ARAB REPUBLIC OF EGYPT

THE INSTITUTE OF NATIONAL PLANNING

Memo.(1497)

A Model For Distribution Of Resources in Multi-Projects

By

Dr. Mohd. Yehia Abdel Rahman

March 1989

CAIRO
SALAH SALEM St-NASR CTTY

CONTENTS

INTRODUCTION :

(*)
" At this time, computationally feasible models for optimizing the leveling of resources do not exist "

<<<...>>>

One of the common problems which faces a production or a construction firm, is to distribute its resources on its different projects. There will be no problem, if the resources were enough to cover the needs of all projects.

The problem will arises, if one project or more could not be accomplished at its exact time. In this case, some subsidiary problems could be created; such as :

i) the posterior project/ s -if any - will be retarded;
ii) the firm may pay some overtaxes as penalties for retardation;
iii) the confidence and trust in that firm will be chaked and changed ;
iv) the total cost of the project/ s , will augment with the time (the opportunity and indirect costs are directly proportional with the time).

To overcome the preceeding problems, the firm planners have to put their hands on the bottlenecks; then reevaluate and modify their plans.

(*) Ref. (5); pp. 545

There are many reasons, that the project(s) may accomplished after its calculable time. Some of these reasons, could be summarized in the following points :

i) the absence of scientific techniques and managements;
ii) the shortage of resources during the execution ;
iii) the wrong distribution of resources on the projects;
iv) the negligence of control for some sensible and critical activities ;
v) the unavoidable and unexpected exterior factors as climate, power, ..., etc.

In this research, we will consider the PERT or CPM technique as a starting step to formulate the mathematical relations of our model. So, the first part of this research will concern to briefly expose the most important parts of PERT or CPM technique as a scientific tool for this kind of problems. An extended and very important part of this technique, is the so-called "Resource leveling and allocation"; where equipment and manpower planning will be put in consideration in this research.

In the second part, we will concentrate our attentions to generalize the use of "Resource allocation" as a preliminary step to formulate our model. In the third part, the computer output result for an example will be presented.

PART - I

PERT and CPM

Two fundamental analytic techniques proposed in the late 1950s for planning, scheduling, and controlling complex projects. PERT which is the synonym of "Project Evaluation and Research Technique" or "Performance Evaluation and Research Technique" or "Program Evaluation and Research Task"; and CPM for "Critical Path Method". Although the two techniques were developed concurrently and independently, they were nearly similar. Aside from minor differences in terminology, notation, and structure, only two major differences usefully distinguished the two methods. First, PERT acknowledged uncertainty in the times to complete activities, while CPM did not. Second, PERT restricted its attention to the time variable, whereas CPM included time - cost tradeoffs.

(1-1) PERT / TIME and PERT / COST :

The common factor in the two techniques -PERT and CPM- is the "job" or the "activity". The project managers must schedule and coordinate the various jobs or activities so that the entire project is completed in time. A complicating factor in carrying out this task is the interdependence of the activities. For example, some activities depend upon the completion of other activities before they can be started. When we realize that projects can have as many as several thousand specific activities, we see why project managers look for procedures that could help them answer questions such as the following :

a) What is the expected project completion date ?
b) What is the scheduled start and completion date for each specific activity ?
c) Which activities are "critical" and must be completed exactly as scheduled in order to keep the project on schedule ?
d) How long can "noncritical" activities be delayed before they cause a delay in the total project ?

As we know, PERT / TIME and CPM can be used to help answering the above questions.

While project time and the meeting of a scheduled completion date are of primary considerations for almost every project, there are many situations in which the cost associated with the project is just as important as time. The technique referred to as PERT / Cost can be used to help plan, schedule, and control project costs. The ultimate objective of a PERT / Cost system is to provide information which can be used to maintain project costs within a specified budget. In this term, PERT / Cost technique offers different alternatives schedules (Time vis-a-vis Cost) for achieving a project.

The first step in a PERT / Cost control system is to break the entire project in to components that are convenient in terms of measuring and controlling costs. The related activities which are under the control of one department, subcontractor, etc.; are often grouped together to form what are referred to as "work packages". By identifying costs of each work package, a project manager can use a PERT / Cost system to help plan, schedule, and control project costs.

In this technique, three categories of cost could be identified for any given project :

1) <u>Direct costs</u> :

Associated with the commitment of resources (labor, materials, equipment, and so forth) to activities ;

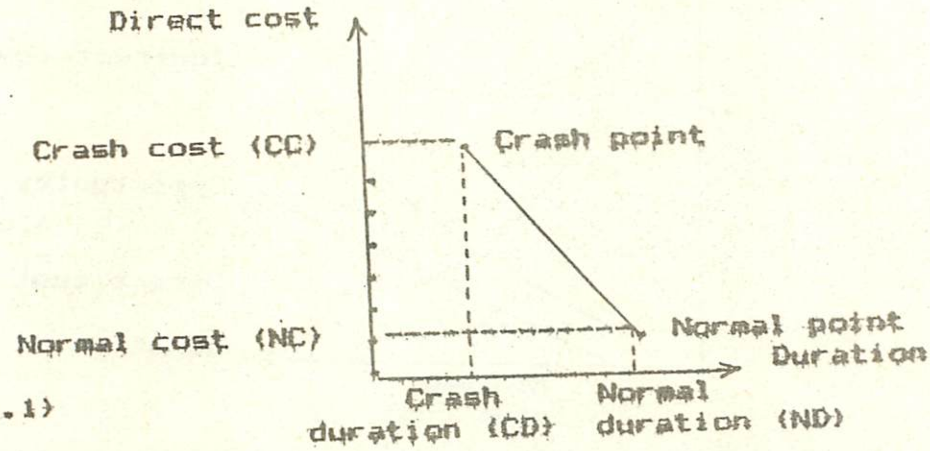
2) <u>Indirect or Overhead costs</u> :

Such as expenses associated with utilities, administration, and supervision; and

3) <u>Opportunity costs</u> :

Such as penalties for completing a project beyond a certain date or bonuses (benefits or negative costs) for completing a project prior to a specified date.

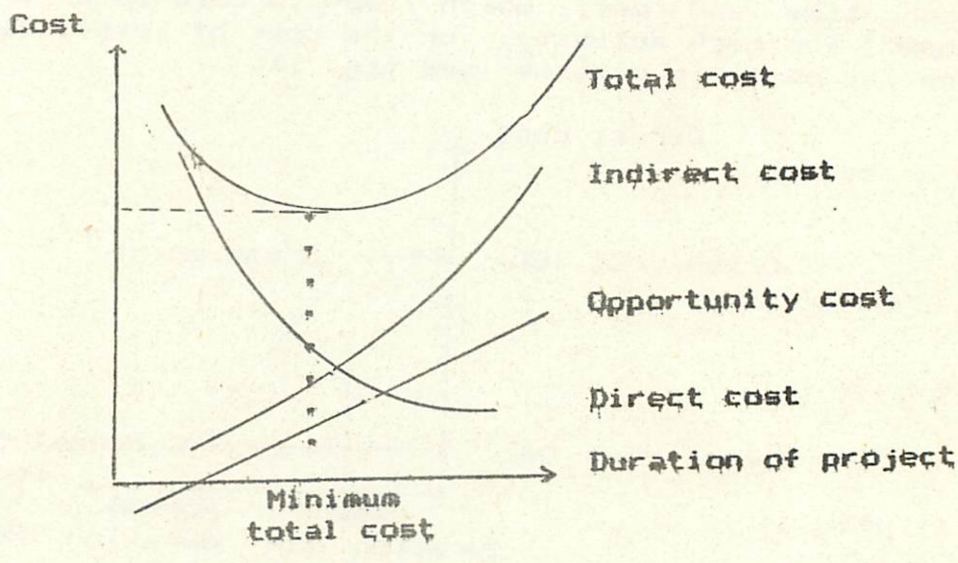
In this approach, for every job or activity there are four data items must be associated. However; its normal, crash time and cost, which leads to calculate the "Cost slope" for each activity, or the cost of expediting this activity per unit of time (see fig. 1).



(Fig.1)

$$\text{Cost slope} = \left| \frac{CC - NC}{CD - ND} \right|$$

Given the determination of a critical path based on normal times, the act of "crashing" involves the sequential time compression of critical activities from normal durations to crash durations. The end result of this effort is a set of schedules yielding direct cost as a discrete function of project duration. When this function is combined with the indirect and opportunity cost functions, a discrete version of the total cost function can be constructed;(Fig. 2).



(Fig. 2)

(I - 2) Kinds of floats :

Every activity in a project is defined or characterized by two event (point of time), the first determines its start and the second for its finish. At every event, there are two types of times to be calculated, **Earliest Event Time (EET)** and **Latest Event Time (LET)**. For critical activities, these types of times are equal.

Suppose an activity of starting event i, finishing event j and of **duration** D_{ij}. There are three kinds of floats can be calculated (if desired). Namely ;

a) **Total float**

Which is the maximum time by which the activity may be expanded **without affecting project completion.**

i.e ; Total float = $LET - EET_i - D_{ij}$

b) **Free float**

Which is the maximum time by which the activity may be expanded **without affecting any subsequent activity.**

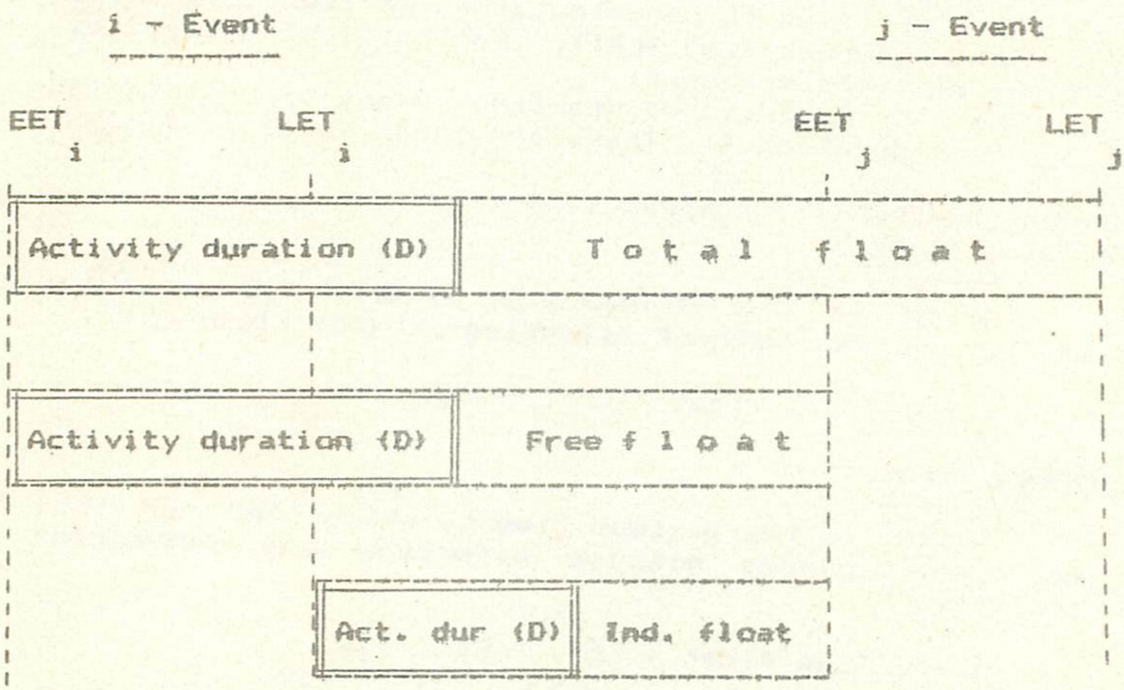
i.e ; Free float = $EET_j - EET_i - D_{ij}$

c) **Independant float**

Which is the maximum time by which the activity may be expanded **without affecting any other activity.**

i.e ; Independant float = $EET_j - LET_i - D_{ij}$

The above three kinds of floats could be represented graphically as follow (*) :



For any critical activity, the three kinds of float are equal to zero.

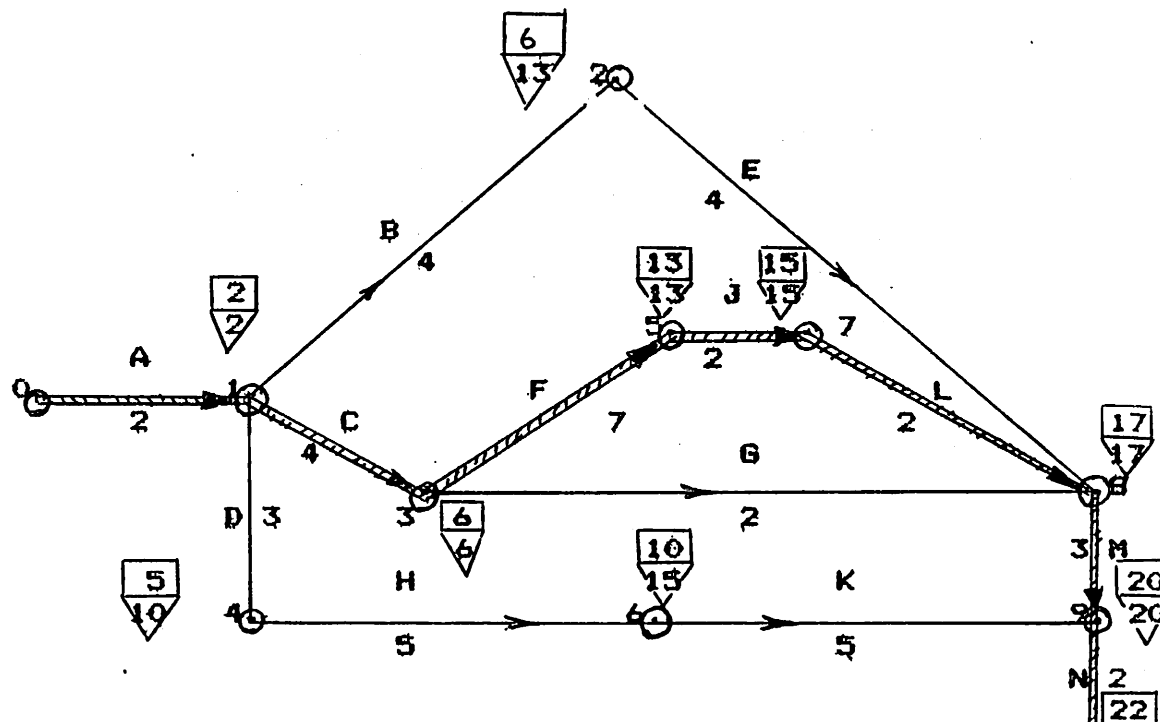
(*) Patrick O. Malone ,"The basics of critical path method", course notes CN. 821. September 1976 - International Bank for Reconstruction and Development.

(I - 3) Time Chart :

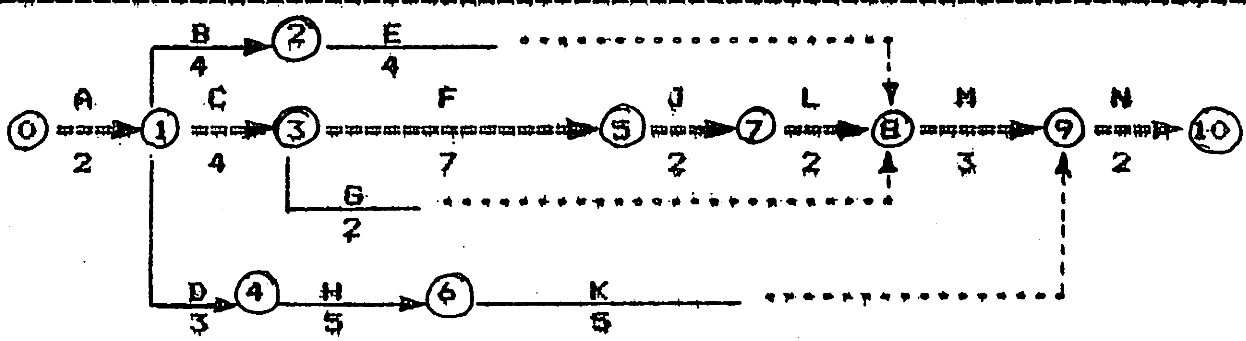
The time chart readily identifies scheduling flexibilities because of the visual case in identifying the total float associated with each non critical activity. The first step to design the time chart is the scheduling of critical activities. These activities are located as those having zero total float. Then they are plotted sequentially, the length of each line corresponds to the duration of the corresponding activity. The dummy activities need not be plotted, as they take up zero time. Each non critical activity is plotted separately. The beginning and end of each line correspond, respectively, to the earliest starting time (EET_i) and the latest finishing time (LET_j) for the given activity. The total float for each non critical activity is represented by a dotted line.

For example, consider the following network of a project (Fig. 3). After calculating the total completion time of the project, and identifying both critical and non critical activities, a horizontal line with a scale proportional to that time, is plotted.

This line (doubled line) is divided in to parts. Each part represent a critical activity. The non critical activities are represented by continuous thin lines. Their lengths were taken by the same scale of the critical path (Fig. 4).


A
2
B
4
C
4
D 3
E
4
F
7
G
2
H
5
J
2
K
5
L
2
3 M
N 2

(Fig. 3)


A
2
B
4
C
4
D
3
E
4
F
7
G
2
H
5
J
2
K
5
L
2
M
3
N
2

(Fig. 4)

The role and importance of time chart will be evident in calculating the total resources required at every period during the execution of the project. It is also essential in PART II, when we have to formulate the proposed model for leveling the resources.

(I - 4) Resource leveling and allocation :

Basic models assume unlimited resource availability, which is clearly unrealistic for many enviroments. Activities on parallel paths in a network, may compete for the same resources. Consequently, limited resources may force sequential scheduling of activities which are not related by precedence relationships.

There are two basic approaches to scheduling with limited resources : resource leveling and resource allocation. In resource leveling, peak resource requirements are smoothed to reduce maximum capacity needs - consistent with meeting the scheduled completion date.

In resource allocation the duration of the project is minimized (usually beyond the scheduled completion date provided by the basic model) subject to appropriate resource and sequential constraints.

The basic issue involves scheduling non critical path activities at times which minimize fluctuations in expected needs for the resource. The resources that are to be leveled or allocated will be of different types (for example, money, labor, machinery, and so forth). Furthermore, capacities in various work centers (each resource is a work center) may or may not be transferable to other work centers or activities.

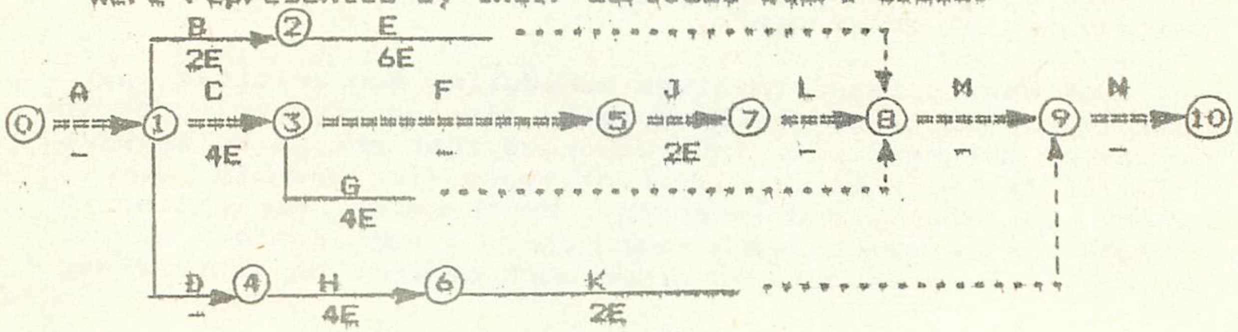
For example, a particular type of skilled labor may be appropriate only for certain activities.

As mentioned before, that leveling of resources depends on the float of the non critical activities of the project. The idea is to shift or to alleviate the resources from the points which have a high density of resources (peaks) to less density points via the non critical activities.

For example, suppose the number of engineers required at every unit of time in the previous project, like the following :

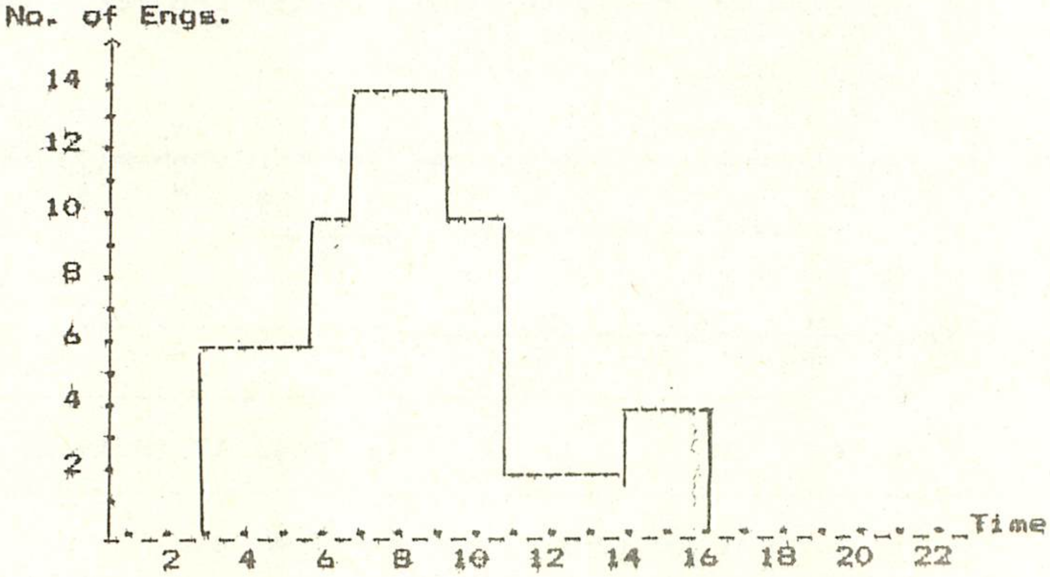
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
-	-	6	6	6	10	14	14	10	10	2	2	2	4	4	-	-	-	-	-	-	-

(Figure 5) represent the time - chart, and the activity requirements from the engineers as resources. The activities were represented by their earliest start times.



(Fig. 5)

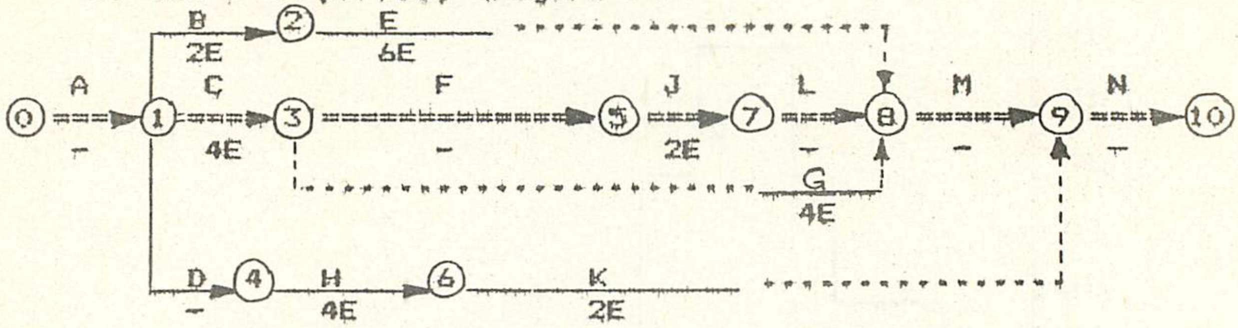
The total number of engineers required at each period in the project is represented as in (Figure 6).



(Fig. 6)

It is clear, that the maximum number of engineers is 14 at periods 7,8 only in the project; while there is no need for any engineers (resources)1,2,16 and over. The project managers, have to pay salaries for these 14 engineers allover the project duration. The responsability of the project manager now, is to change the execution period of some non critical activities in order to attain a minimum number of engineers that could be employed, during the same project duration. The following changes could be happened in order to redistribute the engineers (resources).

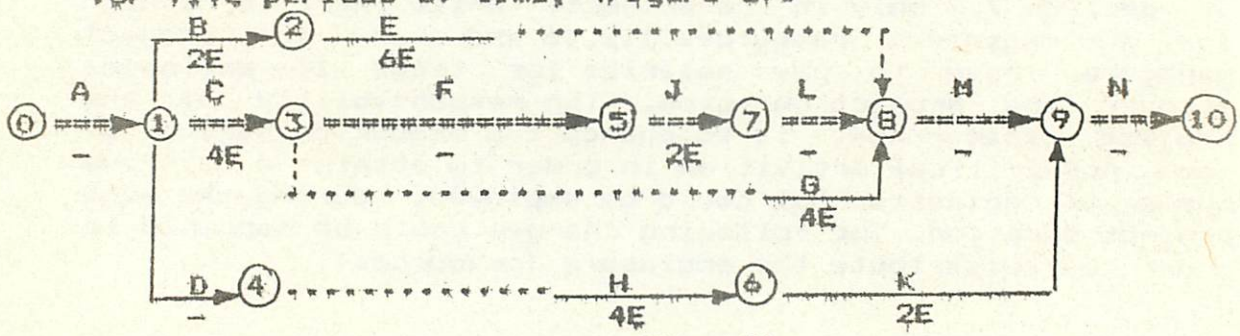
i- Postponing the execution of activity (3-8), to start at the begining of the 16-th period of the project, instead of the 7-th period; (Figure 7).



1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
–	–	6	6	6	10	10	10	10	10	2	2	2	4	4	4	4	–	–	–	–	–

(Fig. 7)

ii- Postponing the execution of activities (4-6),(6-9) for five periods of time; (Figure 8).



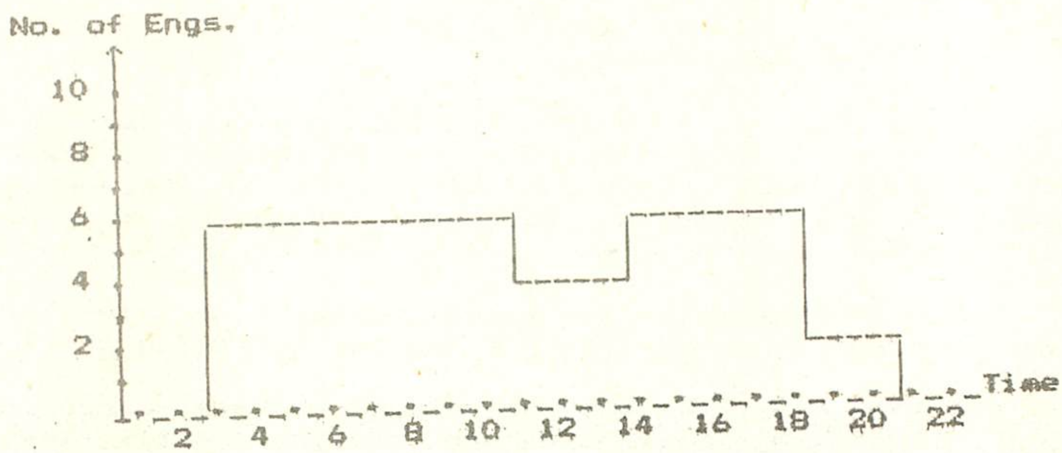
(Fig. 8)

The redistributed number of engineers will be as in (Fig. 9).

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
-	-	6	6	6	6	6	6	6	6	4	4	4	6	6	6	6	2	2	2	-	-

(Fig. 9)

The final results of these changes leads us to minimize the number of engineers from 14 to 6 only, which nearly works allover the project duraction; (Figure 10).



(Fig. 10)

PART - II

Introduction

As mentioned before, that till now, computationally feasible models for optimizing the distribution of resources do not exist. In the following, we will try to formulate the problem and to solve it using the linear programming technique. Before discussing the model, let us try to expose the problem and its constituents.

(II - 1) The problem:

Consider a firm with limited resources (manpower or machinery or tools, ..etc.), signed a contract to accomplish the execution of one or more project/s in a given period. How can this firm distribute their resources in order to satisfy the requirements of every activity in the project/s?

To advance towards the formulation of the problem mathematically, we have to prepare the following preliminary steps:

i- for every project, calculate its total duration, its critical path, its non critical activities, and their total floats.

ii- Prepare a global time - chart for the project/s, clarifying on it, the duration, the required resources and the total float for each non critical activity.

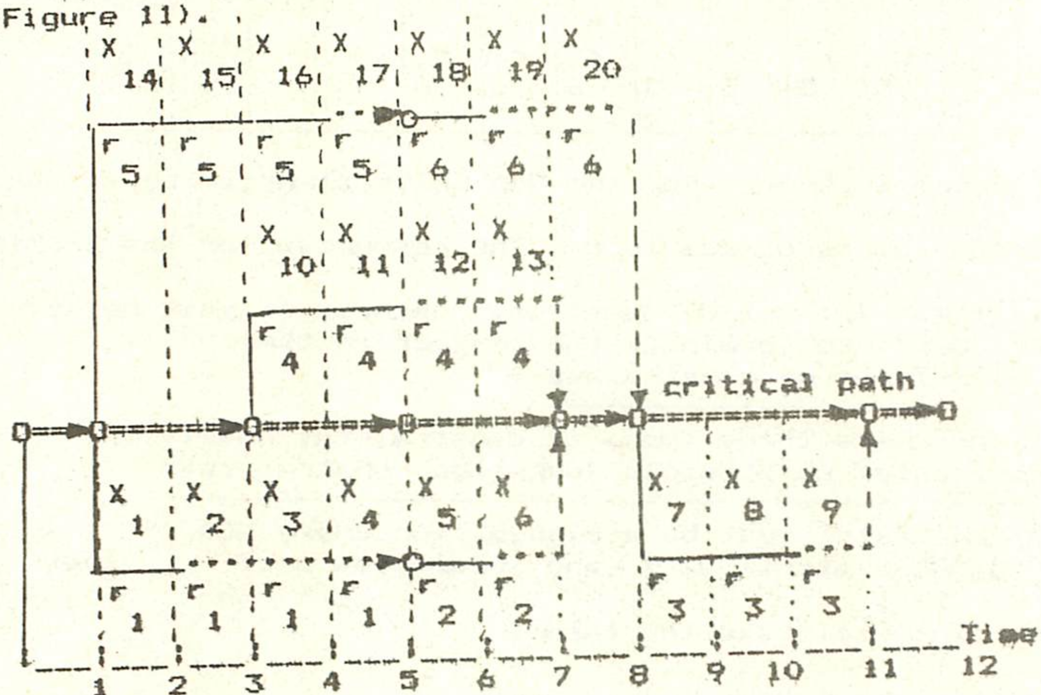
iii- For the non critical activities and their floats, define every unit of time by a variable X_i.

iv- Determine the available maximum number of resources assigned for the execution of the non critical activities R. This could be done by subtracting the maximum resources required for a critical activity from the total resources of the firm.

(II - 2) The mathematical models

For simplicity, we will consider only one project. The mathematical formulations which we are going to drive, can be applied for the case of more than one project.

Suppose a project, its time - chart is represented as in (Figure 11).



(Fig. 11)

The resources required at each period of the project, will be as in the following table :

	0–1	1–2	2–3	3–4	4–5	5–6	6–7	7–8	8–9	9–10	10–11	11–12
Resources Required		r_1 + r_5	r_1 + r_5	r_1 + r_4 + r_5	r_1 + r_4 + r_5	r_2 + r_4 + r_6	r_2 + r_4 + r_6	r_6	r_3	r_3	r_3	

Suppose that, for the Non Critical Activity i (NCA_i); the required resources is r_i . The resources for the critical activities are out of question, because it must be secured if we decide to terminate the project in time.

1 - Types of constraints :

There are three types of constraints; namely :

i) Horizontal constraints (duration constraints):

A constraint must be assigned, for every NCA. If the NCA_i starts at X_m and terminates at X_n , then its constraint will take the form :

$$\sum_{s=m}^{n} X_s = D_i \quad (D_i \text{ is the duration of } NCA_i)$$

In our representation, the horizontal constraints will be :

$$X_1 + X_2 + X_3 + X_4 = 1 \quad = \text{Duration of first NCA.}$$

$$X_5 + X_6 = 1 \quad = \text{Duration of second NCA.}$$

$$X_7 + X_8 + X_9 = 2$$

$$X_{10} + X_{11} + X_{12} + X_{13} = 2$$

$$X_{14} + X_{15} + X_{16} + X_{17} = 3$$

$$X_{18} + X_{19} + X_{20} = 1$$

ii) Vertical constraints (Resources constraints) :

Also, a constraint must be assigned for every unit of time whenever a NCA(s) exists. So;

$$r_i X_j \leqslant r_i \quad \text{(resources required at time i)}$$

$$r_1 X_1 + r_5 X_{14} \leqslant r_1 + r_5 \quad \text{(for 2-nd \& 3-rd periods)}$$

$$r_1 X_2 + r_5 X_{15} \leqslant r_1 + r_5 \quad \text{(for 4-th \& 5-th periods)}$$

$$r_1 X_3 + r_4 X_{10} + r_5 X_{16} \leqslant r_1 + r_4 + r_5$$

$$r_1 X_4 + r_4 X_{11} + r_5 X_{17} \leqslant r_1 + r_4 + r_5$$

$$r_2 X_5 + r_4 X_{12} + r_6 X_{18} \leqslant r_2 + r_4 + r_6$$

$$r_2 X_6 + r_4 X_{13} + r_6 X_{19} \leqslant r_2 + r_4 + r_6$$

$$r_6 X_{20} \leqslant r_6 \quad ; \quad r_3 X_7 \leqslant r_3$$

$$r_3 X_8 \leqslant r_3 \quad ; \quad r_3 X_9 \leqslant r_3$$

If R (step iv in II-1) is greater than or equal to each right-hand side of the above constraints, there will be no problem and the firm could easily accomplish the project/s in time. The problem may arises, if R is less than one or more of the right-hand side of the above constraints. This is our case, in which the model have to find a solution. The solution could be attained for some extent of R. It is evident that, there is no solution for the problem if R equals zero or below some value.

iii) <u>The non-negativity constraints :</u>

In this model, the value of X_i not only must be greater or equal to zero, but also must be less than or equal to one.

In fact X_i must be a boolean variable ;i.e. it must take either the value 0 or 1; and the problem have to be solved by applying one of the (0 - 1) integer programming techniques which is not available for the time being. So, we will consider that this type of constraints will take the form :

$$X_i \leqslant 1 \quad \forall i$$

2 - The objective function :

It is of a particular type. At its optimum value, it must take a value which is already known and could be calculated, This value is equal to :

$$\sum_{i=1}^{L} D_i r_i \qquad (1)$$

Where L is the number of NCAs, D_i is the duration of NCA_i, and r_i is its required resources.

From the horizontal constraints, each one represent a NCA. This means that X_i of a constraint is equal to the duration of the corresponding NCA. So, the objective function of the mathematical model will take the form :

$$f(X) = r_1 X_1 + r_1 X_2 + r_1 X_3 + r_1 X_4 + r_2 X_5 + r_2 X_6 +$$

$$r_3 X_7 + r_3 X_8 + r_3 X_9 + r_4 X_{10} + r_4 X_{11} + r_4 X_{12} +$$

$$r_4 X_{13} + r_5 X_{14} + r_5 X_{15} + r_5 X_{16} + r_5 X_{17} + r_6 X_{18} +$$

$$r_6 X_{19} + r_6 X_{20}$$

If the problem is solved to maximize f(X), the solution will be attained and stopped at a value for f(X) equal to that in (1). But if the problem was to minimize f(X), we will expect a smaller value for f(X) than in the case of maximization. This is due to the absence of one or more of the basic variables, which affects on the objective function. This case is expected and examined, since the 3-rd type of constraints permit to give the value of zero for the basic variables which still remain after escaping of the artificial variables.

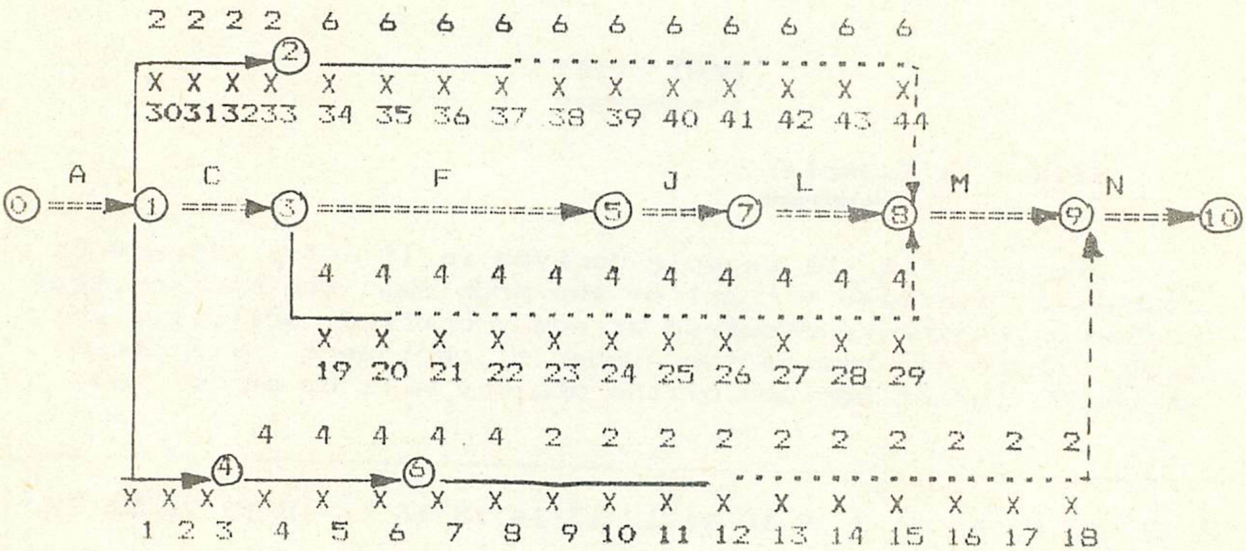
PART - III

(III - 1) Example:

Considering the example derived in (I - 3); (figure 5) represent the time - chart of the problem. Subtracting the number of engineers required at every critical activity; the table which represents the number of engineers (resources) at the different periods of the project will be as follow:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
-	-	2	2	2	6	14	14	14	14	12	12	12	12	12	12	12	2	2	2	-	-

Denoting every period, for the non critical activities in (Figure 5) by a variable X_i (Figure 12);



(Fig. 12)

The problem could be formulated as follows :

$$\text{Min } f(x) = 0X_1 + 0X_2 + 0X_3 + 4X_4 + 4X_5 + 4X_6 + 4X_7 + 4X_8 + 2X_9 + 2X_{10} + 2X_{11}$$

$$+2X_{12} + 2X_{13} + 2X_{14} + 2X_{15} + 2X_{16} + 2X_{17} + 2X_{18} + 4X_{19} + 4X_{20} + 4X_{21} + 4X_{22}$$

$$+4X_{23} + 4X_{24} + 4X_{25} + 4X_{26} + 4X_{27} + 4X_{28} + 4X_{29} + 2X_{30} + 2X_{31} + 2X_{32} + 2X_{33} + 6X_{34}$$

$$+6X_{35} + 6X_{36} + 6X_{37} + 6X_{38} + 6X_{39} + 6X_{40} + 6X_{41} + 6X_{42} + 6X_{43} + 6X_{44}$$

Such that :

$0X_{1} + 2X_{30} \leq 2$; $0X_{2} + 2X_{31} \leq 2$;

$0X_{3} + 2X_{32} \leq 2$; $4X_{4} + 2X_{33} \leq 6$;

$4X_{5} + 4X_{19} + 6X_{34} \leq 14$; $4X_{6} + 4X_{20} + 6X_{35} \leq 14$

$4X_{7} + 4X_{21} + 6X_{36} \leq 14$; $4X_{8} + 4X_{22} + 6X_{37} \leq 14$

$2X_{9} + 4X_{23} + 6X_{38} \leq 12$; $2X_{10} + 4X_{24} + 6X_{39} \leq 12$

$2X_{11} + 4X_{25} + 6X_{40} \leq 12$; $2X_{12} + 4X_{26} + 6X_{41} \leq 12$

$2X_{13} + 4X_{27} + 6X_{42} \leq 12$; $2X_{14} + 4X_{28} + 6X_{43} \leq 12$

$2X_{15} + 4X_{29} + 6X_{44} \leq 12$; $2X_{16} \leq 2$

$2X_{17} \leq 2$; $2X_{18} \leq 2$

$X_{1} + X_{2} + X_{3} = 3$; $X_{4} + X_{5} + X_{6} + X_{7} + X_{8} = 5$

$X_{9} + X_{10} + X_{11} + X_{12} + X_{13} + X_{14} + X_{15} + X_{16} + X_{17} + X_{18} = 5$

$X_{19} + X_{20} + X_{21} + X_{22} + X_{23} + X_{24} + X_{25} + X_{26} + X_{27} + X_{28} + X_{29} = 2$

$$X_{30} + X_{31} + X_{32} + X_{33} = 4$$

$$X_{34} + X_{35} + X_{36} + X_{37} + X_{38} + X_{39} + X_{40} + X_{41} + X_{42} + X_{43} + X_{44} = 4$$

$$X_i \leqslant 1 \quad \forall i$$

(III - 2) Results:

The problem was solved in 2 cases :

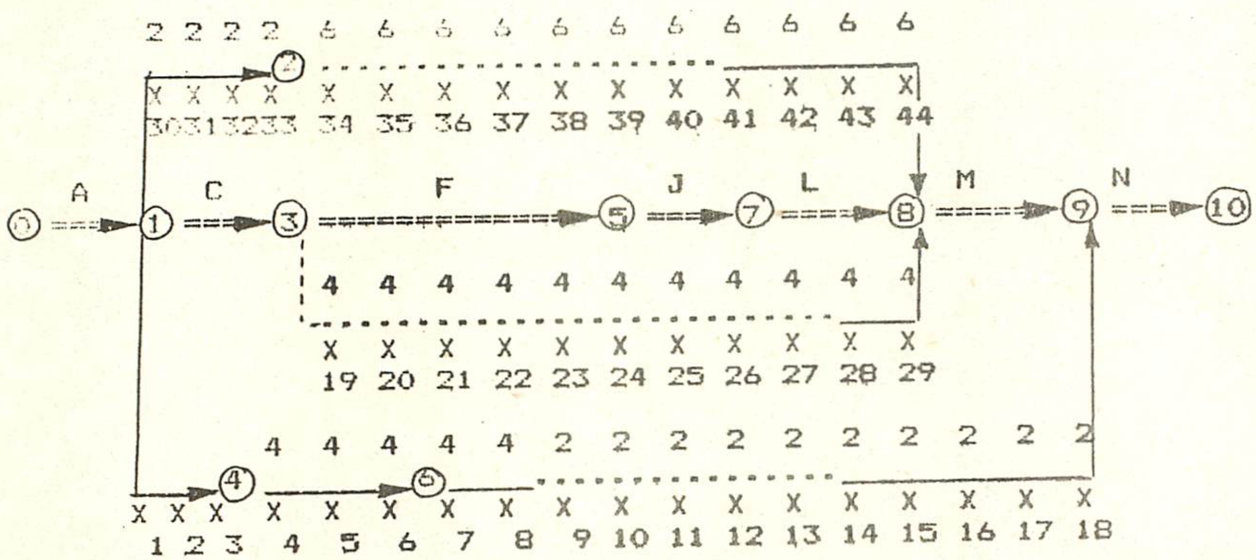
i - Normal case.
ii- Assuming that, the maximum number of resources available at any period is not greater than 6 .

RESULTS
for
NORMAL CASE

```
ITER  1        f(X)=  0

ITER  2        f(X)=  6
X 44= 1
ITER  3        f(X)=  12
X 43= 1   X 44= 1
ITER  4        f(X)=  18
X 42= 1   X 43= 1   X 44= 1
ITER  5        f(X)=  24
X 41= 1   X 42= 1   X 43= 1   X 44= 1
ITER  6        f(X)=  24
X 41= 1   X 42= 1   X 43= 1   X 44= 1   X 40= 0
ITER  7        f(X)=  26
X 33= 1   X 41= 1   X 42= 1   X 43= 1   X 44= 1   X 40= 0
ITER  8        f(X)=  28
X 32= 1   X 33= 1   X 41= 1   X 42= 1   X 43= 1   X 44= 1   X 40= 0
ITER  9        f(X)=  30
X 31= 1   X 32= 1   X 33= 1   X 41= 1   X 42= 1   X 43= 1   X 44= 1   X 40= 0
ITER  10       f(X)=  32
X 30= 1   X 31= 1   X 32= 1   X 33= 1   X 41= 1   X 42= 1   X 43= 1   X 44= 1
X 40= 0
ITER  11       f(X)=  36
X 30= 1   X 31= 1   X 32= 1   X 29= 1   X 33= 1   X 41= 1   X 42= 1   X 43= 1
X 44= 1   X 40= 0
ITER  12       f(X)=  40
X 30= 1   X 31= 1   X 32= 1   X 28= 1   X 29= 1   X 33= 1   X 41= 1   X 42= 1
X 43= 1   X 44= 1   X 40= 0
ITER  13       f(X)=  40
X 30= 1   X 31= 1   X 32= 1   X 28= 1   X 29= 1   X 33= 1   X 41= 1   X 42= 1
X 43= 1   X 44= 1   X 27= 0   X 40= 0
ITER  14       f(X)=  42
X 30= 1   X 31= 1   X 32= 1   X 18= 1   X 28= 1   X 29= 1   X 33= 1   X 41= 1
X 42= 1   X 43= 1   X 44= 1   X 27= 0   X 40= 0
ITER  15       f(X)=  44
X 30= 1   X 31= 1   X 32= 1   X 17= 1   X 18= 1   X 28= 1   X 29= 1   X 33= 1
X 41= 1   X 42= 1   X 43= 1   X 44= 1   X 27= 0   X 40= 0
ITER  16       f(X)=  46
X 30= 1   X 31= 1   X 32= 1   X 16= 1   X 17= 1   X 18= 1   X 28= 1   X 29= 1
X 33= 1   X 41= 1   X 42= 1   X 43= 1   X 44= 1   X 27= 0   X 40= 0
ITER  17       f(X)=  48
X 30= 1   X 31= 1   X 32= 1   X 15= 1   X 16= 1   X 17= 1   X 18= 1   X 28= 1
X 29= 1   X 33= 1   X 41= 1   X 42= 1   X 43= 1   X 44= 1   X 27= 0   X 40= 0
ITER  18       f(X)=  48
X 30= 1   X 31= 1   X 32= 1   X 15= 1   X 16= 1   X 17= 1   X 18= 1   X 28= 1
```

```
X 29= 1   X 33= 1   X 41= 1   X 42= 1   X 43= 1   X 44= 1   X 27= 0   X 40= 0
ITER  19      f(X)=  50
X 30= 1   X 31= 1   X 32= 1   X 14= 1   X 15= 1   X 16= 1   X 17= 1   X 18= 1
X 28= 1   X 29= 1   X 33= 1   X 41= 1   X 42= 1   X 43= 1   X 44= 1   X 27= 0
X 40= 0
ITER  20      f(X)=  50
X 30= 1   X 31= 1   X 32= 1   X 14= 1   X 15= 1   X 16= 1   X 17= 1   X 18= 1
X 28= 1   X 29= 1   X 33= 1   X 41= 1   X 42= 1   X 43= 1   X 44= 1   X 27= 0
X 40= 0
ITER  21      f(X)=  50
X 30= 1   X 31= 1   X 32= 1   X 14= 1   X 15= 1   X 16= 1   X 17= 1   X 18= 1
X 28= 1   X 29= 1   X 33= 1   X 41= 1   X 42= 1   X 43= 1   X 44= 1   X 13= 0
X 27= 0   X 40= 0
ITER  22      f(X)=  54
X 30= 1   X 31= 1   X 32= 1   X 14= 1   X 15= 1   X 16= 1   X 17= 1   X 18= 1
X 8= 1   X 28= 1   X 29= 1   X 33= 1   X 41= 1   X 42= 1   X 43= 1   X 44= 1  X
 13= 0   X 27= 0   X 40= 0
ITER  23      f(X)=  58
X 30= 1   X 31= 1   X 32= 1   X 14= 1   X 15= 1   X 16= 1   X 17= 1   X 18= 1
X 7= 1   X 8= 1   X 28= 1   X 29= 1   X 33= 1   X 41= 1   X 42= 1   X 43= 1   X
 44= 1   X 13= 0   X 27= 0   X 40= 0
ITER  24      f(X)=  62
X 30= 1   X 31= 1   X 32= 1   X 14= 1   X 15= 1   X 16= 1   X 17= 1   X 18= 1
X 6= 1   X 7= 1   X 8= 1   X 28= 1   X 29= 1   X 33= 1   X 41= 1   X 42= 1   X
 43= 1   X 44= 1   X 13= 0   X 27= 0   X 40= 0
ITER  25      f(X)=  66
X 30= 1   X 31= 1   X 32= 1   X 14= 1   X 15= 1   X 16= 1   X 17= 1   X 18= 1
X 5= 1   X 6= 1   X 7= 1   X 8= 1   X 28= 1   X 29= 1   X 33= 1   X 41= 1   X 42
= 1   X 43= 1   X 44= 1   X 13= 0   X 27= 0   X 40= 0
ITER  26      f(X)=  70
X 30= 1   X 31= 1   X 32= 1   X 4= 1   X 14= 1   X 15= 1   X 16= 1   X 17= 1  X
 18= 1   X 5= 1   X 6= 1   X 7= 1   X 8= 1   X 28= 1   X 29= 1   X 33= 1   X 41=
 1   X 42= 1   X 43= 1   X 44= 1   X 13= 0   X 27= 0   X 40= 0
ITER  27      f(X)=  70
X 30= 1   X 31= 1   X 32= 1   X 4= 1   X 14= 1   X 15= 1   X 16= 1   X 17= 1  X
 18= 1   X 3= 1   X 5= 1   X 6= 1   X 7= 1   X 8= 1   X 28= 1   X 29= 1   X 33=
 1   X 41= 1   X 42= 1   X 43= 1   X 44= 1   X 13= 0   X 27= 0   X 40= 0
ITER  28      f(X)=  70
X 30= 1   X 31= 1   X 32= 1   X 4= 1   X 14= 1   X 15= 1   X 16= 1   X 17= 1  X
 18= 1   X 2= 1   X 3= 1   X 5= 1   X 6= 1   X 7= 1   X 8= 1   X 28= 1   X 29=
 1   X 33= 1   X 41= 1   X 42= 1   X 43= 1   X 44= 1   X 13= 0   X 27= 0   X 40=
 0
ITER  29      f(X)=  70
X 30= 1   X 31= 1   X 32= 1   X 4= 1   X 14= 1   X 15= 1   X 16= 1   X 17= 1  X
 18= 1   X 1= 1   X 2= 1   X 3= 1   X 5= 1   X 6= 1   X 7= 1   X 8= 1   X 28= 1
  X 29= 1   X 33= 1   X 41= 1   X 42= 1   X 43= 1   X 44= 1   X 13= 0   X 27= 0
  X 40= 0
```



2 2 2 2 6 6 6 6 6 6 6 6 6 6 6
X X X X X X X X X X X X X X X
30 31 32 33 34 35 36 37 38 39 40 41 42 43 44
A
C
F
J
L
M
N
0
1
2
3
4
5
6
7
8
9
10
4 4 4 4 4 4 4 4 4 4 4
X X X X X X X X X X X
19 20 21 22 23 24 25 26 27 28 29
4 4 4 4 4 2 2 2 2 2 2 2 2 2 2
X X X X X X X X X X X X X X X X X X
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18

RESULTS
for
MAX. RESOURCES = 6

ITER 1 f(X)= 0

ITER 2 f(X)= 6
X 44= 1

ITER 3 f(X)= 12
X 43= 1 X 44= 1

ITER 4 f(X)= 18
X 42= 1 X 43= 1 X 44= 1

ITER 5 f(X)= 24
X 41= 1 X 42= 1 X 43= 1 X 44= 1

ITER 6 f(X)= 24
X 41= 1 X 42= 1 X 43= 1 X 44= 1 X 40= 0

ITER 7 f(X)= 26
X 41= 1 X 42= 1 X 43= 1 X 44= 1 X 33= 1 X 40= 0

ITER 8 f(X)= 28
X 32= 1 X 41= 1 X 42= 1 X 43= 1 X 44= 1 X 33= 1 X 40= 0

ITER 9 f(X)= 30
X 31= 1 X 32= 1 X 41= 1 X 42= 1 X 43= 1 X 44= 1 X 33= 1 X 40= 0

ITER 10 f(X)= 32
X 30= 1 X 31= 1 X 32= 1 X 41= 1 X 42= 1 X 43= 1 X 44= 1 X 33= 1
X 40= 0

ITER 11 f(X)= 36
X 30= 1 X 31= 1 X 32= 1 X 41= 1 X 42= 1 X 43= 1 X 44= .3333333 X
29= 1 X 33= 1 X 40= .6666667

ITER 12 f(X)= 38
X 30= 1 X 31= 1 X 32= 1 X 41= 1 X 42= 1 X 43= .6666667 X 44=
.3333333 X 29= 1 X 33= 1 X 28= .5 X 40= 1

ITER 13 f(X)= 40
X 30= 1 X 31= 1 X 32= 1 X 41= 1 X 42= 1 X 43= .3333334 X 44=
.3333333 X 39= .3333334 X 29= 1 X 33= 1 X 28= 1 X 40= 1

ITER 14 f(X)= 40
X 30= 1 X 31= 1 X 32= 1 X 41= 1 X 42= 1 X 43= .3333334 X 44=
.3333333 X 39= .3333334 X 29= 1 X 33= 1 X 28= 1 X 27= 0 X 40= 1

ITER 15 f(X)= 42
X 30= 1 X 31= 1 X 32= 1 X 41= 1 X 42= 1 X 43= .3333334 X 44=
.3333333 X 18= 1 X 39= .3333334 X 29= 1 X 33= 1 X 28= 1 X 27= 0 X
40= 1

ITER 16 f(X)= 44
X 30= 1 X 31= 1 X 32= 1 X 41= 1 X 42= 1 X 43= .3333334 X 44=
.3333333 X 17= 1 X 18= 1 X 39= .3333334 X 29= 1 X 33= 1 X 28= 1 X
27= 0 X 40= 1

ITER 17 f(X)= 46
X 30= 1 X 31= 1 X 32= 1 X 41= 1 X 42= 1 X 43= .3333334 X 44=
.3333333 X 16= 1 X 17= 1 X 18= 1 X 39= .3333334 X 29= 1 X 33= 1 X
28= 1 X 27= 0 X 40= 1

ITER 18 f(X)= 48
X 30= 1 X 31= 1 X 32= 1 X 41= 1 X 42= 1 X 43= .3333334 X 15=
.9999999 X 16= 1 X 17= 1 X 18= 1 X 39= .6666666 X 29= 1 X 33= 1 X
28= 1 X 27= 0 X 40= 1

ITER 19 f(X)= 48
X 30= 1 X 31= 1 X 32= 1 X 41= 1 X 42= 1 X 43= .3333334 X 15= 1 X
16= 1 X 17= 1 X 18= 1 X 39= .6666666 X 29= 1 X 33= 1 X 28= 1 X 27
= 2.980232E-08 X 40= 1

ITER 20 f(X)= 50
X 30= 1 X 31= 1 X 32= 1 X 41= 1 X 42= 1 X 43= 5.960465E-08 X 15= 1
X 16= 1 X 17= 1 X 18= 1 X 39= .9999999 X 29= 1 X 33= 1 X 14=
.9999998 X 28= 1 X 27= 2.980232E-08 X 40= 1

ITER 21 f(X)= 50
X 30= 1 X 31= 1 X 32= 1 X 41= 1 X 42= 1 X 43= 1.986821E-08 X 15= 1
X 16= 1 X 17= 1 X 18= 1 X 39= .9999999 X 29= 1 X 33= 1 X 14=
.9999999 X 28= 1 X 38= 3.973643E-08 X 27= 2.980232E-08 X 40= 1

ITER 22 f(X)= 54
X 30= 1 X 31= 1 X 32= 1 X 41= 1 X 42= 1 X 43= 1.986821E-08 X 15= 1
X 16= 1 X 17= 1 X 18= 1 X 8= 1 X 39= .9999999 X 29= 1 X 33= 1 X
14= .9999999 X 28= 1 X 38= 3.973643E-08 X 27= 2.980232E-08 X 40= 1

ITER 23 f(X)= 58
X 30= 1 X 31= 1 X 32= 1 X 41= 1 X 42= 1 X 43= 1.986821E-08 X 15= 1
X 16= 1 X 17= 1 X 18= 1 X 7= 1 X 8= 1 X 39= .9999999 X 29= 1 X
33= 1 X 14= .9999999 X 28= 1 X 38= 3.973643E-08 X 27= 2.980232E-08 X
40= 1

ITER 24 f(X)= 62
X 30= 1 X 31= 1 X 32= 1 X 41= 1 X 42= 1 X 43= 1.986821E-08 X 15= 1
X 16= 1 X 17= 1 X 18= 1 X 6= 1 X 7= 1 X 8= 1 X 39= .9999999 X 29
= 1 X 33= 1 X 14= .9999999 X 28= 1 X 38= 3.973643E-08 X 27=
2.980232E-08 X 40= 1

ITER 25 f(X)= 66
X 30= 1 X 31= 1 X 32= 1 X 41= 1 X 42= 1 X 43= 1.986821E-08 X 15= 1
X 16= 1 X 17= 1 X 18= 1 X 5= 1 X 6= 1 X 7= 1 X 8= 1 X 39=
.9999999 X 29= 1 X 33= 1 X 14= .9999999 X 28= 1 X 38= 3.973643E-08
X 27= 2.980232E-08 X 40= 1

ITER 26 f(X)= 70
X 30= 1 X 31= 1 X 32= 1 X 4= 1 X 41= 1 X 42= 1 X 43= 1.986821E-08
X 15= 1 X 16= 1 X 17= 1 X 18= 1 X 5= 1 X 6= 1 X 7= 1 X 8= 1 X 39
= .9999999 X 29= 1 X 33= 1 X 14= .9999999 X 28= 1 X 38= 3.973643E-08
X 27= 2.980232E-08 X 40= 1

ITER 27 f(X)= 70
X 30= 1 X 31= 1 X 32= 1 X 4= 1 X 41= 1 X 42= 1 X 43= 1.986821E-08
X 15= 1 X 16= 1 X 17= 1 X 18= 1 X 3= 1 X 5= 1 X 6= 1 X 7= 1 X 8=
1 X 39= .9999999 X 29= 1 X 33= 1 X 14= .9999999 X 28= 1 X 38=
3.973643E-08 X 27= 2.980232E-08 X 40= 1

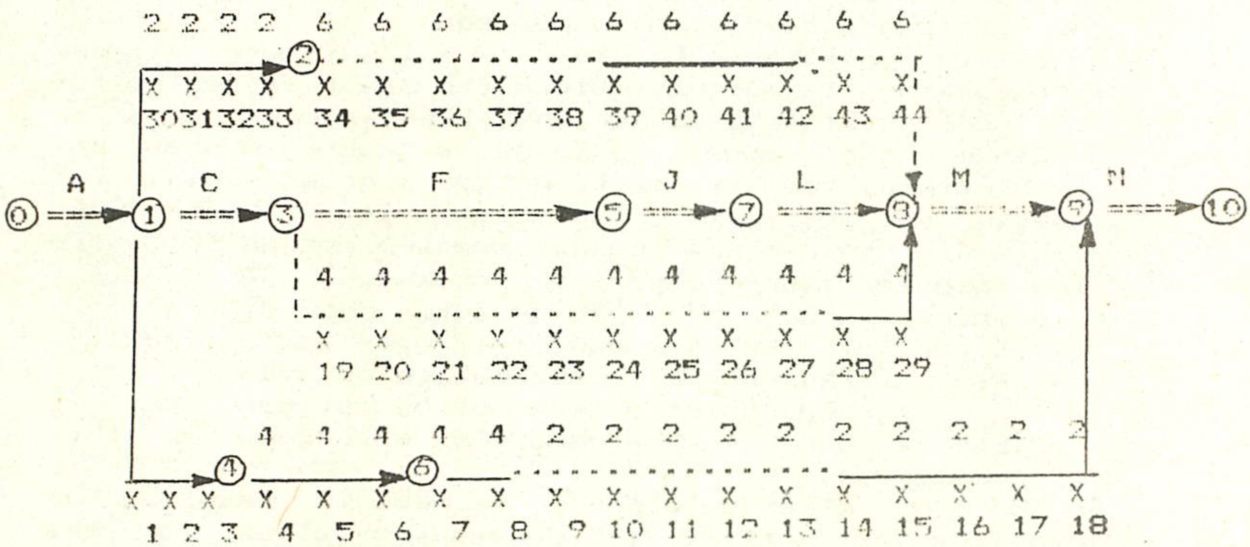
ITER 28 f(X)= 70
X 30= 1 X 31= 1 X 32= 1 X 4= 1 X 41= 1 X 42= 1 X 43= 1.986821E-08
X 15= 1 X 16= 1 X 17= 1 X 18= 1 X 2= 1 X 3= 1 X 5= 1 X 6= 1 X 7=
1 X 8= 1 X 39= .9999999 X 29= 1 X 33= 1 X 14= .9999999 X 28= 1 X
38= 3.973643E-08 X 27= 2.980232E-08 X 40= 1

ITER 29 f(X)= 70
X 30= 1 X 31= 1 X 32= 1 X 4= 1 X 41= 1 X 42= 1 X 43= 1.986821E-08
X 15= 1 X 16= 1 X 17= 1 X 18= 1 X 1= 1 X 2= 1 X 3= 1 X 5= 1 X 6=
1 X 7= 1 X 8= 1 X 39= .9999999 X 29= 1 X 33= 1 X 14= .9999999 X
28= 1 X 38= 3.973643E-08 X 27= 2.980232E-08 X 40= 1

ITER 30 f(X)= 70
X 30= 1 X 31= 1 X 32= 1 X 4= 1 X 25= 2.980232E-08 X 41= 1 X 42= 1
X 43= 1.986821E-08 X 15= 1 X 16= 1 X 17= 1 X 18= 1 X 1= 1 X 2= 1 X
3= 1 X 5= 1 X 6= 1 X 7= 1 X 8= 1 X 39= .9999999 X 29= 1 X 33= 1
X 14= .9999999 X 28= 1 X 38= 1.986822E-08 X 27= 0 X 40= 1

ITER 31 f(X)= 70
X 30= 1 X 31= 1 X 32= 1 X 4= 1 X 25= 2.980232E-08 X 41= 1 X 42= 1
X 43= 1.986821E-08 X 15= 1 X 16= 1 X 17= 1 X 18= 1 X 1= 1 X 2= 1 X
3= 1 X 5= 1 X 6= 1 X 7= 1 X 8= 1 X 39= .9999999 X 29= 1 X 33= 1
X 14= .9999999 X 28= 1 X 24= 0 X 38= 1.986822E-08 X 27= 0 X 40= 1

ITER 32 f(X)= 70
X 30= 1 X 31= 1 X 32= 1 X 4= 1 X 25= 2.980232E-08 X 41= 1 X 42= 1
X 43= 1.986821E-08 X 15= 1 X 16= 1 X 17= 1 X 18= 1 X 1= 1 X 2= 1 X
3= 1 X 5= 1 X 6= 1 X 7= 1 X 8= 1 X 39= .9999999 X 29= 1 X 33= 1
X 14= .9999999 X 28= 1 X 24= 2.980232E-08 X 44= 1.986822E-08 X 27= 0
X 40= 1


2 2 2 2 6 6 6 6 6 6 6 6 6 6 6
X X X X X X X X X X X X X X X
30 31 32 33 34 35 36 37 38 39 40 41 42 43 44
A
C
F
J
L
M
N
0
1
2
3
4
5
6
7
8
9
10
4 4 4 4 4 4 4 4 4 4 4
X X X X X X X X X X X
19 20 21 22 23 24 25 26 27 28 29
4 4 4 4 4 2 2 2 2 2 2 2 2 2 2
X X X X X X X X X X X X X X X X X X
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18

CONCLUSION

This model is a try to solve this type of problems. There are some points which must be taken in consideration, such as :

1- As mentioned before (in the introduction), there is no mathematical technique to assume an optimum solution for the problem, which require the use of heuristic and combinatoric methods.

2- It is preferable to use a (0 - 1) integer linear programming technique, since the basic variables of the model will take either the value zero or one.

3- Since the model contains a huge number of constraints, so it must include a great number of slack and artificial variables. It will be more desirable to use the compact simplex method to avoid the computer memory capacity problems.

4- A solution could be exist or not, depending on the resources of the firm and if it were sufficient to satisfy all the vertical constraints or not.

5- The obtained solution may not be optimum, but it represents one of the possible solutions in this circumstances.

6- To find other solutions, we have to resolve the model several times with descending values for the amount of resources, or by using computer packages contains an upper and lower bounds for its constraints.

REFERENCES

1-ALFRED V.AHO; JOHN E HOPCROFT; JEFFREY D. ULLMAN [1983]
"DATA STRUCTURES AND ALGORITHMS", ADDISON-WESLEY.

2-BAASE. [1978]
"COMPUTER ALGORITHMS Introduction to Design& Analysis"
ADDISON-WESLEY.

3-BARBARA BUND JACKSON [1973]
"COMPUTER MODELS IN MANAGEMENT", Richard D. Irwin, Inc.

4-BESTOUGEFF H.; GUILPIN C.; JACQUES M. [1975]
"LA TECHNIQUE INFORMATIQUE - Tome 2", MASSON.

5-BUDNICK; MOJENA; VOLLMANN. [1977]
"Principles of Operations Research for Management",
Richard D. Irwin, Inc.

6-DAELLENBACH; GEORGE; McNICKLE; 2-nd Edition; [1983]
"INTRODUCTION TO OPERATIONS RESEARCH TECHNIQUES",
ALLYN AND BACON.

7-DAVID G. LUENBERGER; 2-nd Edition; [1984]
"Linear and Nonlinear Programming", ADDISON-WESLEY.

8-DAVID R. ANDERSON; DENNIS J. SWEENEY; THOMAS A. WILLIAMS
3-rd Edition; [1982]
"An Introduction to MANAGEMENT SCIENCE", WEST.

9-EVANS; ANDERSON; SWEENEY; and WILLIAMS [1984]
"Applied Production and Operations Management". WEST.

10-MORANGE, P. [1969]
"Project informatique et methode Pert", DUNOD economie.

11-PANTUMSINCHAI; HASSAN; GUPTA. [1983]
"Basic Programs FOR Production AND Operations Management", PRENTICE-HALL, INC.

12-PATRICK D. MALONE [1976]
"The basics of critical path method", Course notes CN. 821. International Bank for Reconstruction and Development.

13-SRINATH L.S., [1973]
"PERT and CPM - Principles and Applications", EAST - WEST Press.

14-YEHIA YOUSSEF [1981]
"Pert technique in Petroleum sector - 2 parts "
Internal memo. (in arabic) I.N.P.

<<<....>>>

نموذج لتوزيع موارد محدودة على مجموعة مشروعات

إحدى المشكلات الأساسية التي تواجه كثيرًا من المنشآت الإنتاجيــــة أو الإنشائية ، هو كيفية التنسيق في توزيع مواردها المتاحة على مجموعــــة المشروعات التي تعاقدت على تنفيذها في فترة ما .

فنتيجة لسوء توزيع الموارد ، سنجد أن تلك المنشأة تكون عاجزة على إنهاء مشاريعها في أوقاتها المحددة مما قد يستلزم دفع غرامات ومما قـــد يفقد الثقة في تلك المنشأة ، كما أن تكاليف التنفيذ الكلية لتلك المشروعات ستزداد .

ولصياغة هذا النموذج وتطبيقه ، فلا بد أولاً من حل المشكلة بإستخدام أسلوب المسار الحرج وذلك لمعرفة الأنشطة الحرجة وغير الحرجة فـــي هذا المشروع أو تلك المشروعات ، ثم إستخدام الفائض المتاح في بعـــض الأنشطة الغير حرجة لتغيير موعد تنفيذها بهدف تنفيذ المشروع أو المشروعات في أوقاتها المحددة لها مستخدمين في ذلك الموارد المتاحة .

والجزء الأول من هذا البحث يتعلق بعرض أهم نقاط المسار الحـــرج وخاصة مايطلق عليه إسم توزيع الموارد وتسويتها وفي الجزء الثاني سيتـــم عرض المشكلة ومفرداتها والتغيرات المستخدمة فيها والتي يهـــــدف النموذج إلى الحصول على قيمها المثلى . وفي الجزء الثالث والأخير سنستعرض النتائج التي أمكن الحصول عليها بعد تنفيذ النموذج على الحاسب الآلـــي على أحد الأمثلة الصغيرة . وقد تم تطبيق برنامج الحاسب على حالتين .

- موارد عادية حسب القيم المعطاه في المثال .
- افتراض أن الموارد محددة بقيمة ثابتة أقل من الحالة الأولى .

مطبعة معهد التخطيط القومي