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Planning Manpower and Education With An Equilibrium And Decision Model

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PLANNING MANPOWER AND EDUCATION

WITH AN EQUILIBRIUM AND DECISION MODEL

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We need rules for allocating efficiently both manpower and education services. And we would like that our plan could give us some incentives for inducing people to join the right jobs. So we have to build up a general equilibrium and optimization model.

Again in form of a dynamic Linear Programming Model. Theoretical scheme and some applications.

I.- A DYNAMIC LINEAR PROGRAMMING

(Cf. J. BENARD 1966 and also I. ADELMAN 1966, S. BOWLES 1967-69, VERSLUIS 1970-74).

1) Primal constraints and variables

Now all variables will be dated.

Constraints are the same as before with some peculiarities :

a) Adjonctions of (4.3) and (4.4) for fixed (material) capital requirements and material net investments via (4.1)

b) Substituality is introduced :

1. for material, labour and capital inputs (matrices A_1, N_1 and B_1 are n x m and vector $Z_1(t)$ is an activity vector)

2. for outputs (C_1)

3. for the education system. We may now have several pedogogical techniques for a given educational level

4. and above all for matrix [E] which becomes

$$[\hat{E}] \equiv [e^r_{hq}] = \begin{pmatrix} e^1_{11} & e^2_{11} \\ e^1_{21} & e^2_{21} \\ e^1_{31} & e^2_{31} \end{pmatrix} \quad ....$$

R = number of combinations of skills (Q) and various possibilities education content of a given skill.

$\Longrightarrow \dot{L}(t) \Longrightarrow \tilde{L}(t)$ vector of the mixes of qualifications R x 1 educational content, which we will name "qualifeduc mix"

So it gives :

(4.5) $$\underset{H\times R}{[\tilde{E}]} \; \underset{R\times 1}{\tilde{L}(t)} - \hat{\gamma}\, J(t) \leq 0$$

and in (4.2)

$$[N_1]\, Z_1(t) + [N_2]\, Z_2(t) - \underset{Q\times R}{[\pi]} \; \underset{R\times 1}{\tilde{L}(t)} \leq (t-1)\, \hat{\lambda}\, L_0$$

where $[\pi]$ is an incidence positive matrix regrouping for each q the elements of $\tilde{L}$ in order to get $\dot{L}$

i.e. $[\pi]\,\tilde{L} = \dot{L}$

c) It is no longer necessary to have square transition matrices of type [T], the model being now able to choose among flow of students. But if we want, realistically, to prevent any sudden exhaustion of some flows by the optimization process, it will be wise to keep the previous formalization with both matrices M and T.

d) Terminal conditions on capital stocks both material and human.

2) Objective function

1. For the whole period : maximize the discounted sum of final consumption flows (welfare). Other possible objective function :

2. Max sum of G N P_s

3. Max foreign currencies balances

4. Max labour employment.

3) Meaning of the solution

For every period t , the optimization model selects the values (positive or nul) of all the variables, both educational and economic, which maximize the global discounted value of final consumption for the whole period.

And it does so, by eliminating the inefficient (for the model) techniques or pedagogies, and also the wrong educational contents of skills (though $[\tilde{E}]\ \tilde{L}$).

We can, and must, check the sensibility of solutions to changing the parameters values

- right hand members
- objective function coefficients
- eventually some matrix coefficients

Finally, we get simultaneously

- an equilibrium and optimal solution
- a set of solutions for every subperiod of the whole overperiod, dynamically linked together
- a set of shadow prices through the dual.

II.- THE DAULITY SYSTEM AND ITS ECONOMIC POSSIBLE USE

1) Dual variables, dual constraints and there meaning

The four first vectorial dual variables P , W and both R have a straight forward interpretation : they are shadow prices respectively for produced goods, labour (of various skills) and fixed capital equipment services for the economic (R_1) and for the educational sectors (R_2).

So we have shadow prices (P), shadow Wages (W) and shadow rents (R_1 and R_2).

Less classical are the vectors V , Γ and Ω. They represent the economic value of the human capital accumulated through educational investment at various stages of its completion and use.

So V associated to primal constraint (4.5) is the vectorial value of human capital of the new workers $\hat{\gamma}$ J classified by educative level.

Γ is the same value for the students just leaving the educational system.

Dual constraint (5.8) exhibits a very simple link between these two dual variables since it shows that they are proportionnal through the labour force participation rates $\hat{\gamma}$.

Ω is the vector of human capital values of the students who still participate to the school and university system.

All these dual variables are discounted, since they are directly or indirectly linked to the vector of shadow prices P(t) which, through dual constraint (5.3) is discounted.

Dual relations 5.1 , 5.3 , 5.4 and 5.5 classicaly relate utilities, prices and costs.

Dual relation 5.2 says that the human capital value of students just leaving the school or university system at time t , cannot exceed its total cost in the preceding period t-1 including current material and labour costs, capital costs and human capital costs invested in students Ω (t-1).

Γ , Ω and V are linked by very simple relations (5.7) and (5.8), whilst the shadow wages vector W appears, through (5.6) as a mere combination of the elements of V , the human capital value of young workers just leaving the educational system.

2) The possible use of dual variables and relations

Strangely enough, the dual relations of an optimization model are scarecly written and studied. In fact they can be precious as guidelines for implementing planning decisions. As optimal values or shadow prices are equilibrium prices associated to the optimal physical plan, and as dual relations are just the first order conditions of optimum, there is theoretically perfect and one to one correspondance between all the physical quantities solutions of the optimal plan for production, investment, consumption, labour and education on the one hand and all the dual values or shadow prices arising out of the same optimization model.

As far as these shadow prices (shadow wages, etc..) can be translated into market prices and wages, or just influence enough the actual market prices and wages, they might be used as incentives to induce the economic agents to behave consistently with the plan. What is rather difficult and sometimes impossible with arbitrary prices or traditional prices, not related to or even conflicting with planned objectives, would certainly become easier. So the wage structure policy would be related to the manpower policy and to the general economic planning and could be better used both for forecasting and for implementing substitution and optimal allocation of human resources.

Let us underline that this price and wage policy is not to be substituted to physical means and regulations but combined with them in the most efficient way.

III.- SOME APPLICATIONS AND RESULTS

Until now, as far as I know, no general optimization model of that kind has been practically used for actual planning. Probably because of the novelty of the method, the lack of reliable statistical datas in some fields (i.e. matrices E of Ê)and above all, the reluctance of planners and, still more, of educators, towards models.

All these factors are vanishing but still exist.

But numerical computations have been realized at least for three countries.

I'll leave aside the BOWLES' model (4) which has been applied to Northern Nigeria and to Greece, because it deals only with the educative sector linking it with the economy through manpower constraints and through the contribution of education to economic welfare measured by the discounted sum of future differential earnings due to additional education. Apart from this, we have I. ADELMAN's model for Argentina (1966) (1), my own model for France (3) and I. VERSLUIS application of my model to Peru (5).

All these applications were "laboratory-experiments", but gave some precious learnings.

1) Size of programs

I. ADELMAN's model covered 4 five-years period from 1950 to 1970 and had 400 primal endogenous variables for 284 constraints. Its resolution lasted 40 minutes with a IBM 7094 computer.

My own model was solved for four different coverages and horizons. For the largest horizon (6 three-years periods -1967/85-) and without including professional training it had 368 primal endogenous variables for 292 constraints. When it included professional training, it grew up to more than 400 primal variables and to more than 300 constraints. This last model needed 70 minutes of computarization with an IBM 360/50.

VERSLUIS' application of my model to Peru covers 5 five-years periods (1964-89). It has between 350 and 400 primal endogenous variables as well as primal constraints.

2) Numerical and policy results

All the numerical results obtained emphazize the rigidity and the weight of the educational system upon the economic system. The most recent exercice (VERSLUIS' one) uses alternatively three objective functions :

1. maximizing the discounted sum of successive GDP
2. the same for final consumption
3. maximizing final consumption under an additional constraint of minimum labour employment.

Very interestingly the calculations show that the first policy (maximizing GDP) implies a sudden and big jump of agriculture and services production after the second period, which is not realistic. The second policy (maximizing consumption) endeavours a more regular growth and ends with pratically the same GDP level. The third policy, which institutes a labour employment floor, sacrifices a minor output of GDP than the second policy in the first period and reaches slightly higher levels in the last periods. So the third policy seems to be the best.

The three policies lead to a substantial development of both primary and general secondary educations, but only the first one implies some (limited) increase of teacher training and University enrollments. These are stagnating in the other two policies and it is the same for secondary technical education in any alternative. This last result corroborates what S. BOWLES and I. ADELMAN had already arrived at in their own studies.

Of course all these numerical exercices have their value depending on the statistical material accuracy and many improvment have to be done yet.

However, looking at the various matrices and coefficients, we may notice that in ordinary forecasts or projections we currently use many of them. So the task is more to systematize and to reinforce our material than to create it from scratch. And modelization helps us to check the consistency of many datas.

So, here as in many other fields, the main virtuye of economic models is perhaps pedagogical, as it complies us, and the policy makers, to take a wider and more consistent view of intricated phenomenas.

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LIST OF SYMBOLS

ECONOMY

$[I - A_1]$ = matrix of input output

$[N_1]$ = matrix of labour coefficients

X = vector of outputs

$\bar{Y}$ = vector fo final demand (exogenous)

MANPOWER

[E] = matrix of educational content of the various occupational labour

$\dot{L}$ = vector of the new entrants on the labour market

L_{t-1} = vectorial stock of available labour force in time t-1

$\hat{\lambda}$ = diagonal matrix of rates of survival of this stock of available labour force

$\hat{J}$ = vector of demand of new manpower by the economic system

./...

J = vector of students (graduated or not) going out from the educational system and classified by educational levels. It is the vector of supply of manpower by the educational system.

$\hat{Y}$ = diagonal matrix of participation rates of these students to the labour force

S = vector of the stock of students present in the school system (enrollments)

$\hat{S}$ = vector of the transition flows of students from one educational level to another one

$[A_2]$ = material input matrix of the educational system (inputs per student)

$[N_2]$ = teaching and other staff matrix of the educational system (per student)

$[T]$ = flow of students transition matrix (line Markovian)

$[M+]$ = positive incidence matrix for flow of students (departure matrix)

$[M-]$ = negative incidence matrix (arrival matrix)

$\bar{D}$ = vector of demographic entries into the school system (per educational levels)

$\hat{\sigma}$ = diagonal matrix of students death rates

STRUCTURE OF THE OPTIMIZATION MODEL

<u>PRIMAL MODEL</u>

$$\text{Max } \Phi = \sum_{t=1}^{T} \alpha(t)\, \bar{U}'(t)\, Y(t) \qquad (4.0)$$

$$- [C_1 - A_1]\, Z_1(t) + [A_2]\, S(t) + Y(t) + \dot{K}_1(t) + \dot{K}_2(t) \leq -\bar{Y}(t) \qquad (4.1) \qquad P(t) \geq 0$$

$$[N_1]\, Z_1(t) + [N_2]\, S(t) - [\pi]\, \tilde{L}(t) \leq (t-1)\, \hat{\lambda}\, L_o \qquad (4.2) \qquad W(t) \geq 0$$

$$[B_1]\, Z_1(t) - \sum_{\theta=1}^{t} [I-(t-\theta)\, \hat{\delta}_1]\, \dot{K}_1(\theta) \leq [I-(t-1)\, \hat{\delta}_1]\, \bar{K}_1(o) \qquad (4.3) \qquad R_1(t) \geq 0$$

$$[B_2]\, S(t) - \sum_{\theta=1}^{t} [I-(t-\theta)\, \hat{\delta}_1]\, \dot{K}_2(\theta) \leq [I-(t-1)\, \hat{\delta}_2]\, \bar{K}_2(o) \qquad (4.4) \qquad R_2(t) \geq 0$$

$$[\tilde{E}]\, \tilde{L}(t) - \hat{\gamma}\, J(t) \leq 0 \qquad (4.5) \qquad V(t) \geq 0$$

$$[M+]\, \tilde{S}(t) - [I-T']\, [I-\hat{\sigma}]\, \dot{S}_{t-1} + J(t) = 0 \qquad (4.6) \qquad \Gamma(t) \gtrless 0$$

$$S(t) - [M-]\, \tilde{S}(t) - [T']\, [I-\hat{\sigma}]\, S_{t-1} = \bar{D}(t) \qquad (4.7) \qquad \Omega(t) \gtrless 0$$

$$- K_i(T) \leq - \underline{K}_i \quad i = 1,2 \qquad (4.8)$$

$$- S(T) \leq - \underline{S} \qquad (4.9)$$

DUAL MODEL

$$\text{Min } \Psi = \sum_{t=1}^{T} \left[- P'(t)\ \bar{Y}(t) + N'(t)\ (t-1)\ \hat{\lambda}\ L_o + \sum_{i=1}^{2} R'_i(t)\ [I-(t-1)\ \hat{\delta}_i]\ \bar{K}_i(o) \right] + \Omega'(t)\ \bar{D}(t) \qquad (5.0)$$

$$- P'(t)\ [C_1-A_1] + W'(t)\ [N_1] + R'_1(t)\ [B_1] \geq 0 \qquad (5.1) \quad Z_1(t)$$

$$P'(t)\ [A_2] + W'(t)\ [N_2] + R'_2(t)\ [B_2] + \Omega'(t) - \Omega'(t+1)\ [T']\ [I-\hat{\sigma}] + \Gamma'(t+1)\ [I-T']\ [I-\hat{\sigma}] \geq 0 \qquad (5.2) \quad S(t) \geq 0$$

$$P'(t) \geq \alpha(t)\ \bar{U}'(t) \qquad (5.3) \quad Y(t)$$

$$P'(t) - \sum_{\theta=t}^{T} R'_1(\theta)\ [I-(\theta-t)\ \hat{\delta}_1] \geq 0 \qquad (5.4) \quad \dot{K}_1(t)$$

$$P'(t) - \sum_{\theta=t}^{T} R'_2(\theta)\ [I-(\theta-t)\ \hat{\delta}_2] \geq 0 \qquad (5.5) \quad \dot{K}_2(t)$$

$$- W'(t)\ [\pi] + V'(t)\ [\tilde{E}] \geq 0 \qquad (5.6) \quad \tilde{L}(t)$$

$$- \Omega'(t)\ [M-] + \Gamma'(t)\ [M+] \geq 0 \qquad (5.7) \quad \tilde{S}(t)$$

$$- V'(t)\ \hat{\gamma} + \Gamma'(t) \geq 0 \qquad (5.8) \quad J(t)$$