1974/75-79/80	5.21
1969/70-79/80	4.99

What are the distinctions between the two concepts?

(i) The first concept includes decreasing annual rates of growth opposite to the second one. This has to be taken as the main difference.

(i i) Following from point (i) the average annual rate of growth till 1974/75 is greater than that for the period from 1974/75 till 1979/80 so far the first concept is concerned, but considering the second concept the average rate till 1974/75 is less than the other one.

(iii) Table 8 gives the differences* between the projections with regard to the two concepts

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* We substracted the values of the second concept from those of the first one. For getting the percentages the former difference was divided over the value of the second concept.

Year	Differences	
	Mio. L.E.	%
1969/70	- 7.9	- 0.41
1970/71	7.0	0.34
1971/72	19.1	0.90
1972/73	29.3	1.32
1973/74	36.8	1.58
1974/75	41.5	1.69
1975/76	42.7	1.66
1976/77	40.0	1.48
1977/78	32.6	1.15
1978/79	20.0	0.67
1979/80	1.3	0.04

Table (8)

Im the next section we are going to deal with the projection of the expenditure spent by urban and rural population, respectively. On the ground of the two concepts we will finally get two alternatives for the expenditure, just mentioned.

3.2. The projection of total expenditure spent by urban and rural population.

This projection is taken as the decomposition of the expenditure for the whole population which can only be done on the basis of some assumptions.

What are the assumptions ?

(1) The shares of expenditure spent by the urban population (γ'_t) increase over time whereas those related to the rural population (γ''_t) decrease.*

$$\gamma'_{t-1} < \gamma'_t \qquad \gamma''_{t-1} > \gamma''_t \tag{11}$$

Or in other words the expenditure of the urban population grow more rapidly than those of the whole population, but the expenditure of the rural population increase more slowly than those for the whole population. Hence, we have

$$r'_e > r_e \qquad r''_e < r_e \tag{12}$$

* One dash added to the symbol refers to urban population, two dashes to rural population. If there is no dash the symbol is related to the whole population.

r_e , r'_e and r''_e denote annual rates of growth of expenditure from period (t-1) to t.

Apparently, this assumption is in line with some actual tendencies taking place in the society as for example the migration of people from the contryside to the big towns which yields a quicker growth of urban population opposite to that of the rural population. Another point is the industrialization of the national economy being closely connected with the development of urban areas. Finally, it has to be mentioned that the assumption is also backed by the figures in table 4.

(2) The per capita total expenditure for urban population as well as for rural population increase over time. At least for the whole population this assumption is illustrated by the figures (in current prices) of table 1 as the annual rates of growth are higher than those of the population. There is no doubt thatthis tendency will be continued in the future time, hence we assumed it also for the two parts of the population referring to figures of table 5. Subsequently we have the two unequalities *

$$r'_e > r'_p \qquad r''_e > r''_p \qquad (13)$$

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* See appendix.

where r_p denotes the rate of growth of population from period (t-1) to t.

According to (12) and (13) there is a first interval for r''_e

$$r''_p < r''_e < r_e \tag{14}$$

(3) The rural population's total expenditure per capita should increase more rapidly than those for the urban population. The clues to this presumption are firstly the figures in table 5 (rates of growth) and secondly the slower growth of rural population which might bring about such a tendency. Besides, the forementioned relation would be able to contribute to bridging gradually the gap between the living conditions in urban and rural areas. It has to be reminded of considering the expenditure as an approximative indicator for income.

Now, there is another unequality*

$$0 < \frac{r'_e - r'_p}{1 + r'_p} < \frac{r''_e - r''_p}{1 + r''_p} \tag{15}$$

leading to an interval for r'_e

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* see appendix.

$$0 < r'_e < r'_p + \frac{r''_e - r''_p}{1 + r''_p} (1+ r'_p) \tag{16}$$

(4) The per capita total expenditure for the urban population (X'_t) must be greater than that for the rural population (X''_t), that is

$$X'_{t-1} \cdot \frac{1 + r'_e}{1 + r'_p} > X''_{t-1} \cdot \frac{1 + r''_e}{1 + r''_p}$$

or*

$$r'_e > \frac{X''_{t-1}}{X'_{t-1}} \cdot \frac{1 + r''_e}{1 + r''_p} (1+r'_p) - 1 \tag{17}$$

By the assumptions we have the unequalities required for our purpose.

From (16) and (17) one obtains

$$\frac{X''_{t-1}}{X'_{t-1}} \cdot \frac{1 + r''_e}{1 + r''_p} (1+r'_p) - 1 < r'_e < r'_p + \frac{r''_e - r''_p}{1 + r''_p} (1+r'_p) \tag{18}$$

By means of the equation*

$$r_e = r'_e \, \gamma'_{t-1} + r_e'' \cdot \gamma''_{t-1} \tag{19}$$

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* See appendix.

it follows

$$r_e' = \frac{1}{\gamma'_{t-1}} \cdot (r_e - r_e'' \cdot \gamma''_{t-1}) \qquad (20)$$

Introducing the bounds of (14) to (20) we get

$$r_e < r_e' < \frac{1}{\gamma'_{t-1}} \cdot (r_e - r_p'' \cdot \gamma''_{t-1}) \qquad (21)$$

For getting feasible sulutions for r_e' and r_e'' the intervals (18) and (21) have to lap over into each other , i.e.

$$r_e < \frac{r_e'' - r_p''}{1 + r_p''} \; (1 + r_p') + r_p'$$

and

$$\frac{X''_{t-1}}{X'_{t-1}} \cdot \frac{1+r_e''}{1+r_p''} \; (1+r_p') - 1 < \frac{1}{\gamma'_{t-1}} \; (r_e - r_p'' \gamma''_{t-1})$$

The last two unequalities give another interval for r_e''

$$\frac{r_e - r_p'}{1+r_p'} (1+r_p'') + r_p'' < r_e'' < \left[\frac{1}{\gamma'_{t-1}} \; (r_e - r_p'' \gamma''_{t-1}) + 1 \right] \frac{X'_{t-1}}{X''_{t-1}} \cdot \frac{1+r_p''}{1+r_p'} - 1 \qquad (22)$$

It can be proved* that the upper bound of (22) is greater than r_e so far the period till 1980 is concerned, hence it has to be replaced by the upper bound of (14) and we get the final interval for r''_e

$$\frac{r_e - r'_p}{1 + r'_p} \left(1+ r''_p \right) + r''_p < r''_e < r_e \qquad (23)$$

Except r''_e all variables of (23) are known, r_e can be taken from table 6 and r'_p as well as r''_p from table 2.

The following tables 9 (concept 1) and 10 (concept 2) give both the bounds (lower and upper bounds) and the mean value between the two bounds.

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* See appendix.

Limits for r''_e (concept 1)

Years	Lower bounds% two	one	upper bounds % two	one	mean %
	decimals		decimals		
0	1	2	3	4	5
64/65	3.21	3.2	5.56	5.6	4.40
65/66	3.20	3.2	5.50	5.5	4.35
66/67	3.14	3.1	5.44	5.4	4.25
67/68	3.08	3.1	5.38	5.4	4.25
68/69	3.02	3.0	5.32	5.3	4.15
69/70	2.96	3.0	5.26	5.3	4.15
70/71	2.80	2.8	5.20	5.2	4.00
71/72	2.74	2.7	5.14	5.1	3.90
72/73	2.68	2.7	5.08	5.1	3.90
73/74	2.62	2.6	5.02	5.0	3.80
74/75	2.56	2.6	4.96	5.0	3.80
75/76	2.36	2.4	4.90	4.9	3.65
76/77	2.30	2.3	4.84	4.8	3.55
77/78	2.24	2.2	4.78	4.8	3.50
78/79	2.18	2.2	4.72	4.7	3.45
79/80	2.12	2.1	4.66	4.7	3.40

Table (9)

Limits for r''_e (concept 2)

Years	Lower bounds %		Upper bounds %		mean rate %
	two decimal	one decimal	two decimal	one decimal	
0	1	2	3	4	5
69/70	2.18	2.2	4.46	4.5	3.35
70/71	2.16	2.2	4.54	4.5	3.35
71/72	2.24	2.2	4.63	4.6	3.40
72/73	2.32	2.3	4.71	4.7	3.50
73/74	2.40	2.4	4.79	4.8	3.60
74/75	2.49	2.5	4.88	4.9	3.70
75/76	2.42	2.4	4.96	5.0	3.70
76/77	2.48	2.5	5.02	5.0	3.75
77/78	2.59	2.6	5.13	5.1	3.85
78/79	2.66	2.7	5.21	5.2	3.95
79/80	2.75	2.8	5.30	5.3	4.05

Table 10

By the aid of the estimates for r''_e it is possible to get similar estimates for r'_e allowing for equ. (20). For the sake of clearness it will be better to re-write equ. (20) adding the periods the symbols are related to.

$$r'_e\,(t-1,t) = \frac{1}{\gamma'_{t-1}} \left[r_e\,(t-1,t) - r''_e\,(t-1,t)\ \gamma''_{t-1} \right] \qquad (24)$$

The r's are related to any change from a basic year to the next one whereas the γ's are related to the basic year. $r_e(t-1,t)$ and $r''_e\,(t-1,t)$ are known for all periods (see tables 6 and 7 as well as 9 and 10). The calculations ought to start in 1964/65 on the ground of having figures about γ' and γ''(see table 4) for that year. Hence, it is easy to get projections for the γ's concerning the following years using the term*

$$\gamma'_t = \gamma'_{t-1} \cdot \frac{1 + r'_e(t-1,t)}{1 + r_e(t-1,t)} \qquad \gamma''_t = 1 - \gamma'_t \qquad (25)$$

The calculation starts with equ. (24) for 1964/65 to gain an estimate for r'_e (t-1,t) related to the year forementioned. After that equ. (25) will give a figure for γ' with respect to 1965/66 and again equ. (24) will be applied yielding a value of r'_e (t-1,t) for 1965/66 and so forth.

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* See appendix.

There is one particular point as regards the estimation of $r'_e\ (t-1,t)$ because two alternatives are thinkable. An unique estimate for $r'_e(t-1,t)$ might be reached if the mean value (table 9 column 5) between the lower and upper bounds (table 9 columns 2 and 4) of $r''_e\ (t-1,t)$ is taken into account.

As an example we give the calculations for the first two years

1964/65 $$r'_e\ (64/65-65/66) = \frac{1}{0.5293}\ (0.0556 - 0.0440.0.4707) = 0.0659$$

$$\gamma'_{65/66} = 0.5293\ \frac{1 + 0.0659}{1 + 0.0556} = 0.5344$$

$$\gamma''_{65/66} = 0.4656$$

1965/66

$$r'_e\ (65/66-66/67) = \frac{1}{0.5344}\ (0.0550 - 0.0435\ .\ 0.4656) =$$

$$= 0.0649$$

$$\gamma'_{66/67} = 0.5344\ \frac{1+ 0.0649}{1+ 0.0550} = 0.5393$$

$$\gamma''_{66/67} = 0.4607$$

Table 11 contains r'_e (column 3) and γ'_t (column 4) as well as γ''_t (column 5).

Estimates for r'_e, γ'_t and γ''_t

Year	r'_e %		r'_e %	γ'_t %	γ''_t %
	lower bound	upper bound			
0	1	2	3	4	5
64/65	5.56	7.65	6.59	52.93	47.07
65/66	5.50	7.50	6.49	53.44	46.56
66/67	5.44	7.39	6.45	53.93	46.07
67/68	5.38	7.34	6.31	54.44	45.56
68/69	5.32	7.23	6.28	54.92	45.08
69/70	5.26	7.15	6.15	55.42	44.58
70/71	5.20	7.12	6.15	55.89	44.11
71/72	5.14	7.02	6.10	56.39	43.61
72/73	5.08	6.92	5.97	56.90	43.10
73/74	5.02	6.83	5.92	57.38	42.62
74/75	4.96	6.74	5.80	57.87	42.13
75/76	4.90	6.75	5.79	58.33	41.67
76/77	4.84	6.64	5.74	58.82	41.18
77/78	4.78	6.55	5.66	59.32	40.68
78/79	4.72	6.46	5.56	59.81	40.19
79/80	4.66	6.36	5.49	60.29	39.71

Table 11

Additionally, there is another possibility for getting estimates of $r_e'(t-1,t)$ which gives certain bounds for it by using the bounds of $r_e''(t-1,t)$. Then there must also be bounds for γ_t' one of them (the lower bound) being constant because of the relation

upper bound of $r_e''(t-1,t) = r_e(t-1,t) =$ lower bound of $r_e'(t-1,t)$

which yields applied to equ. (25)

lower bound of $\gamma_t' = \gamma_{t-1}' = \gamma_{64/65}' = 0.5293$

for all t. For our opinion this does not represent an useful solution, hence we did another operation. The initial value for γ_t' equals $\gamma_{64/65}' = 0.5293$ being simultaneously the lower bound for the year 1965/66, whereas the upper bound comes from equ. (25).

The final γ' for 1965/66 was fixed as the mean between the lower (0.5293) and the upper bound which again was taken as a lower bound for the next year (1966/67) and so forth. It is a matter of fact that these estimates for the γ's are nearly the same as those given in table 11 (column 4 and 5) based on an unique value for $r_e'(t-1,t)$ as it was explained before.

Again, we want to illustrate the calculations.

1964/65 $$0.0556 < r'_e < \frac{1}{0.5293}(0.0556 - 0.0321 \cdot 0.4707) = 0.0765$$

$$0.5293 < \gamma'_{65/66} < 0.5293 \cdot \frac{1 + 0.0765}{1 + 0.0556} = 0.5398$$

$$\gamma'_{65/66} = ½(0.5293 + 0.5398) = 0.5346$$

$$\gamma''_{65/66} = 0.4654$$

1965/66 $$0.0550 < r'_e < \frac{1}{0.5346}(0.0550 - 0.0320 \cdot 0.4654) = 0.0750$$

$$0.5346 < \gamma'_{66/67} < 0.5346 \frac{1 + 0.0750}{1 + 0.0550} = 0.5447$$

$$\gamma'_{66/67} = ½(0.5346 + 0.5447) = 0.5397$$

$$\gamma''_{66/67} = 0.4603$$

The bounds for r'_e are given in table 11 (column 1 and 2).

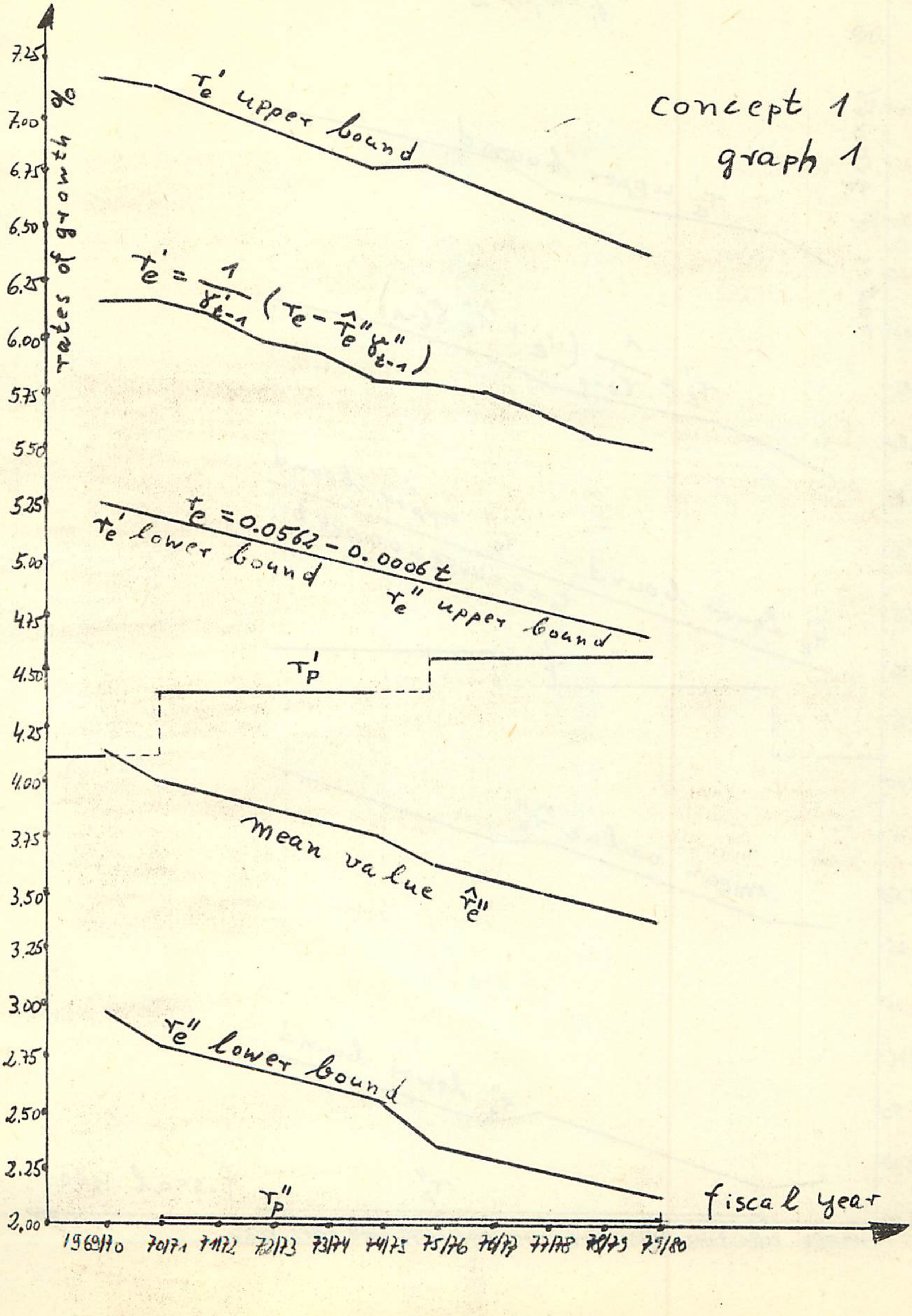
The proceeding we described before refers to the first concept because the time period starting in the year 1964/65 was taken into account. This very year provided us with some information about the γ's stemming from the family budget survey carried out in that year.

As regards the second concept allowing for the time period from 1969/70 till 1979/80 there are no information about the γ's with respect to any of the years concerned for after 1964/65 no family budget survey was implemented. But in order to handle the second concept in the same way as the first one we need at any rate a figure for the γ's in the starting year. Having such a figure the same proceeding can be applied to the second concept.

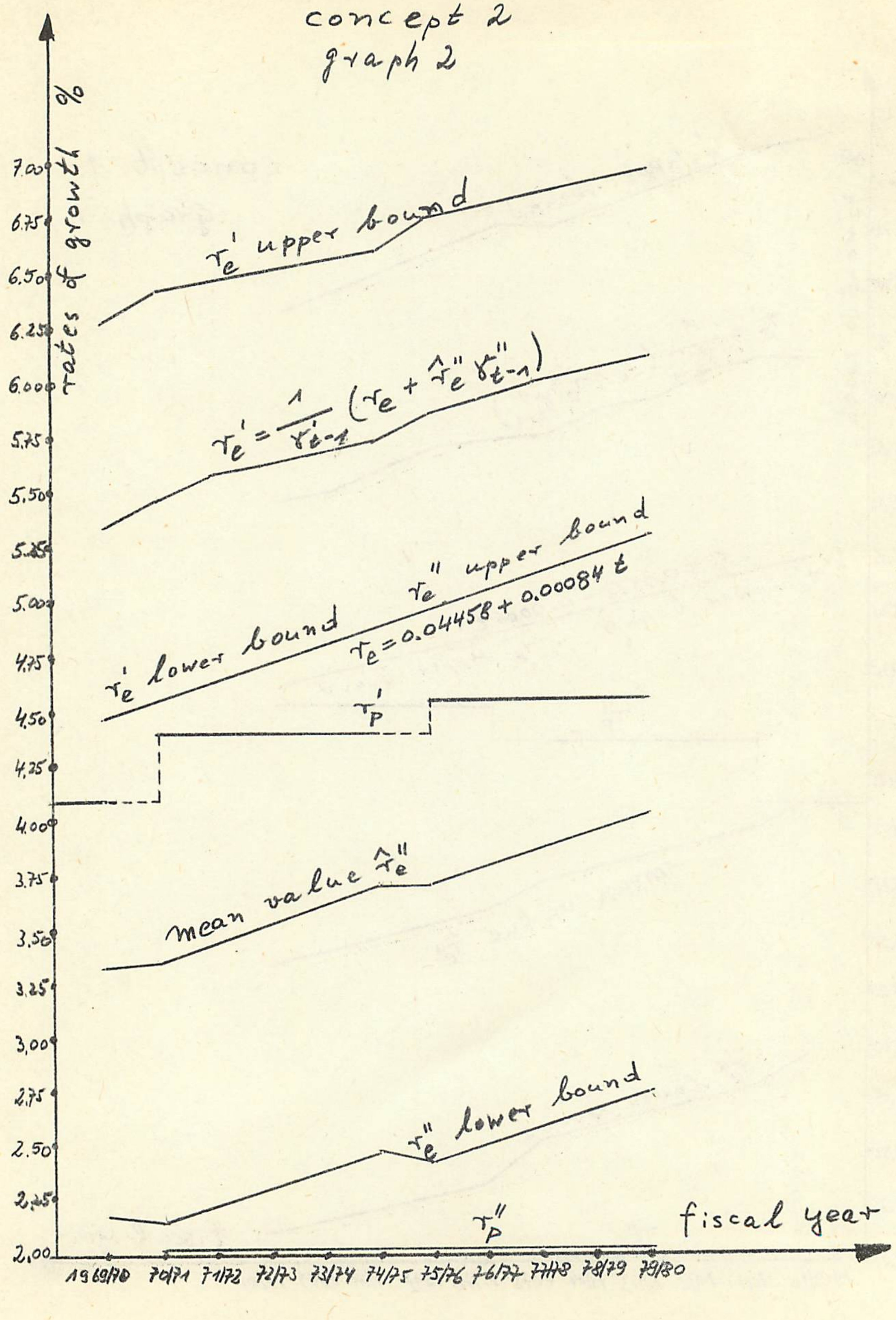
On the ground of the lack of other possibilities we adopted the estimate for $\gamma_{69/70}$ we obtained by the first concept. Hence, the initial $\gamma'_{69/70}$ equals 55.42% (See table 11, column 4, row 6). Except this particularity the second concept was handled in the same manner as the first concept.

Table 12 contains the estimates for r'_e, γ'_t and γ''_t. The graphs 1 (concept 1) and 2 (concept 2) show a graphical representation of the problems.

concept 1
graph 1



concept 2
graph 2



Estimates for r'_e, γ'_t and γ''_t

(concept 2)

Year	r'_e % Lower bound	upper bound	r'_e %	γ'_t %	γ''_t %
	1	2	3	4	5
69/70	4.46	6.29	5.35	55.42	44.55
70/71	4.54	6.42	5.47	55.89	44.11
71/72	4.63	6.47	5.58	56.38	43.62
72/73	4.71	6.51	5.62	56.89	43.11
73/74	4.79	6.56	5.68	57.38	42.62
74/75	4.88	6.61	5.73	57.86	42.14
75/76	4.96	6.76	5.86	58.33	41.67
76/77	5.02	6.79	5.91	58.83	41.17
77/78	5.13	6.87	6.00	59.32	40.68
78/79	5.21	6.91	6.05	59.81	40.19
79/80	5.30	6.97	6.12	60.28	39.72

Table (12)

In tables 13 (concept 1) and 14 (concept 2) we have the projections for the expenditure from 1964/65 to 1979/80 and from 1969/70 to 1979/80 respectively. The projection of expenditure spent by urban population is based on the equation.

$$Z'_t = Z_t \cdot \gamma'_t$$

and for rural population

$$Z''_t = Z_t - Z'_t = Z_t\,(1 - \gamma'_t)$$

Annual total expenditure for the whole population, urban and rural population.
(concept 1)

Year	Expenditure Mio L.E.		
	Total	Urban	Rural
0	1	2	3
64/65	1,475.9	781.2	694.7
65/66	1,559.7	833.5	726.2
66/67	1,647.2	888.3	758.9
67/68	1,738.5	946.4	792.1
68/69	1,833.5	1,007.1	826.6
69/70	1,932.9	1,071.2	861.7
70/71	2,036.1	1,138.0	898.1
71/72	2,143.5	1,208.7	934.8
72/73	2,255.1	1,283.2	971.9
73/74	2,370.9	1,360.4	1,010.5
74/75	2,491.1	1,441.6	1,049.5
75/76	2,615.7	1,525.7	1,090.0
76/77	2,744.8	1,614.5	1,130.3
77/78	2,878.4	1,707.5	1,170.9
78/79	3,016.6	1,804.2	1,212.4
79/80	3,159.3	1,904.7	1,254.6

Table (13)

Annual total expenditure for the whole population, urban and rural population.

(concept 2)

Year	Expenditure Mio L.E.		
	Total	Urban	Rural
0	1	2	3
69/70	1,940.0	1,075.1	864.9
70/71	2,029.1	1,134.1	895.0
71/72	2,124.4	1,197.7	926.7
72/73	2,225.8	1,266.3	959.5
73/74	2,334.1	1,339.3	994.8
74/75	2,449.6	1,417.3	1,032.3
75/76	2,573.0	1,500.8	1,072.2
76/77	2,704.8	1,591.2	1,113.6
77/78	2,845.8	1,688.1	1,157.7
78/79	2,996.6	1,792.3	1,204.3
79/80	3,158.0	1,903.6	1,254.4

Table (14)

In 1972 the government of the ARE decided to change the financial year in such a way that as from the 1st of January 1973 the financial year equals the calendar year. Due to this decision the estimates of expenditure and the rates of growth were recalculated according to the following method. The calender year 1970 was supposed to be the starting point of the operations. Z_t (t = 1, 2, ...) may denote the expenditure of the old fiscal year and C_t (t = 1, 2, ...) those of the calender year. We assumed that the expenditure related to the first half of the calender year 1970 equals $\frac{1}{2} Z_1$.

The growth rate r_1 with respect to the first fiscal year 1969/70 indicates the increase ($r_1 > 0$) of the expenditure 1969/70 up to 1970/71 including a certain change of the expenditure concerning the first half of the calendar 1970. The amount from 1.1.70 till 30.6.70 was supposed to have increased up to the end of 1970 to $\sqrt{1 + r_1}$, because r_1 is related to a whole year but we are considering only half a year. Hence, we have for 1970

$$C_1 = \tfrac{1}{2} Z_1 + \tfrac{1}{2} Z_1 \cdot \sqrt{1 + r_1} = \tfrac{1}{2} Z_1 (1 + \sqrt{1 + r_1}) \qquad (26)$$

In order to get an estimate for C_2 we assume that C_1 goes up by a certain rate which has to be specified.

There are two rates which can be applied to the amount of 1970, namely r_1 and r_2 (denoting the growth from 1970/71 to 1971/72). On the ground of dealing with rates of growth a reasonable solution would be the geometric mean between $C_1(1 + r_1)$ and $C_1 (1 + r_2)$ yielding

$$C_2 = C_1 \sqrt{(1 + r_1)(1 + r_2)}$$

or in general

$$C_t = C_{t-1} \sqrt{(1+r_{t-1})(1+r_t)} \quad (t=2,3,\ldots) \qquad (27)$$

where r_t and r_{t-1} stand for the rates of growth related to the old fiscal year.

According to equ. (26) and (27) the calculations were carried out as regards expenditure for the total and urban population* whereas the term

$$C''_t = C_t - C'_t \qquad (t=1,2,\ldots)$$

was used as to the rural population. The results can be seen in tables 15 (concept 1) and 16 (concept 2).

* As an estimate for r'_t we did not use its bounds but the unique values from tables 11 (column 3) and 12 (column 3).

Total expenditure with respect to calenbar year

(concept 1)

year	total		Urban		Rural
	absolute Mio L.E.	rate of growth %	absolute Mio L.E.	rate of growth %	Mio. L.E.
1970	1,958.1	5.21	1,087.8	6.15	870.3
71	2,060.1	5.17	1,154.7	6.11	905.4
72	2,166.6	5.12	1,225.3	6.02	941.3
73	2,277.5	5.07	1,299.1	5.92	978.4
74	2,393.0	4.98	1,376.0	5.88	1,017.0
75	2,512.2	4.93	1,456.9	5.80	1,055.3
76	2,636.1	4.88	1,541.4	6.78	1,094.7
77	2,764.7	4.83	1,630.5	5.69	1,134.2
78	2,898.2	4.74	1,723.3	5.59	1,174.9
79	3,035.6	4.69	1,819.6	5.55	1,216.0
80	3,178.0	-	1,920.6	-	1,257.4

Table 15

Total expenditure with respect to calendar year

(Concept 2)

year	Total				rural mio L.E.
	absolute Mio L.E.	rate of growth %	absolute Mio. L.E.	rate of growth %	
1970	1,961.4	4.49	1,089.4	5.41	872.0
71	2,049.5	4.59	1,148.3	5.55	901.2
72	2,143.6	4.69	1,212.0	5.59	931.6
73	2,244.1	4.74	1,279.8	5.64	964.3
74	2,350.5	4.83	1,352.0	5.69	998.5
75	2,464.0	4.93	1,428.9	5.78	1,035.1
76	2,585.5	4.97	1,511.5	5.88	1,074.0
77	2,714.0	5.07	1,600.4	5.97	1,113.6
78	2,851.6	5.17	1,695.9	6.02	1,155.7
79	2,999.0	5.26	1,798.0	6.07	1,201.0
80	3,156.7	-	1,907.1	-	1,249.6

Table 16

APPENDIX

In the appendix we want to give proofs for some of the equations and unequalities, respectively, used in the paper.

1. Relations (13).

The starting point is the presumption that there is an increase of per capita expenditure for urban as well as for rural population over time.

Let r'_e and r''_e be the annual rates of growth of per capita expenditure from period (t-1) to t then

$$r'_c > 0 \qquad r''_c > 0 \tag{1}$$

$$1 + r'_c > 1 \qquad 1 + r''_c > 1 \tag{1a}$$

From the equation (2)

$$1 + r'_c = \frac{Z'_t}{p'_t} : \frac{Z'_{t-1}}{p'_{t-1}} = \frac{Z'_t}{Z'_{t-1}} : \frac{p'_t}{p'_{t-1}} = \frac{1 + r'_e}{1 + r'_p} \tag{2}$$

and taking into consideration (1a) one gets

$$1 + r'_e > 1 + r'_p$$

and finally $$r'_e > r'_p$$

Z'_t , Z'_{t-1} , p'_t and p'_{t-1} denote expenditure and population related to urban areas with respect to (t-1) and t.

In the same manner $r''_e > r''_p$ can be proved.

2. Relation (15)

The assumption is termed as

$$0 < r'_c < r''_c$$

According to (2)

$$r'_c = \frac{1 + r'_e}{1 + r'_p} - 1 = \frac{r'_e - r'_p}{1 + r'_p}$$

and

$$r''_c = \frac{r''_e - r''_p}{1 + r''_p}$$

hence

$$0 < \frac{r'_e - r'_p}{1 + r'_p} < \frac{r''_e - r''_p}{1 + r''_p}$$

3. Relation (17).

The per capita expenditure for the urban population X'_t as well as for the rural population X''_t with respect to period t is given by

$$X'_t = X'_{t-1} (1 + r'_c) \qquad X''_t = X''_{t-1} (1 + r''_c)$$

and allowing for (2) the equation can be re-written as

$$X'_t = X'_{t-1} \frac{1 + r'_e}{1 + r'_p} \qquad X''_t = X''_{t-1} \frac{1 + r''_e}{1 + r''_p}$$

Following from the assumption concerned we get

$$X'_{t-1} \frac{1+r'_e}{1+r'_p} > X''_{t-1} \frac{1+r''_e}{1+r''_p}$$

which gives immediately relation (17).

4. Relation (19).

In more details equation (19) may be written as

$$\frac{Z_t - Z_{t-1}}{Z_{t-1}} = \frac{Z'_t - Z'_{t-1}}{Z'_{t-1}} \cdot \frac{Z'_{t-1}}{Z_{t-1}} + \frac{Z''_t - Z''_{t-1}}{Z''_{t-1}} \cdot \frac{Z''_{t-1}}{Z_{t-1}}$$

Because of

$$Z'_t + Z''_t = Z_t \qquad Z'_{t-1} + Z''_{t-1} = Z_{t-1}$$

we get an identity indicating the truth of (19).

5. Relation (22) and (23)

It was mentioned that the upper bound of (23) is greater than r_e so far the period till 1980 is concerned. For this reason the upper bound of (23) has been replaced by r_e because r''_e was due to be less than r_e. We want to justify this replacement.

The following unequality has to be proved

$$r_e < \left[\frac{1}{\gamma'_{t-1}} (r_e - r''_p \gamma''_{t-1}) + 1\right] \frac{X_{t-1}}{X''_{t-1}} \cdot \frac{1+r''_p}{1+r'_p} - 1 \qquad (3)$$

If this relation is true at least till 1980 the upper bound of r''_e must be equal to r_e.

Term (3) can be re-written as

$$1+r_e < \left[\frac{1}{\gamma'_{t-1}} \quad (r_e - r''_p \, \gamma''_{t-1}) + 1 \right] \frac{X'_{t-1}}{X''_{t-1}} \cdot \frac{1+r''_p}{1+r'_p} \tag{4}$$

From unequality (21) it is known that

$$1+r_e < \frac{1}{\gamma'_{t-1}} \quad (r_e - r''_p \, \gamma''_{t-1}) + 1 \tag{5}$$

If there is a proof for

$$\frac{X'_{t-1}}{X''_{t-1}} \cdot \frac{1+r''_p}{1+r'_p} > 1 \tag{6}$$

then (4) is true

The per capita expenditure X'_t is equal to

$$X'_{t-1} = \frac{Z'_{t-1}}{P'_{t-1}}$$

for all t and naturally, there is a similar term for X''_t. Hence, from (6).

$$\frac{Z'_{t-1}}{P'_{t-1}(1+r'_p)} > \frac{Z''_{t-1}}{P''_{t-1}(1+r''_p)}$$

and

$$\frac{Z'_{t-1}}{P'_t} > \frac{Z''_{t-1}}{P''_t}$$

$$\frac{Z'_{t-1}}{Z''_{t-1}} > \frac{P'_t}{P''_t} \tag{7}$$

Term (7) equals

$$\frac{\gamma'_{t-1}}{\gamma''_{t-1}} > \frac{\beta'_t}{\beta''_t} \qquad (8)$$

following from

$$\frac{\gamma'_{t-1}}{\gamma''_{t-1}} = \frac{Z'_{t-1}}{Z_{t-1}} : \frac{Z''_{t-1}}{Z_{t-1}} \qquad \frac{\beta'_t}{\beta''_t} = \frac{P'_t}{P_t} : \frac{P''_t}{P_t}$$

β'_t and β''_t denote the proportion of urban population and rural population, respectively, in the whole population.

The lefthand side of (8) is greater than unity according to the first assumption and to the figures in table 4.

As to the righthand side of (8) we know that in the past time β'_t was increasing whereas β''_t was decreasing over time and that β'_t was less than β''_t. From population projection* it is known that this tendency, the so-called urbanisation, will be continuing till 1980 and that β'_t is always less than β''_t at least until 1980.

Subsequently, the lefthand side in (8) is greater than unity opposite to the righthand side being less than unity. Hence, unequality (8) is true (till 1980). i.e. unequality (3) must be true, too.

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* INP-memo. no. 642, series 25, 1971 (arabic).

6. Relation (25)

Equation (25) can be written as

$$\frac{Z'_t}{Z_t} = \frac{Z'_{t-1}}{Z_{t-1}} \cdot \left[\frac{Z'_t}{Z'_{t-1}} : \frac{Z_t}{Z_{t-1}} \right]$$

which is equal to the identity

$$\frac{Z'_t}{Z_t} = \frac{Z'_t}{Z_t}$$